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ANALYTICAL ANNEX FOR "A META-ANALYSIS OF NPL'S IMPACT CASE STUDIES; CHARTING NPL'S ECONOMIC AND SOCIETAL BENEFIT MECHANISMS"

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Analytical Annex for “A Meta-Analysis of NPL’s Impact Case Studies; Charting NPL’s Economic and Societal Benefit Mechanisms”

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ABSTRACT

This document details a range of statistics and statistical tools used to analyse the Case Study Database. There is a set of summary statistics for each of the Sectors from 2015-2018, detailing annualised income, papers, and staffing levels, along with the number of case studies produce by each sector between 2000-2020. The 2nd section of the annex details how the analysis was conducted for each line of the table in section 6 for the sectors. It is with the scores produced here that each sector is positioned on the Technological Lifecycle and Triangle model. This process was mirrored for the Activities detailed in section 7, with no differences between the process of analysis.

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1 SUMMARY STATISTICS CONCERNING THE SECTORS

Sector	Annualised Staff	Annualised Papers	Annualised Non-NMS Income	Annualised NMS income	Total number of Case studies
Advanced Manufacturing	246	132	£9.5 million	£16.4 million	133
Life Sciences & Health	182	117	£8.8 million	£14.9 million	87
Energy & Environment	139	79	£8.7 million	£14.6 million	89
Digital & Quantum Technologies	190	125	£8.6 million	£13.3 million	48

Sector	Papers per staff member	Non-NMS Income per staff member	NMS income per staff member	Case studies by staff member
Advanced Manufacturing	0.54	£39k	£67k	0.54
Life Sciences & Health	0.97	£49k	£82k	0.48
Energy & Environment	0.57	£62k	£71k	0.64
Digital & Quantum Technologies	0.66	£45k	£70k	0.25

2 EXTRA INFORMATION CONCERNING THE ANALYSIS

2.1 HOW WE SCORED THE SECTORS FOR EACH QUESTION WITHIN THE TABLE

Underlying this paper was analysis conducted which looked at a range of different parameter vis-à-vis the case study database. Examples of these lie within the category column within the table in section 1.3. The following will explain what the categories mean, and how the sectors were ranked.

2.1.1 Swann Mechanisms

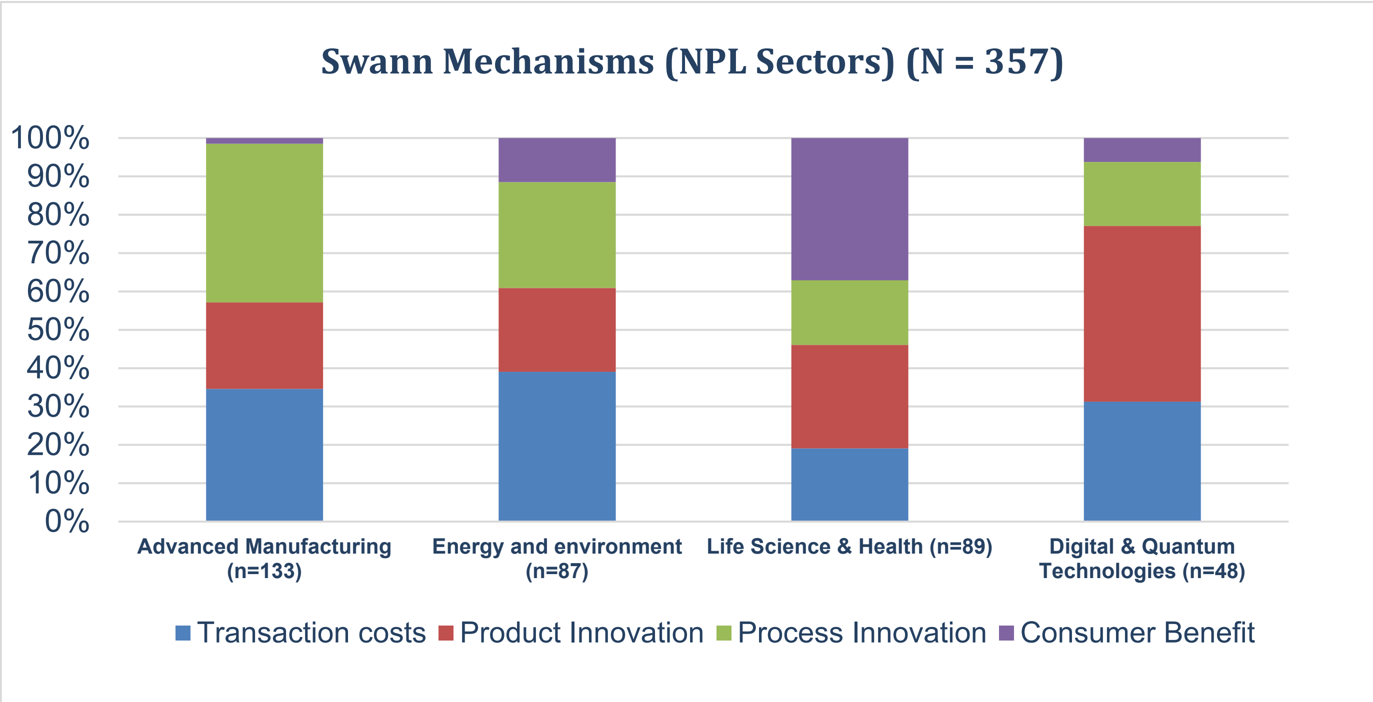
For Swann Mechanisms, each case study was given an indicator for the impact mechanism through which the work detailed provided impact to the economy. In total there were 4 impact mechanisms:

- ***Transaction costs*** includes case studies where NPL's product or service has reduced the cost associated with trading a good or service that our beneficiaries would incur due to trade
- **Product innovation** includes case studies where NPL has enabled/assisted the beneficiary to create a new product or NPL has created a new product.
- ***Process innovation*** entails case studies where we have influenced how the product /service of the beneficiary is made/delivered.
- ***Consumer benefit*** case studies concerned with: improving the Quality of life of the consumer; implementing Health & Safety standards in the workplace; civic impact (impact to the citizens of the state).
-

The results were as follows:

	Transaction costs	Product Innovation	Process Innovation	Consumer Benefit
Advanced Manufacturing	46	30	55	2
Energy & Environment	34	19	24	10
Life Sciences & Health	17	24	15	33
Digital & Quantum Technologies	15	22	8	3
Total	**112**	**95**	**102**	**48**

By tabulating these results, we obtained the following graph:



Only three of the four impact mechanisms were used to provide underlying information for the three themes of impact. Process Innovation relates closely to Direct Impact in NPL's context, as this often entails Measurement Services and Consultancy work, where NPL is helping customers with their metrological needs. Transaction Costs and Consumer Benefit relate to Indirect Benefits for different reasons. For Transaction Costs, this would be where NPL is helping to reduce uncertainties in measurement, closely linked to the concept of Fan-out, where traceable measurements can be disseminated down the calibration chain. Consumer Benefit entails work where NPL has looked to improve the lives of consumers, either through health benefits directly in the form of working with the NHS or by providing aid to Health & Safety initiatives. This was done by asking "questions" of the sectors and see where they scored with respect to the NPL average, assessing whether the sector displays higher than average (being the NPL average overall) levels of case studies for the respective sector. The first stage of analysis can be done by generating a quotient[1] for the sectors for

1 Quotients (or Location quotients) a way of quantifying how concentrated a particular category is in a sector as compared to NPL as a whole. It can reveal what makes a particular Sector "unique" in comparison to the NPL average. An example of how it is calculated is as follows (For Advanced Manufacturing's Transaction costs case studies:

$$1.10 = \frac{\dfrac{\#Transaction\ Costs\ Case\ Studies\ for\ Adv, Manuf}{\#Case\ Studies\ for\ Adv, Manuf}}{\dfrac{\#Transaction\ Costs\ Case\ Studies}{\#Case\ Studies}} = \frac{46/133}{112/357}$$

each mechanism. This provides us with a way of see how concentrated a mechanism is within each sector. Calculating the quotients produced the following result:

Quotient	Transaction costs	Process Innovation	Consumer Benefit
Advanced Manufacturing	1.10	1.45	0.11
Energy & Environment	1.25	0.97	0.85
Life Sciences & Health	0.61	0.59	2.76
Digital & Quantum Technologies	1.00	0.58	0.46

By assessing the quotients, we can see clear and obvious differences between the sectors. When assessing for Transaction Costs, Life Sciences and Health are particularly weak, while the remaining sectors range from following the NPL average to potentially higher than average. There is also an indication that that the Advanced Manufacturing is particularly strong in Process Innovation, while Life Sciences & Health and Digital & Quantum are particularly weak in Process Innovation. Finally, all the sectors seem particularly weak on Consumer Benefit, apart from Life Sciences and Health, which appears to be much stronger at this mechanism.

Following on from assessing the quotients, we can use a chi square test of independence[2] to further prove an association between the sectors and the impact mechanisms. This is calculated by comparing the observed case studies to the expected (calculated by multiplying the total for each sector by the total for each indicator, divided by the total number of case studies). This produced the following:

Observed	Transaction costs	Product Innovation	Process Innovation	Consumer Benefit
Advanced Manufacturing	46	30	55	2
Energy & Environment	34	19	24	10
Life Sciences & Health	17	24	15	33
Digital & Quantum Technologies	15	22	8	3

Expected	Transaction costs	Product Innovation	Process Innovation	Consumer Benefit
Advanced Manufacturing	41.7	35.4	38.0	17.9
Energy & Environment	27.3	23.2	24.9	11.7
Life Sciences & Health	27.9	23.7	25.4	12.0
Digital & Quantum Technologies	15.1	12.8	13.7	6.5

The chi square values are computed by the following:

$$\chi^2 = \sum \frac{(Observed\ Value\ -\ Expected\ Value)^2}{Expected\ Value}$$

After running the test, the chi square value for the variables in question was found to be 82.1 (3 S.F), which was greater than the critical value and produced a p-value of 0.00. This allowed for the null hypothesis (that there is no relationship between Sectors and Impact mechanisms) to be rejected. Once this was done, t-tests could be used to assess which sectors were significantly higher or lower than the NPL average for specific mechanisms.

2 The Chi-Square Test for Association is **used to determine if there is any association between two variables**. It is really a hypothesis test of independence. The null hypothesis is that the two variables are not associated, i.e., independent. The alternate hypothesis is that the two variables are associated.

For the t-tests, we used two different distributions to confirm significance can be assessed. We used the Poisson[3] and the Binomial[4] Distributions, with the tables shown producing the p values from the tests. Furthermore, we used a colour scheme to indicate if a sector is statistically significantly higher or lower than the NPL average at the 95%, 90% or not significant, either in a positive or negative way as follows:

95% +ve
90% +ve
70% +ve (1 S.D.)
No Significance
70% +ve (1 S.D.)
90% +ve
95% -ve

P value - Poisson	Transaction costs	Process Innovation	Consumer Benefit
Advanced Manufacturing	0.51	0.01	0.00
Energy & Environment	***<u>0.20</u>***	0.86	0.62
Life Sciences & Health	0.04	0.04	0.00
Digital & Quantum Technologies	0.99	***<u>0.12</u>***	***<u>0.17</u>***

P value - Binomial	Transaction costs	Process Innovation	Consumer Benefit
Advanced Manufacturing	0.42	0.00	0.00
Energy & Environment	***<u>0.12</u>***	0.84	0.59
Life Sciences & Health	0.01	0.01	0.00
Digital & Quantum Technologies	0.99	0.07	***<u>0.14</u>***

At first look, we see mostly similar results from the quotients in a formalized way. We see the negative p-value for Transaction costs for Life Sciences and Health, following the quotient's indications, proving significance. We also see significance with reference to Process Innovation, with Advanced Manufacturing having a far greater concentration of Process Innovation than the NPL average, with Life Sciences and Digital both lower concentration than average. Finally, we see Life Sciences and Health holding much higher than average levels of consumer benefit, while Advanced Manufacturing has significantly lower levels of consumer benefit.

To implement these values within the table, we used the quotients to prescribe a value to the sector for each Mechanism. To provide further context, we would colour-code each input using the colour codes as seen above, using the p-values to match. The use of t-testing5 is useful for showing significance, but due to the nature of the case studies themselves, significance can be difficult to find. This is due to a range of factors, such as the sample sizes of the sectors, with Digital & Quantum technologies being much smaller than the other three sectors in terms of sectors.

On top of the normal 95% and 90% significance indicators, a 70% indicator was included to show where a sector resided within 1 standard deviation of the mean. This isn't describing the sector as being significantly different to the mean, rather it assesses if the sector is tending towards being significantly higher or lower than the NPL average. These trends, seen within the case studies during analysis, can allow for hypotheses to be constructed about the sectors that could be tested with further evidence. These values are in Bold, Italics and Underline to specify that they are spurious now and require more evidence.

There is reason to believe that Energy & Environment could be significantly higher than the NPL average with respect to Transaction costs, and Digital & Quantum could be significantly lower than average with respect to Consumer Benefit.

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3 The Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

4 the Binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments, each asking a yes–no question, and each with its own Boolean-valued outcome: success (with probability p) or failure (with probability q = 1 − p).

5 A t-test is a type of inferential statistic used to determine if there is a significant difference between the means of two groups, which may be related in certain features.

2.1.2 Breadth of Impact

Breadth of impact was used to categorize the case studies by the size of the market affected. Case studies default to being a non-niche case study, while an indicator is used to show if the case study impacts a niche market.

The main criteria when assessing breadth of impact are:

- How significant is the beneficiary in its sector (market share)?
- How significant is the outcome to the sector?
- Is the sector niche?
- How significant is the outcome to the beneficiaries?

In order for a case study to be deemed as having a large breadth of impact, three of the above criteria had to be met.

For simplicity, it was assumed that all the niche and non-niche markets detailed within the database were of the same size. While this is untrue in the real world, it would be much more complex to quantify all the markets involved within the database without this assumption. The indicator simply addresses the breadth of NPL's impact, rather than the actual size of the industries that NPL is involved with.

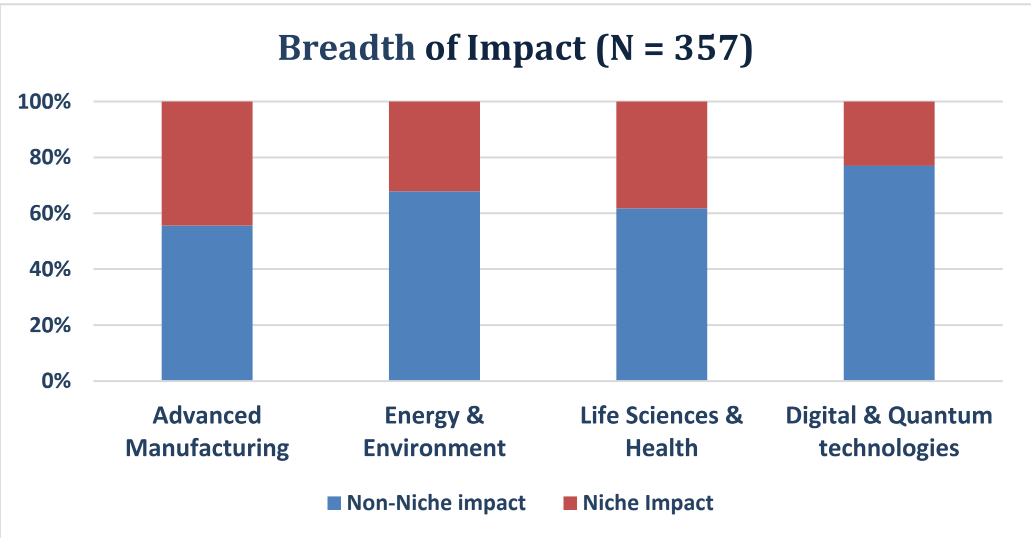
The main criteria when assessing breadth of impact are:

- How significant is the beneficiary in its sector (market share)?
- How significant is the outcome to the sector?
- Is the sector niche?
- How significant is the outcome to the beneficiaries?

The results for this were as follows:

Sector	Non-Niche impact	Niche Impact
Advanced Manufacturing	74	59
Energy & Environment	59	28
Life Sciences & Health	55	34
Digital & Quantum technologies	37	11

With the graph as such:



For this question, an assessment is made to see if the respective sector has higher than average levels of non-niche impact. This would suggest that the sector conducts research with a broad level of impact on a large segment of the Economy. This is likely to provide a great level of impact in the present as well as the future. Firstly, quotients were used to assess if there were noticeable differences between the sectors:

Quotients	Non-Niche
Advanced Manufacturing	0.88
Energy & Environment	1.08
Life Sciences & Health	0.98
Digital & Quantum Technologies	1.22

Here we can see some notable differences, with Digital & Quantum appearing notably higher than average while Advanced Manufacturing appears lower than average. To confirm a relationship between the Sectors and Niche/Non-Niche case studies, a chi square test was run, which produced a chi square value greater than the critical value (95% confidence), producing a p-value of 0.044, leading to the null hypothesis being rejected.

After this, the two t-tests were run in the data, using the same colour scheme to indicate positive and negative correlations:

P value - Poisson	Non-Niche
Advanced Manufacturing	***0.28***
Energy & Environment	0.57
Life Sciences & Health	0.88
Digital & Quantum Technologies	***0.22***

P value - Binomial	Non-Niche
Advanced Manufacturing	0.08
Energy & Environment	0.35
Life Sciences & Health	0.81
Digital & Quantum Technologies	0.04

Here we see some differences in the results between the sectors and the tests. Advanced manufacturing appears to have a lower-than-average level of Non-Niche case studies, while Digital & Quantum has a higher-than-average level of non-niche case studies. This was seen by the labelling.

2.1.3 Customer-Collaborations relationship

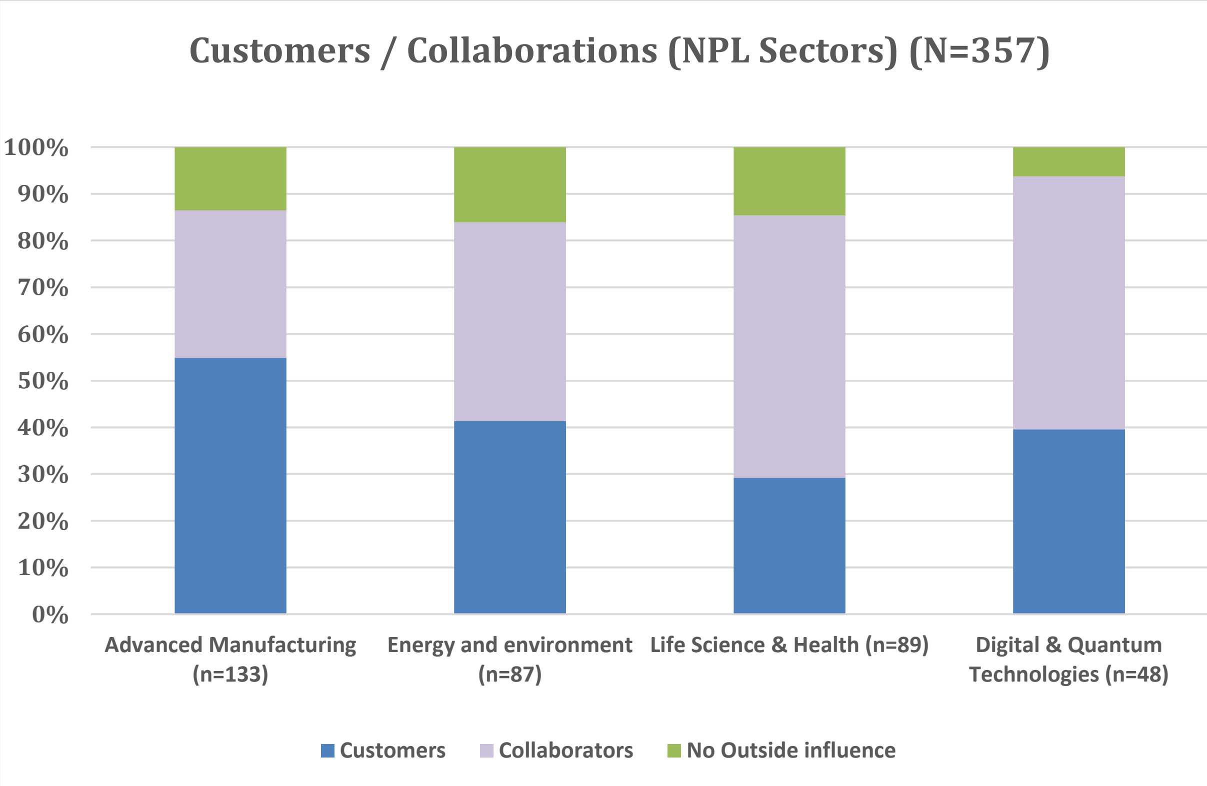
We have classified the nature of each case study's project. This was done by assessing the relationship between NPL and the other organisations involved with the project in the case study. If the relationship was of a collaborative nature, NPL would be seen working alongside an organisation (or organisations) in order to produce knowledge, learning from one another. Often, the outputs would be jointly owned, be it products or services or merely knowledge, IP or peer-reviewed papers produced.

If the case study was about a customer, NPL would be working on behalf of the party detailed in the case study, delivering a product or service to them. Furthermore, the database also details case studies where there was neither a customer nor a collaborator. However, it may be that some of this case studies may not actually detail if there is outside help/influence on the projects. For the table, case studies involving customers would indicate high levels of direct impact, as NPL would often be paid directly for the work being conducted, suggesting the party paying for the work believes that it would contribute greatly to the current business. This would often involve a measurement service or consultancy work.

Looking at the raw numbers of case studies with customers, collaborators and “No Outside Influence”, we see the following:

Observed	Collaborators	Customers	No Outside influence	Total
Advanced Manufacturing	42	75	16	133
Energy & Environment	37	36	14	87
Life Sciences & Health	50	26	13	89
Digital & Quantum Technologies	26	17	5	48

When represented as a graph:



Here we see some notable differences between the sectors, particularly when assessing levels of Customers, with some sectors having high levels of Customers when compared to the rest. Using quotients, we can get a sign if there is any significance:

Percentages	Customers
Advanced Manufacturing	1.31
Energy & Environment	0.96
Life Sciences & Health	0.68
Digital & Quantum Technologies	0.82

Here we see the potential for significance for Advanced Manufacturing and Life Sciences & health. This is further backed up by the chi square test for association, which produces a chi square score larger than the critical value, with a p-value of 0.00, leading to the null hypothesis being rejected. This allows for the analysis to be taken a step further, using the two t-tests as detailed before.

P value - Poisson	Customers
Advanced Manufacturing	0.02
Energy & Environment	0.80
Life Sciences & Health	0.05
Digital & Quantum Technologies	0.42

P value - Binomial	Customers
Advanced Manufacturing	0.00
Energy & Environment	0.74
Life Sciences & Health	0.01
Digital & Quantum Technologies	0.28

As seen with the quotients, there is significance when looking at Advanced Manufacturing and Life Sciences and Health. It can also be hypothesized that Digital & Quantum may have lower than average levels of customer-based case studies, due to the 30% significance indicated by the Binomial t-test.

2.1.4 Time Profile of impact

Unlike the previous indicators, time profile of impact allows for assessments of impact over time. This works along breadth of impact to assess the scope of a project for firms/industries. For this, there are four main categories:

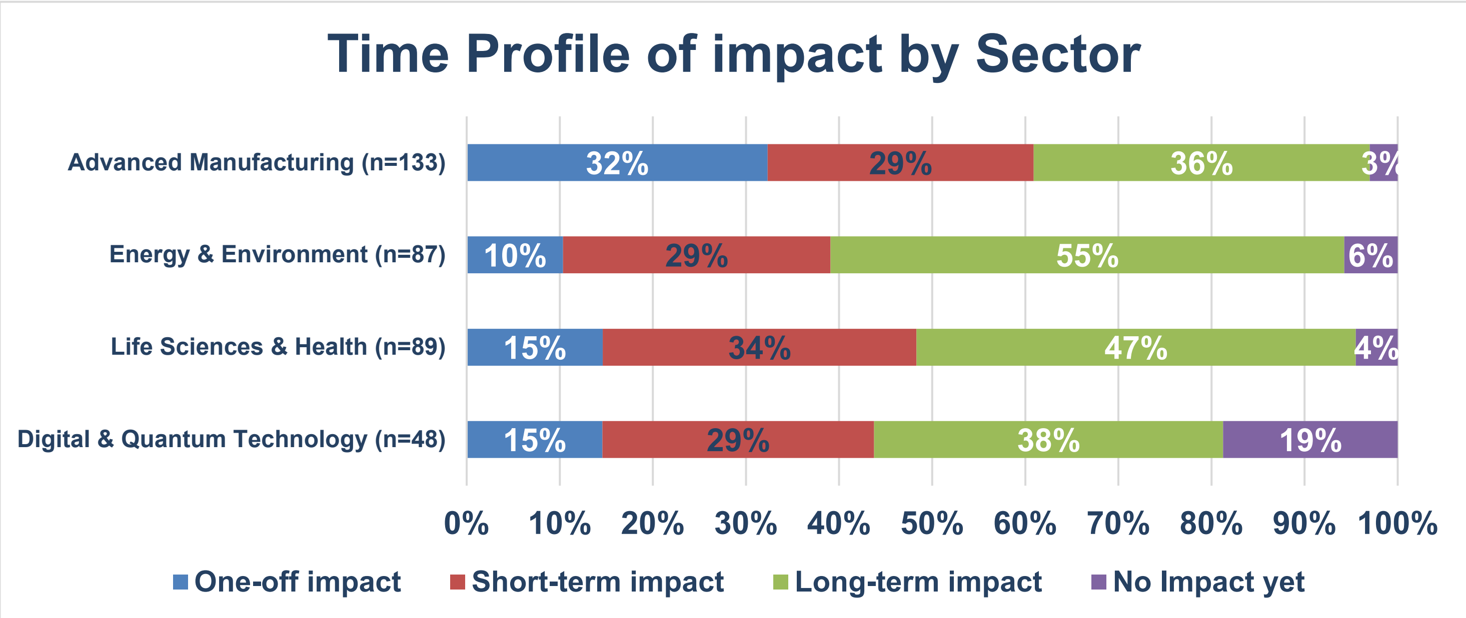
- One-off impact
  - Case studies where the impact is felt by a firm for a short time, with little outside scope
- Short-term impact
  - Case studies where the impact is felt for a short time, often by a number of firms, which may be providing benefits to this day
- Long-term impact
  - Case studies where the impact is felt for a significant duration, often by many firms across an industry, helping a great number for organisations
- No Impact yet
  - Case studies where there is no current economic impact, however there is potential for impact in the future

For the table, **the longer-term impact indicators** (Long-term and No Impact Yet) are used to provide indication towards which sectors seem to be focusing on longer-term impacts rather than the here-and-now.

The tabulated raw results are as follows:

Sector	One-off impact	Short term impact	Long-term impact	No Impact yet
Advanced Manufacturing	43	39	47	4
Energy & Environment	9	25	48	5
Life Sciences & Health	13	30	42	4
Digital & Quantum Technology	7	13	19	9

Presented graphically:



When looking at the sectors from the graph, there are some notable differences. All the sectors apart from Advanced Manufacturing have greater than 50% of their case studies in the longer-term impact groups, suggesting Advanced Manufacturing may be the outlier of the four. Using quotients, this can be better assessed:

Quotients	Long-term impact	No Impact yet
Advanced Manufacturing	0.81	0.49
Energy & Environment	1.26	0.93
Life Sciences & Health	1.08	0.73
Digital & Quantum Technologies	0.91	3.04

Here we see Energy & Environment along with Digital & Quantum scoring strongly on the longer-term impact indicators, while Advanced Manufacturing seems to perform poorly. After running the Chi-Square test of independence, we see a p-value of 0.0026, indicating a relationship between the sectors and the time profile of impact. This allows for t-tests to be run on the indicators, producing the following results:

P value - Poisson	Longer-term impact
Advanced Manufacturing	0.06
Energy & Environment	***0.14***
Life Sciences & Health	0.81
Digital & Quantum Technologies	0.41

P value - Binomial	Longer-term impact
Advanced Manufacturing	0.09
Energy & Environment	0.01
Life Sciences & Health	***0.28***
Digital & Quantum Technologies	0.48

Using the t-tests we see how Advanced Manufacturing has significantly lower than average levels of long-term impact case studies, while Energy & Environment has higher than average levels. It can also be hypothesized that Life Sciences and Health has higher than average longer-term impact.

2.1.5 Externalities

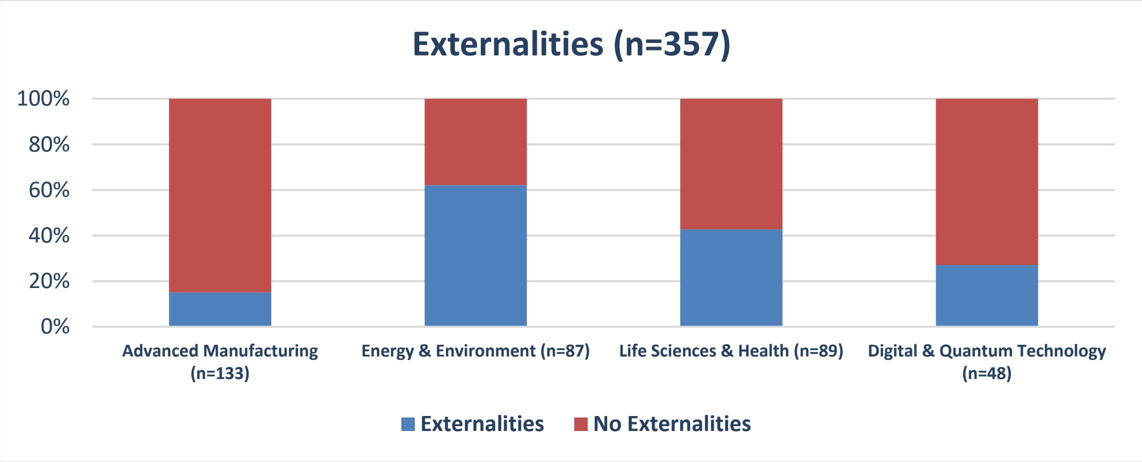
This section highlights case studies where the impact from the case study has not only affected the party we are directly involved with, but also affects unrelated third parties who were not involved in the market transaction. (i.e. Would/did the outcome benefit a group of individuals who did not pay NPL for its work or the beneficiary for their product or service?)

Externalities directly corresponds to the indirect benefits themes, assessing the spillover effects of certain types of research.

The initial tabulation of results is as follows:

Observed	Externalities	No Externalities
Advanced Manufacturing	20	113
Energy & Environment	54	33
Life Sciences & Health	38	51
Digital & Quantum Technologies	13	35

The corresponding graph is:



Once again, we see significant differences between the sectors, with a large range of proportions, with Advanced Manufacturing having <20% of its case studies displaying externality effects, meanwhile Energy & Environment has >60% of its case studies indicating externalities.

Quotients	Externalities
Advanced Manufacturing	0.43
Energy & Environment	1.77
Life Sciences & Health	1.22
Digital & Quantum Technologies	0.77

Using the quotients, we also seen a similar pattern. Advanced Manufacturing and, to a lesser extent, Digital & Quantum display lower than average levels of externalities. Meanwhile Energy & Environment and, to a lesser extent, Life Sciences and Health indicate higher than average levels of externalities. With a p-value for the chi-square test of independence being 0.00, this further indicates a relationship between sectors and the level of externalities, allowing for the t-tests to be run:

P value - Poisson	Externalities
Advanced Manufacturing	0.00
Energy & Environment	0.00
Life Sciences & Health	***0.22***
Digital & Quantum Technologies	0.35

P value - Binomial	Externalities
Advanced Manufacturing	0.00
Energy & Environment	0.00
Life Sciences & Health	***0.13***
Digital & Quantum Technologies	***0.25***

As seen at the start of this section, the belief that Advanced Manufacturing and Energy & Environment were significantly different to the mean has been proven, though Digital & Quantum and Life Sciences and Health weren't. However, given that the p-values are less than 0.3 for at one of the t-tests, the significance can be hypothesized. Life Sciences and Health can be hypothesized as having higher than average levels of externalities, while Digital & Quantum can be hypothesized as having lower than average levels of externalities

2.1.6 Products

When looking at the case studies, a set of "products" can be seen by assessing the type of output. Looking at common themes within the database, we have been able to assign each case study to a product group. The products groups are

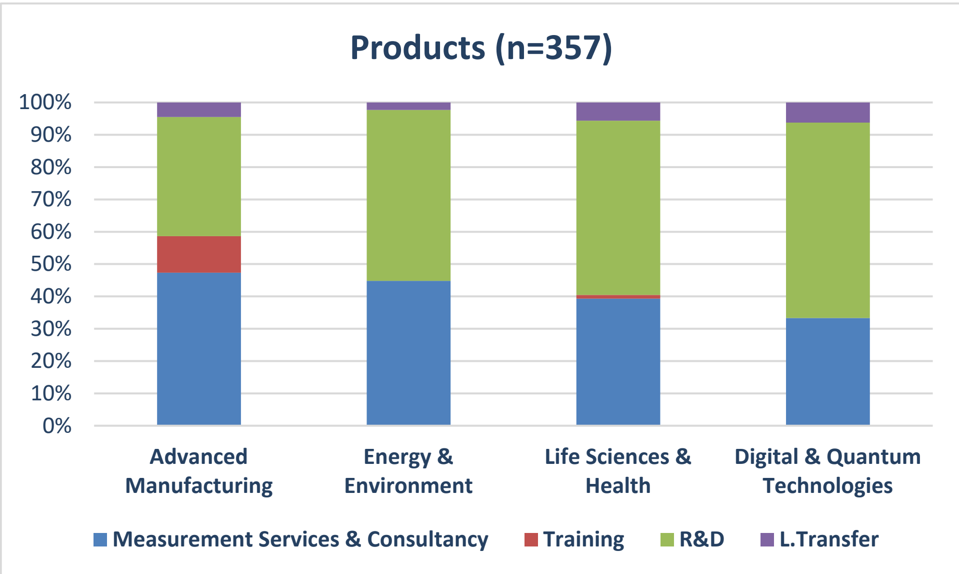
- License Transfer – this entails case studies where our IP and know-how has been passed to a beneficiary, for the beneficiary to exploit the resource.
- Consultancy – this involves case studies where the beneficiary sought expert information or advice from us for an external project. (E.g. Standards consultation).
- Measurement services – this are for case studies where we performed a measurement service such as calibrations for a beneficiary.
- Training – this includes case study where our educational product/service was used by the beneficiary.
- R&D – Where NPL conducts research, for a range of potential reasons
    - In-House R&D – This includes case studies where research was conducted by our scientists without direction from outside commercial influence.
    - Collaborative R&D – This is where NPL works alongside a commercial organisation to find an innovative solution to a problem
    - Contract R&D – This includes case studies where NPL was tasked with finding an innovative solution to a problem for a commercial customer

For the analysis table, R&D and Measurement Services & Consultancy were both used for different themes. Measurement Services & Consultancy are useful proxies for direct benefits, as they are products that tend to be paid for by customers. R&D tends to be focused on generating impacts in the future, as the IP generated must be turned into an impactful item.

Looking at the raw numbers for the products, we see the following

	Measurement Services & Consultancy	Training	R&D	License Transfer
Advanced Manufacturing	63	15	49	6
Energy & Environment	39	0	46	2
Life Sciences & Health	35	1	48	5
Digital & Quantum Technologies	16	0	29	3

Tabulating these results produced the following graph:



By looking at the graphs, there are some notable differences, the proportion of R&D and Measurement Services case studies appears to increase and decline respectively as you go across the sectors.

Quotient	Measurement Services & Consultancy	R&D
Advanced Manufacturing	1.11	0.76
Energy & Environment	1.05	1.10
Life Sciences & Health	0.92	1.12
Digital & Quantum Technologies	0.78	1.25

When assessing the quotients, we see a similar pattern to the graphs. When the chi square test of association was run, a p-value of 0.07 was found. This suggests that there is some statistical significance between the sectors and products, which allows for the two t-tests to be run.

P value - Poisson	Measurement Services & Consultancy	R&D
Advanced Manufacturing	0.05	0.38
Energy & Environment	0.68	0.43
Life Sciences & Health	0.76	0.31
Digital & Quantum Technologies	0.36	***0.18***

P value - Binomial	Measurement Services & Consultancy	R&D
Advanced Manufacturing	0.00	0.47
Energy & Environment	0.46	***0.20***
Life Sciences & Health	0.96	0.05
Digital & Quantum Technologies	0.39	0.02

After running the t-tests we see that Advanced Manufacturing is clearly significantly stronger than the NPL average at Measurement Services & Consultancy, while Life Sciences and Health and Digital & Quantum both score highly on R&D. With further evidence, it can be hypothesized that Energy & Environment would likely score positively on R&D, given its result in the Binomial t-test.

2.2 HOW WE CALCULATED THE POSITIONS OF EACH SECTOR

By using the quotients to construct the table as seen in section 1.3, we were able to calculate the geometric means[6] of each theme of impact for each sector. The following indicators were used for each Impact Theme:

Direct Impact			Indirect Impact			Future Impact		
Process Innovation	Customers	Measurement Services or Consultancy	Externalities	Customer Benefits	Transaction Costs	Longer-Term impact	R&D	Breadth of impact

For each Sector, the relevant quotient scores were used then computed to generate the geometric mean. For Advanced Manufacturing's Direct Impact, the following calculation was done:

$$1.28 = \sqrt[3]{1.45 * 1.31 * 1.11}$$

By using geometric means, we were able to assess output relative of each sector to one another holding the NPL mean as 1, as seen in the quotients. This led to the following table:

	Advanced Manufacturing	Life Sciences & Health	Energy & Environment	Digital & Quantum Technologies
Direct Impact	1.28	0.72	0.99	0.72
Indirect Impact	0.38	1.27	1.24	0.71
Future Impact	0.80	1.04	1.13	1.22

Using these values, we were able to plot the Sectors with respect to each other and the NPL average as a whole. In order to construct Current Impact, the three impacts in the table above were described as axis in a 3-dimensional space. (X = Direct Impact, Y = Indirect Impact, Z = Future impact). By doing this, Current Impact could be viewed as a projection on the X-Y plane; a vector whose magnitude is given using Pythagoras theorem:

$$a^2 + b^2 = c^2$$

When re-arranged, the formula looks as follows:

6 The Geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum).

$$\frac{\sqrt{X^2 + Y^2}}{\sqrt{2}} = \text{C}$$

Where C = the current impact and $\sqrt{2}$ divides through to scale for the averages of both X & Y (Both are equal to 1, therefore: $\sqrt{1^2 + 1^2} = \sqrt{2}$)

This produces the following for Current Impact, Along with the Future Impact:

	Advanced Manufacturing	Life Sciences & Health	Energy & Environment	Digital & Quantum Technologies
Current Impact	0.94	1.03	1.12	0.71
Future Impact	0.80	1.04	1.13	1.22

These are points that are subsequently plotted on Figure 2 (page 22) in the main document

2.3 HOW WE CALCULATED THE TRIANGLE MODEL

Using the table as seen in section 2.2 of this note, we are also able to construct the triangle model, allowing for a comparison of the sectors across all 3 impact themes. In order to do this, a similar method to the one used in 2.2 was used. The three impact themes were regarded as axes in a 3-D space. The first step was to find the magnitude of each sector (i.e. the distance of each sector from the origin). This indicates the scale of the output, assuming each impact mechanism is equal to one another. This was done using the following formula (this formula is a generalised version of the one used previously):

$$y_s = \sqrt{\frac{\sum_{i=1}^{n}(x_{is})^2}{n}}, i \in \{Direct\ Impact, Indirect\ Impact, Future\ Impact\}, s \in \{4\ sectors\}$$

Where:
x_{is}= The Quotient Score for the Impact theme and sector in question
n = Number of Impact Themes
y_s= Magnitude of Sector in question

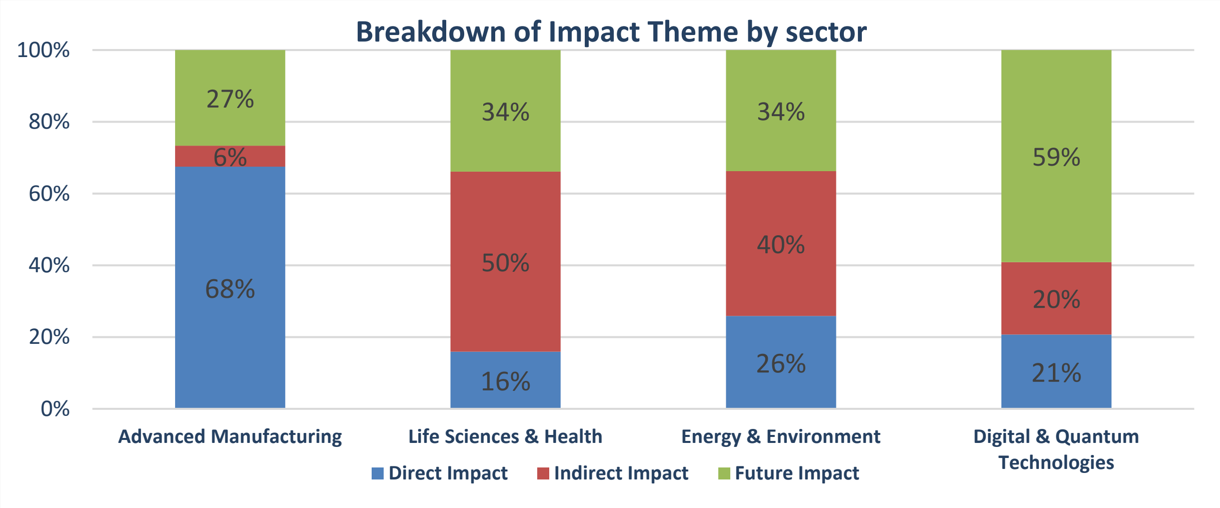
The magnitude generated can then be used to ratio the three forms of impact using the following formula, generating p, the proportion of impact for each sector (s) accounted for by each theme (i):

$$p_{is} = \left(\frac{\left(\frac{x_{is}}{\sqrt{n}}\right)}{y_s}\right)^2 = \frac{x_{is}^2}{\sum_i x_{is}^2}, i \in \{Direct\ Impact, Indirect\ Impact, Future\ Impact\}, s \in \{4\ sectors\}$$

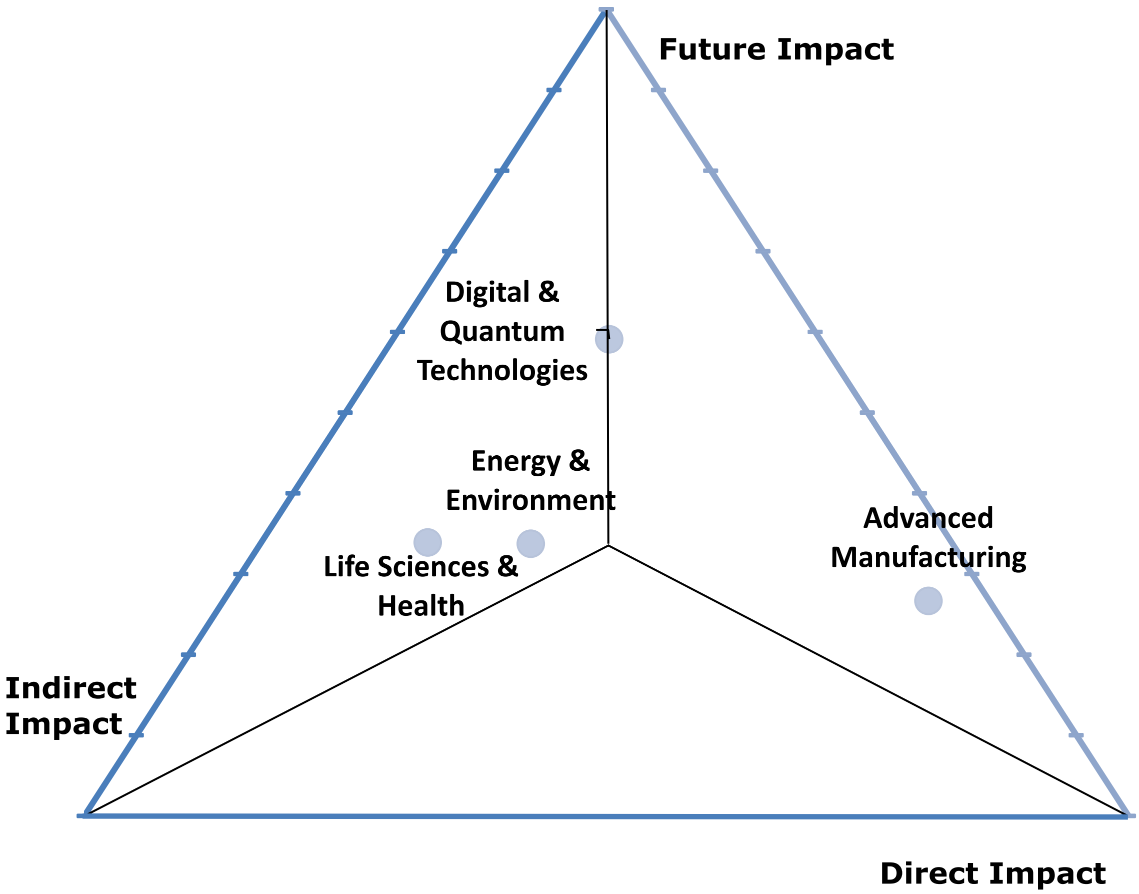
This scales the score for the three impact themes, assuming equal weighting across the three. The formula provides the proportion of impact, for the theme and sector in question as follows:

Ratio	Advanced Manufacturing	Life Sciences & Health	Energy & Environment	Digital & Quantum Technologies
Direct Impact	68%	16%	26%	21%
Indirect Impact	6%	50%	40%	20%
Future Impact	27%	34%	34%	59%

As a graph, the sectors look as follows:



As we assume that all the themes of impact are equal to one another, the centre point of the diagram can be where the proportions are equal to (1/3,1/3,1/3). This allows for a triangle to be constructed using the proportions as seen above for each sector as seen below:



2.4 HOW WE CALCULATED THE CONFIDENCE INTERVALS

To construct the Confidence Intervals, the standard errors are needed for the quotients and impact themes. Once the standard errors have been calculated, the values are multiplied by 2 to produce the interval.

2.4.1 Standard Errors for the Quotient

For the quotient table, the following formulae explains how standard errors were computed.

X = Quotient Value, O = observed value, E = expected value, σ=standard deviation

As seen here, the calculation of the quotients are as follows:

$$X = \frac{O}{E}$$

Following this logic, in order to calculate the subsequent variance, the following must hold true:

$$\frac{\sigma_X}{X} = \sqrt{\left(\frac{\sigma_O}{O}\right)^2 + \left(\frac{\sigma_E}{E}\right)^2}$$

It is assumed that the Observed and Expected variables are independent, therefore $\sigma_{oe} = 0$. It should be noted that the standard error of each variable is ratioed by the variable itself. It should be noted the observed variable is held as a parameter, with no variance, and that the standard deviation of the expected variable can be calculated as follows:

$$\sigma_O = 0$$

$$\sigma_E = \sqrt{E}$$

Therefore, the Formula can be simplified as follows:

$$\frac{\sigma_X}{X} = \frac{\sqrt{E}}{E} = \frac{1}{\sqrt{E}}$$

$$\sigma_X = \frac{X}{\sqrt{E}}$$

The Standard error calculation here can be used to generate the Confidence intervals for the Quotients.

2.4.2 Standard Errors for Arithmetic Means

Using the standard error calculations as seen above for σ_X for each individual quotient score, for each impact theme, the following was done to calculate the standard errors for the means:

$$\frac{\sqrt{\sum_{\mathrm{i}=1}^{3} \sigma_{\mathrm{xi}}^2}}{3}$$

The relevant standard errors were squared, summed together then square-rooted, after which they were divided by the number of indicators within each impact theme, which is three for each.

2.4.3 Standard Errors for Current Impact

Unlike the calculation for the Geometric means, as a projection is used to calculate current Impact, the following must be done to calculate current impacts standard error.

It can be assumed that:

$$f(x, y) = \frac{\sqrt{x^2 + y^2}}{\sqrt{2}}$$

And assuming that holds, the following Taylor expansion can be done:

$$f(x,y) = f(\bar{x},\bar{y}) + (x-\bar{x})\left.\frac{\partial f}{\partial x}\right|_{x,y=\overline{x,y}} + (y-\bar{y})\left.\frac{\partial f}{\partial y}\right|_{x,y=\overline{x,y}}$$

Producing the following calculations for variance

$$Var[f(x,y)] = Var(x).\left(\left.\frac{\partial f}{\partial x}\right|_{x,y=\overline{x,y}}\right)^2 + Var(y)\left(\left.\frac{\partial f}{\partial y}\right|_{x,y=\overline{x,y}}\right)^2$$

Generating the partial derivatives produces the following:

$$\frac{\partial f}{\partial x} = \frac{1}{\sqrt{2}}.\left(\frac{(1/2)}{\sqrt{x^2+y^2}}\right).2x = \frac{x}{2.f(x,y)}$$

Plugging the partial derivatives into the variance formula allows for the following, with the subsequent simplification to generate the Standard Error.

$$Var[f(x,y)] = Var(x).\left(\frac{\bar{x}}{2.f(\bar{x},\bar{y})}\right)^2 + Var(y).\left(\frac{\bar{y}}{2.f(\bar{x},\bar{y})}\right)^2$$

$$SE[f(x,y)] = \sqrt{SE(x)^2.\left(\frac{\bar{x}}{2f(\bar{x},\bar{y})}\right)^2 + SE(y)^2.\left(\frac{\bar{y}}{2f(\bar{x},\bar{y})}\right)^2}$$