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Recommended procedure for the calculation and expression of pressure balance measurement uncertainties

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P R Stuart

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ABSTRACT

This Recommendation was prepared by representatives of 9 national standards laboratories participating in EUROMET Project 220.

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1 INTRODUCTION

This recommendation has been prepared by the participants of EUROMET Project 220; the participants were as follows:

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It gives procedures for the evaluation and subsequent combination of the various component measurement uncertainties of pressure balances, otherwise known as deadweight testers or piston gauges. The pressure medium may be either liquid (usually oil) or gas.

In this document the chosen measure of uncertainty corresponds to the 'standard uncertainty' proposed by ISO Working Group TC69/SC6/WG3. The recommendations are consistent with those of ISO Working Group TAG4/WG3 [1] and also those of the BIPM and WECC working groups on uncertainties [2,3]. The recommendations in this document are intended for use only by national standards laboratories and are not intended to replace those followed by the secondary calibration laboratories accredited by WECC members.

A common feature of all 3 recommendations mentioned in the previous paragraph is the classification of uncertainty contributions into 2 categories - type A and type B. The former are frequently referred to as random uncertainties and, in principle, the calibration laboratory can reduce their magnitude by taking more observations. On the other hand, type B uncertainties correspond broadly to the type formerly referred to as systematic uncertainties. Their magnitude is determined by the quality of the pre-determined data used in calculating the results. These data include the metrological characteristics of the standard and its ancillary equipment, together with the assumed values of such quantities as fluid densities and thermal expansivities. The statistical treatments of type A and type B uncertainty components are different and so they are considered separately in sections 3 and 4.

When combining uncertainty components, whether they are type A or type B, all their values should be converted initially into equivalent single standard deviations (standard uncertainties). After summation by root-sum-of-squares addition the combined standard uncertainty can, if desired, be converted to an "expanded" uncertainty at a specified confidence level by multiplying it by an uncertainty coverage-factor k. The WECC prefers the use of the coverage-factor k=2, which generally corresponds to a confidence level of approximately 95%.

The results of the calibration of a pressure balance by a national standards laboratory are generally expressed in terms of the effective area of the piston-cylinder assembly, which is pressure-dependent, and the masses of the piston unit, the weight-carrier and all the ring-weights. This Recommendation concentrates on the evaluation and expression of the uncertainty in effective area.

Occasionally the results of a calibration may be expressed in terms of the pressure generated by the balance when the piston is supporting specified combinations of weights under defined conditions of temperature and gravity. This case is considered in section 7.

2 THE PRINCIPLE OF THE PRESSURE BALANCE AND ITS CALIBRATION BY CROSS-FLOATING

In a pressure balance the upward force due to pressure acting on the base of a piston located in a closely fitting cylinder is balanced by downward forces due, principally, to the force of gravity acting on a stack of weights supported by the piston. The equilibrium condition may be expressed by the equation:

$$F(p,T) \;=\; p \;.\; A(p,T) \tag{1}$$

where:	F(p,T)	is the sum of the downward forces
	p	is the pressure acting on the base of the piston
	T	is the temperature of the piston-cylinder assembly
and	A(p,T)	is its effective cross-sectional area.

F(p,T) is thus a function of the supporting pressure. A(p,T) is also a function of pressure because the piston and cylinder both distort on the application of pressure; the effective area also changes with temperature as a consequence of thermal expansion.

The total downward force F(p,T) acting on the piston is, in practice, the sum of many components and may be expressed by the equation:

$$\begin{aligned} F(p,T) = g\cos\theta &\left\{ m_p\left(1-\frac{\rho_a}{\rho_p}\right) + m_w\left(1-\frac{\rho_a}{\rho_w}\right) + m\left(1-\frac{\rho_a}{\rho_m}\right) + M\left(1-\frac{\rho_a}{\rho_M}\right)\right\} \\ &- g.\cos\theta.v.\rho_f\left(1-\frac{\rho_a}{\rho_f}\right) + s.c + g.\rho_f\left(1-\frac{\rho_a}{\rho_f}\right)h.A(p,T) + P_r.A\,(p,T) \end{aligned} \tag{2}$$

where:	g	is the local acceleration due to gravity
	θ	is the angle, intended to be zero, between the piston axis and the vertical

m_p	is the true mass of the piston
ρ_a	is the density of ambient air
ρ_p	is the density of the piston
m_w	is the true mass of the weight-carrier
ρ_w	is the density of the weight carrier
m	is the additional true mass necessary to balance the minimum calibration pressure
ρ_m	is the density of the added masses necessary to balance at the minimum calibration pressure
M	is the additional true mass necessary to balance that part of the pressure in excess of the minimum calibration pressure
ρ_M	is the density of the masses added to achieve balance at pressures above the minimum calibration pressure
v	is the buoyancy volume, if any, of the piston, depending on the level of the base of the piston relative to the datum level of the pressure balance
ρ_f	is the density of the pressure medium
s	is the surface tension of the pressure medium
c	is the circumference of the piston
h	is the distance of the base of the piston below the reference level at which the pressure is to be measured
and P_r	is the reference pressure; this is normally zero in the gauge mode and as measured by the vacuum gauge in the absolute mode.

The effective cross-sectional area of the piston-cylinder assembly of a pressure balance is normally determined by the "cross-floating" method. The pressure balance under test (the "testee") is connected to a standard pressure balance and a pressure generator. The loads on both pistons are adjusted until, according to the working fluid, they are hydrostatically or pneumatically balanced at the required test pressure. In this condition there is no flow of fluid through the connecting pipe and both pistons should be descending at their natural fall rates, ie the rates at which they would descend if they were isolated by valving off. When thus balanced, and provided that both piston bases are at the same level, the pressure P_t acting on the base of the piston of the testee is equal to the pressure P_s under the base of the piston of the standard, ie:

$$P_t = P_s \tag{3}$$

From equations 1 and 3, we obtain

$$\frac{F_t(p,T)}{A_t(p,T)} = \frac{F_s(p,T)}{A_s(p,T)} \tag{4}$$

Hence

$$\frac{A_t(p,T)}{A_s(p,T)} = \frac{F_t(p,T)}{F_s(p,T)} = R(p,T) \tag{5}$$

where **R(p,T)** is the ratio of the downward forces acting on the testee and the standard.

From equations 4 and 5:

$$A_t(p,T) = A_s(p,T).\frac{F_t(p,T)}{F_s(p,T)} \tag{6}$$

which may otherwise be stated as:

$$A_t(p,T) = A_s(p,T).\ R_t(p,T) \tag{7}$$

The effective area of the testee is thus expressible in terms of the effective area of the standard at the pressure and temperature of each crossfloat observation. It is normal practice, however, to convert effective areas to the values which they would have at a constant reference temperature T_r (usually 20°C), rather than to express them at the various temperatures at which the crossfloat observations were taken. The conversions of effective areas to those at the reference temperature are made using the equations:

$$A_t(p,T_r) = \frac{A_t(p,T_t)}{[1 + \alpha_t(T_t - T_r)]} \tag{8}$$

and

$$A_s(p,T_r) = \frac{A_s(p,T_s)}{[1 + \alpha_s(T_s - T_r)]} \tag{9}$$

where α_t and α_s are the *area* thermal expansivities of the piston-cylinder assemblies of the testee and standard respectively. If the piston and cylinder of an assembly are made from different materials, then the mean value of area expansivities of the piston and cylinder should be used.

Putting $T = T_r$ in equation 5 and combining with equations 8 and 9 one obtains:

$$F_t(p,T_r) = \frac{F_t(p,T)}{[1 + \alpha_t(T_t - T_r)]} \tag{10}$$

and

$$F_s(p,T_r) = \frac{F_s(p,T)}{[1 + \alpha_s(T_s - T_r)]} \tag{11}$$

From equations 6, 8 and 9 one obtains:

$$\frac{A_t(p,T_r)}{A_s(p,T_r)} = R(p,T).\frac{[1 + \alpha_s(T_s - T_r)]}{[1 + \alpha_t(T_t - T_r)]} \tag{12}$$

which may be re-arranged to give:

$$A_t(p,T_r) = A_s(p,T_r).\ R(p,T).\frac{[1 + \alpha_s(T_s - T_r)]}{[1 + \alpha_t(T_t - T_r)]} \tag{13}$$

which is the basis of the conversion of measured effective area to that at the reference temperature.

3 TYPE A UNCERTAINTY COMPONENTS

The results of an effective area determination are usually expressed in two parts:

i) The effective area, **A(0,T_r)**, at zero applied pressure, corrected to the reference temperature **T_r**, and

ii) the coefficient or coefficients quantifying the dependence of the effective area on the applied pressure.

However, for some low pressure gas balances the change of effective area with pressure is negligible and it is sufficient to give a mean value for the effective area over the whole of the calibrated range. The repeatability of such a balance may be quantified by the experimental standard deviation of the values of effective area obtained.

The uncertainty in the mean value of the effective area may be quantified by the experimental standard deviation of the mean, otherwise known as the standard error of the mean, which is obtained by dividing the experimental standard deviation of the values of effective area by the square root of the number of observations.

The experimental results from a crossfloat calibration nearly always have **p**, the applied pressure measured at the datum level, as the independent variable, **x**. The dependent variable, **y**, may be either **A(p,T_r)**, the calculated value of the effective area at the reference temperature or, alternatively, **R(p,T_r)**, the calculated ratio of the net downward forces which would be supported at crossfloat equilibrium and at the reference temperature by the pressure acting on the pistons of the standard and the instrument being calibrated. In view of this choice of dependent variable, the general case will be considered and the dependent variable will be represented by the symbol **y(p,T_r)**.

Except in the case of low pressure gas balances, already discussed above, the procedure starts with finding the polynomial which fits the experimental results best according to the criterion of Gauss, ie the sum of the squares of the residuals is a minimum. Such a general polynomial expression has the form:

$$y(p,T_r) = a + b\,p + c\,p^2 + d\,p^3 + \ldots\ldots \tag{14}$$

It is the responsibility of the staff of the calibration laboratory to decide, on the basis of past experience, whether any apparent non-linearity in the results is a chance occurrence or whether it is a consequence of a real physical cause, such as a re-entrant design of cylinder.

Provided that any non-linearity is considered to be random or insignificant, a simple 1st degree polynomial of the following form may be used:

$$y(p,T_r) = a + b\,p \qquad (15)$$

The curve-fitting method of Gauss, which is described more fully in Appendix B, will then yield the following quantities:

a) $\bar{\mathbf{p}}$, the estimated mean value of **p**

b) $\bar{\mathbf{E}}_p$, the experimental standard deviation in $\bar{\mathbf{p}}$, the estimated mean value of **p**

c) $\bar{\mathbf{y}}$, the estimated mean value of $\mathbf{y(p,T_r)}$

d) $\bar{\mathbf{E}}_y$, the experimental standard deviation in $\bar{\mathbf{y}}$, the estimated mean value of $\mathbf{y(p,T_r)}$

e) **b**, the gradient of the best-fit straight line passing through the centroid ($\bar{\mathbf{p}}$, $\bar{\mathbf{y}}$). This is the line which gives the minimum value for the sum of the squares of the residuals (ie for **V**, the variance of the residuals)

f) $\bar{\mathbf{E}}_b$, the experimental standard deviation in the estimated value of **b**

g) **V**, the variance of the residuals

V is a measure of the repeatability of the *measurement process* and one would not expect it to change significantly if the number of measurements is increased. On the other hand, $\bar{\mathbf{E}}_p$, $\bar{\mathbf{E}}_y$ and $\bar{\mathbf{E}}_b$ are the experimental standard deviations of the best estimates of the values of $\bar{\mathbf{p}}$, $\bar{\mathbf{y}}$ and **b**; one would expect these to be reduced by a factor of approximately $\sqrt{\mathbf{n}}$ if the number of measurements is increased by a factor **n**.

Using $\bar{\mathbf{p}}$, $\bar{\mathbf{y}}$ and **b**, it is a simple matter to calculate the intercept of the fitted straight line with the **y**-axis.

$\mathbf{E}_y$, the experimental standard deviation in the estimated value of $\mathbf{y(p,T_r)}$, varies with **p** and is a minimum at the centroid, where it is equal to $\bar{\mathbf{E}}_y$. The experimental standard deviation in the estimated value of $\mathbf{y(p,T_r)}$ at any other pressure **p** will be larger and is given by:

$$E_y(p) = \sqrt{[\,\bar{E}_y^2 + [(\bar{p}-p).\bar{E}_b]^2\,]} \qquad (16)$$

Thus the experimental standard deviation in the estimated value of the intercept (ie at **p = 0**) is given by:

$$E_y(0) = \sqrt{[\,\bar{E}_y^2 + [\bar{p}.\bar{E}_b]^2\,]} \qquad (17)$$

Since $\mathbf{E}_y$ is a function of **p**, it should be quoted either for selected values of **p** or, otherwise, its maximum value over the whole of the certificated pressure range should be stated.

If $\mathbf{y(p,T_r)}$ represents $\mathbf{R(p,T_r)}$, rather than $\mathbf{A(p,T_r)}$, then $\mathbf{E_y(p)}$ and $\mathbf{E_y(0)}$ may easily be converted into $\mathbf{E_A(p,T_r)}$ and $\mathbf{E_A(0,T_r)}$, ie the standard errors in the estimated values of $\mathbf{A(p,T_r)}$ and $\mathbf{A(0,T_r)}$. If required, these data can be used in combination with data for the values of the masses associated with the pressure balance to calculate the pressures generated by the balance with specified weight combinations and at a specified temperature and value of **g** (see section 7).

It should be noted that, so far as type A uncertainties are concerned, $E_A(p,T_r)$ will generally be less than $E_A(0,T_r)$ throughout the calibrated pressure range.

As pointed out in the recommendations of the WECC Working Group on Uncertainties, the values of the experimental standard deviations obtained as above may be underestimates, particularly if the number of observations is less than 10. The number of observations should therefore be always at least 10. If, in exceptional circumstances, there are unavoidably less than 10 observations, then the experimental standard deviations should be multiplied by a factor which is based on both the Student distribution and the Normal distribution. This factor also depends on the number of observations and on the uncertainty coverage factor k by which the uncertainty values will subsequently be multiplied (see WECC Document 19-1990, Appendix B Table 1).

The method of combining the type A and type B uncertainties to produce a combined uncertainty will be discussed in section 5.

4 TYPE B UNCERTAINTY COMPONENTS

The analysis of the uncertainty components for the results of a pressure balance calibration is a complicated process, so it is useful to start with a summary before considering the details.

From equation 6, the relative standard uncertainty in $A_t(p,T)$ will be given by the root-sum-of-squares of the relative standard uncertainties in $A_s(p,T)$, $F_t(p,T)$ and $F_s(p,T)$. This being so, the method proposed in this Recommendation for determining the relative standard uncertainty in $A_t(p,T)$ has the following steps:

i) Evaluate the uncertainties for the downward forces acting on the piston of the pressure balance being calibrated. These uncertainties are of 3 types:

   a) those present at the minimum calibration pressure

   b) those proportional to the amount by which the pressure exceeds the minimum calibration pressure

   c) those proportional to the total operating pressure

ii) Repeat (i) above for the standard pressure balance

iii) Evaluate the relative uncertainty in the ratio of the forces as a function of pressure (this ratio is also the ratio of the effective areas)

iv) Incorporate the type A relative uncertainties in the force ratio

v) Incorporate the pressure-dependent uncertainties in the effective area of the standard

vi) multiply by the chosen k factor.

The downward forces on the pistons take the form previously given by equation 2. It is convenient to re-arrange the right-hand side of this equation into two parts - those force components which are acting at the minimum calibration pressure and those which are proportional to the additional mass M necessary to balance pressures in excess of the minimum, ie:

$$F(p,T) = g \cos\theta \left\{ m_p \left(1 - \frac{\rho_a}{\rho_p}\right) + m_w \left(1 - \frac{\rho_a}{\rho_w}\right) + m \left(1 - \frac{\rho_a}{\rho_m}\right) - v.\rho_f \left(1 - \frac{\rho_a}{\rho_f}\right)\right\}$$

$$+ s.c + g.\rho_f \left(1 - \frac{\rho_a}{\rho_f}\right) .h.A(p,T) + g.\cos\theta.M \left(1 - \frac{\rho_a}{\rho_M}\right) \tag{18}$$

The quantities on the right-hand side of equation 18 are known as influence quantities, since they all can have an influence the value of **F(p,T)**. The relationship between an error δQ in one of these influence quantities Q and the resulting error δF in the force **F(p,T)** may be expressed in the form:

$$\delta F = C_{QF} \cdot \delta Q \tag{19}$$

where C_{QF} is referred to as the sensitivity coefficient. Sensitivity coefficients may be determined by partial differentiation, ie

$$C_{QF} = \frac{\partial F}{\partial Q} \tag{20}$$

Alternatively, they may be found by the use a computer program in which each influence parameter in turn is taken in several increments through its mean value in a FOR-NEXT sequence and the corresponding changes in F calculated.

The sensitivity coefficients for the influence quantities in equation 18 are given as partial differentials in Appendices A.1, A.2 and A.3.

As stated in section 2, it is normal practice to quote values of effective area corrected to a constant reference temperature T_r, rather than those at the varying temperature at which the crossfloat observations were taken. In determining the *uncertainties* associated with the conversions of the effective areas to those at the reference temperature (equations 8 and 9) it is necessary to take into account the uncertainties in the thermal expansivities and in the measurement of temperature. As in the case of the force uncertainties, the conversion requires the calculation of sensitivity coefficients. These coefficients are given in Appendix A.2.

Using equation 13 it is next possible to work out, for each test pressure **p**, the amount by which the uncertainty in $A_t(p,T_r)$ exceeds that of $A_s(p,T_r)$. The uncertainty of the latter is itself pressure-dependent, being in part determined by the uncertainty in the pressure distortion coefficient of the standard. It is necessary that the uncertainty in the pressure distortion coefficient is determined in advance, either experimentally against another standard or by calculation using elasticity theory (see section 6).

The evaluation and summation of uncertainty components is clearly a very complex process but it can be greatly eased with the aid of a computer. One method is to use software which already exists in the form of spreadsheet packages. Another method is to determine, for each of a number of pressures in the working range, the amount by which $\mathbf{A_t(p,T_r)}$ changes when each influence quantity is changed by its uncertainty. The many individual changes are then summed by root-sum-of-squares addition.

The process of uncertainty summation by linked spreadsheets is illustrated by the example in the following section.

5 EXAMPLE OF A BUDGET OF UNCERTAINTY COMPONENTS

The example chosen for this section relates to the calibration of a pressure balance having a steel piston-cylinder assembly and cast-iron ring-weights against a standard having a tungsten carbide piston-cylinder assembly and stainless-steel ring weights. There is also a significant height difference between the two pressure balances.

Many of the uncertainty components covered by the spreadsheets (Appendix C) are very small and could be ignored without significantly affecting the value of the total. However, since their inclusion by no means overtaxes the capabilities of a modern personal computer they have been retained. This guards against the possibility of their being significant for unforeseen developments in design or for changes in operating conditions.

All type B uncertainty components are converted into single, standard uncertainties. A final spreadsheet links and sums the data obtained by the preceding spreadsheets and enables the incorporation of the pressure-dependent uncertainties in the effective area of the standard and also the type A (random) uncertainties arising in the calibration process.

The spreadsheets in Appendix C (on pages 20 to 27) represent an example as follows.

No 1. Basic data on the testee, the standard and the operating conditions, required in the subsequent spreadsheets.

No 2A. The uncertainty in the downward force acting on the piston of the testee at the minimum calibration pressure. Column 1 lists the influence quantities, column 2 their units of measurement and column 3 their approximate values. The uncertainties are entered in either column 4, 5, 6 or 7, according to the confidence level at which they have been quoted. In column 8 the entries in columns 4, 5, 6 or 7 are converted into single standard uncertainties. Column 9 gives the sensitivity coefficients of the influence quantities in newtons per unit of measurement (see Appendix A.1). Column 10 is the product of columns 8 and 9, giving the uncertainty in force. Column 11 is the variance, ie the square of column 10. The sum of the variances is given at the foot of this column and its square-root, given at the foot of column 3, is the total uncertainty in the force at the minimum calibration pressure. The bottom row in this spreadsheet provides an allowance for possible extraneous forces, eg magnetic and electrostatic forces.

No 2B. The uncertainty in the additional downward force required to act on the piston of the testee for each pascal of applied pressure above the minimum calibration pressure. The column headings are similar to those of spreadsheet 2A except that columns 9, 10 and 11 relate to the uncertainty in the additional force needed to balance each pascal of applied pressure above the minimum calibration pressure.

No 2C. The additional *relative* uncertainty in the total downward force acting on the piston of the testee over the whole calibration pressure range, after correction to the reference temperature, as a consequence of uncertainties in temperature measurement and in the tilt of the axis of the piston-cylinder assembly.

The effect of tilt needs special consideration (see Appendix A.3), since the errors due to tilt are proportional to the square of the angle of tilt and the result is thus always biassed in one direction. The pressure required to balance a pressure balance at a given load is reduced as the angle of tilt is increased. This applies to both the standard and the instrument being calibrated.

The information from spreadsheets 2A, 2B and 2C is fed into spreadsheet 6 to enable the calculation of the relative uncertainty in the downward force on the piston of the testee instrument at each nominal calibration pressure, corrected to the reference temperature. This is shown in columns 2 and 3 of spreadsheet 6.

No 3A. The uncertainty in the downward force acting on the piston of the standard at the minimum calibration pressure.

No 3B. The uncertainty in the additional downward force required to act on the piston of the standard for each pascal of applied pressure above the minimum calibration pressure.

No 3C. The additional *relative* uncertainty in the total downward force acting on the piston of the standard over the whole calibration pressure range, after correction to the reference temperature, as a consequence of uncertainties in temperature measurement and in the tilt of the axis of the piston-cylinder assembly.

The information from spreadsheets 3A, 3B and 3C is fed into spreadsheet 6 to enable the calculation of the relative uncertainty in the downward force on the piston of the standard instrument at each nominal calibration pressure, corrected to the reference temperature. This is shown in columns 4 and 5 of spreadsheet 6.

Column 6 of spreadsheet 6 combines the relative uncertainties of columns 2 to 5 to give their contribution to the relative uncertainty of the total force ratio.

No 4A. The uncertainty in the *difference* between the downward forces on the two pistons at the minimum calibration pressure due to factors which influence them both, such as ambient air density, fluid density and vertical height difference.

No 4B. The uncertainty in the *difference* between the additional downward forces on the two pistons at pressure above the minimum calibration pressure due to factors which influence them both.

The uncertainty in the force *difference* obtained from spreadsheets 4A and 4B has to be split between the two pressure balances. This has been done in proportion to their total loads, ie in proportion to their effective areas, and is displayed in column 7 of spreadsheet 6.

Spreadsheet 5 evaluates the uncertainty in the distortion coefficient of the standard, for use in column 9 of spreadsheet 6 when the distortion coefficient has been calculated using equation 22 in section 6.

From equation 5 the relative uncertainty in the total force ratio is also the relative uncertainty in the ratio of the effective areas, excluding the contribution due to the relative uncertainty in the effective area of the standard. Hence, by the RSS summation of columns 6 and 7 of spreadsheet 6 one can calculate the amount by which the combined type B relative uncertainty of the effective area of the testee exceeds that of the standard.

As already mentioned in discussing the previous spreadsheets, spreadsheet 6 combines the relative uncertainties evaluated in the preceding spreadsheets, adding them by root-sum-of-squares (RSS) summation for each of a series of nominal test pressures. For the purpose of illustration the calibration range has been divided into 10 equal parts. Column 8 is for the Type A (random) relative uncertainties of the calibration (see section 4) while column 9 is for the insertion of the pressure-dependent relative uncertainties in the effective area of the standard. If considered appropriate, the uncertainty in effective area should include a contribution for possible changes due to cylinder clamping forces. Column 10 is the root-sum-of-squares combination expressed a relative standard uncertainty.

Column 10 of spreadsheet 5 shows clearly that, since many uncertainty components are pressure-dependent, the total is also pressure-dependent. If a single value is required for the total, then it should be the maximum value in the certificated range. If necessary, the totals can finally be converted to a different confidence level by multiplying by the appropriate coverage-factor k.

Spreadsheet 7 is included for interest and shows the percentages by which various sources of uncertainty contribute to the total. As is usually the case, the greatest contribution is from the uncertainty in the effective area of the standard. In the example, the Type A uncertainties from the cross-floating are the least significant.

6 UNCERTAINTY IN THE DISTORTION COEFFICIENT OF THE STANDARD

The determination of the uncertainty in the value of the distortion coefficient of the standard is not straightforward. The method used depends on whether the effective area and its pressure dependence have been found by cross-floating, as is usually the case for working standards, or by dimensional metrology and the application of elasticity theory, as is usually the case for primary standards.

In the latter case the value of the uncertainty of the effective area is a minimum at zero applied pressure and increases quadratically (due to RSS summation) as one goes to higher pressures. In the case of determination by cross-floating, the uncertainty in the effective area is a minimum at the mean calibration pressure and increases quadratically as one goes to either higher or lower pressures, as already indicated by equation 16 in section 2. If a standard is used above its calibrated range, as is unavoidable when extending the pressure scale to higher pressures, the uncertainty in its effective area will increase at a rate greater than linearly with pressure.

The dependence of the effective area of the standard on the applied pressure is often expressed by a polynomial of the following form:

$$A_s(p,T_r) = A_s(0,T_r).\{1 + \lambda_{1s}.p + \lambda_{2s}.p^2 + ...\} \qquad (21)$$

$A_s(0,T_r)$ is the effective area at zero applied pressure at the reference temperature and the λ coefficients are known as pressure distortion coefficients.

If the piston-cylinder assembly of the standard is of "simple" form and the piston-cylinder engagement length is long compared with the piston diameter then it is usually sufficient to express the dependence of effective area on pressure by a linear equation.

If the piston and the cylinder are of simple geometry and of the same material, the distortion coefficient λ_1 can be calculated approximately by the following equation:

$$\lambda_1 = \frac{1}{E}\left\{2\nu + \frac{2R^2}{R^2 - R^2}\right\} \qquad (22)$$

where E is the Young's modulus of elasticity

ν is the Poisson's ratio

R is the radius of the piston

and R' is the outer radius of the cylinder.

The sensitivity coefficients for these parameters are given in Appendix A.7. Spreadsheet 5 gives an example of the summation of the uncertainty components. It must always be remembered that an equation such as the above is an approximation only, and that an additional uncertainty component should be introduced to cover inevitable errors. The magnitude of this additional component currently under study in several standards laboratories [4].

7 UNCERTAINTIES FOR GENERATED PRESSURES

When the results of a calibration are expressed in terms of the pressure required to support specified combinations of ring weights, they are often expressed in the form of a table of results. Unless the customer requests otherwise, the quoted pressures are calculated to be those which would be generated under conditions of standard gravity, ie $g = 9.80665 \text{ ms}^{-2}$, and at a specified reference temperature, such as 20°C.

Normally several observations are taken for each set of nominally identical conditions. The effective area is calculated for each observation and the corresponding pressure then calculated from a knowledge of the values of the supported masses. It is appropriate to quote the *mean* value and the *standard deviation* of each set of pressures calculated for nominally identical conditions. These values of *standard deviation* are an appropriate measure of the *repeatability* of the instrument under each set of conditions. The Type A component of the *uncertainty of the mean* may be quantified by the *experimental standard deviation of the mean*, ie the standard deviation divided by the square-root of the number of observations in the set. To this must be added, by root-sum-of-squares summation, the combined Type B uncertainties of the values of the effective areas and masses.

If required, the final combined value of uncertainty can then be multiplied by an *uncertainty coverage-factor*, *k*, the value of which must be stated.

8 REFERENCES

1 International Standards Organisation (ISO). *Guide to the Expression of Uncertainty in Measurement*, 1st edition, June 1992, prepared jointly by ISO/TAG4/WG3, IEC, OIML and BIPM.

2 International Bureau of Weights and Measures (BIPM). Recommendations INC-1 (1980) and CI-1981. *The Expression of Experimental Uncertainties.*

3 West European Calibration Collaboration (WECC). *Guidelines for the Expression of the Uncertainty of Measurement in Calibrations.* WECC Document 19 - 1990.

4 EUROMET Project 256. *Calculation of the elastic distortion of piston-cylinder assemblies in pressure balances and in hydraulic amplifies in force machines.*

9. APPENDICES

A SENSITIVITY COEFFICIENTS OF THOSE INFLUENCE QUANTITIES WHICH GIVE RISE TO TYPE B PRESSURE BALANCE UNCERTAINTY COMPONENTS

A.1 Forces acting on pistons at minimum calibration pressure - (spreadsheets 2A and 3A)

The starting point for the evaluation of the following sensitivity coefficients is equation 2.

Mass of the piston

$$\frac{\partial F}{\partial m_p} = g \cos \theta \left(1 - \frac{\rho_a}{\rho_p}\right)$$

Density of the piston

$$\frac{\partial F}{\partial \rho_p} = \frac{g \cos \theta . m_p . \rho_a}{\rho_p^2}$$

Mass of the weight-carrier

$$\frac{\partial F}{\partial m_w} = g \cos \theta \left(1 - \frac{\rho_a}{\rho_w}\right)$$

Density of the weight-carrier

$$\frac{\partial F}{\partial \rho_w} = \frac{g \cos \theta . m_w . \rho_a}{\rho_w^2}$$

Mass of minimum added load

$$\frac{\partial F}{\partial m} = g \cos \theta \left(1 - \frac{\rho_a}{\rho_m}\right)$$

Density of minimum added load

$$\frac{\partial F}{\partial \rho_m} = \frac{g \cos \theta . m . \rho_a}{\rho_m^2}$$

Buoyancy volume of the piston

$$\frac{\partial F}{\partial V} = - g \cos \theta . \rho_f \left(1 - \frac{\rho_a}{\rho_f}\right)$$

Fluid surface tension

$$\frac{\partial F}{\partial s} = c$$

Fluid head correction

$$\frac{\partial F}{\partial h} = g\,\rho_f \left(1 - \frac{\rho_a}{\rho_f}\right) A\,(p,T)$$

A.2 Forces acting on pistons, proportional to pressure above minimum - (spreadsheets 2B and 3B)

$$F\left(p - p_{min}, T\right) = M \,.\, g \cos\theta \left(1 - \frac{\rho_a}{\rho_M}\right)$$

Mass per additional pascal

$$\frac{\partial F}{\partial M} = g \cos\theta \left(1 - \frac{\rho_a}{\rho_m}\right)$$

Density of added mass

$$\frac{\partial F}{\partial \rho_m} = \frac{M\,g \cos\theta.\rho_a}{\rho_M^2}$$

A.3 Contributions to the relative uncertainty of the total forces acting on the pistons, arising from tilt errors and from the corrections to the reference temperature - (spreadsheets 2C and 3C)

Basic equation:

$$\frac{F\left(p, T_r, 0\right)}{F\left(p, T, \theta\right)} = r = \frac{1}{\cos\theta.\{1 + \alpha\left(T - T_r\right)\}}$$

(r = correction factor for ratio of total forces)

Temperature of P/C

$$\frac{\partial r}{\partial T} = \frac{\alpha}{\cos\,\theta}$$

Area expansion coefficient of P/C

$$\frac{\partial r}{\partial \alpha} = \frac{\left(T - T_r\right)}{\cos\,\theta}$$

Tilt of axis of P/C

$$\frac{\partial r}{\partial \theta} = \frac{\sec\,\theta . \tan\,\theta}{1 + \alpha\left(T - T_r\right)}$$

A.4 Force difference at the minimum calibration pressure due to influences affecting both piston-cylinder assemblies - spreadsheet 4A

Equation 18 is the starting point for the derivation of the following sensitivity coefficients for the force differences, ΔF, resulting from factors affecting both the testee and the standard:

Density of air

$$\frac{\partial(\Delta F)}{\partial \rho_a} = g\left\{\cos\,\theta\left(-\frac{m_p}{\rho_p} - \frac{m_\omega}{\rho_\omega} - \frac{m}{\rho_m} + v\right) - h.A\left(p_{min}, T\right)\right\}_{testee}$$

$$- g\left\{\cos\,\theta\left(-\frac{m_p}{\rho_p} - \frac{m_\omega}{\rho_\omega} - \frac{m}{\rho_m} + v\right) - h.A\left(p_{min}, T\right)\right\}_{standard}$$

Density of fluid

$$\frac{\partial(\Delta F)}{\partial \rho_f} = g\left\{ - v.\cos\,\theta + h\,.\,A\left(p_{min}, T\right)\right\}_{testee}$$

$$- g\left\{ - v.\cos\,\theta + h\,.\,A\left(p_{min}, T\right)\right\}_{standard}$$

Fluid head correction

$$\frac{\partial(\Delta F)}{\partial h} = g \cdot \rho_f \left(1 - \frac{\rho_a}{\rho_f}\right)\left[A_t\left(p_{min}, T\right) - A_s\left(p_{min}, T\right)\right]$$

Acceleration due to gravity

$$\frac{\partial(\Delta F)}{\partial g} = \left[\cos\theta \left\{ m_p\left(1-\frac{\rho_a}{\rho_p}\right) + m_w\left(1-\frac{\rho_a}{\rho_w}\right) + m\left(1-\frac{\rho_a}{\rho_m}\right) - v\,\rho_f\left(1-\frac{\rho_a}{\rho_f}\right)\right\}\right.$$

$$\left. + \rho_f\left(1-\frac{\rho_a}{\rho_f}\right) . \; h \; . \; A(p_{min}, T)\right]_{testee} - \left[\cos\theta \left\{ m_p\left(1-\frac{\rho_a}{\rho_p}\right) + m_w\left(1-\frac{\rho_a}{\rho_w}\right)\right.\right.$$

$$\left.\left. + m\left(1-\frac{\rho_a}{\rho_m}\right) - v\rho_f\left(1-\frac{\rho_a}{\rho_f}\right)\right\} + \rho_f\left(1-\frac{\rho_a}{\rho_f}\right) h \; . \; A\ (p_{min}, T)\right]_{standard}$$

A.5 Force difference proportional to the amount by which the pressure exceeds the minimum calibration pressure, due to influences affecting both piston-cylinder assemblies - spreadsheet 4B

Equation 18 is the starting point for the derivation of the following coefficients:

Density of air

$$\frac{\partial \Delta F}{\partial \rho_a} = g\left(\cos\theta \; . \; \frac{M}{\rho_M}\right)_{testee} - g\left(\cos\theta \; . \; \frac{M}{\rho_M}\right)_{standard}$$

Acceleration due to gravity

$$\frac{\partial \Delta F}{\partial g} = \left[\cos\theta . M\left(1-\frac{\rho_a}{\rho_M}\right)\right]_{testee} - \left[\cos\theta . M\left(1-\frac{\rho_a}{\rho_M}\right)\right]_{standard}$$

A.6 Ratio of total forces, due to influences affecting both piston-cylinder assemblies - (spreadsheet 4C)

From equations 8 and 9:

$$\frac{F_t(p,T_r)}{F_s(p,T_r)} = \frac{F_t(p,T)}{F_s(p,T)} \cdot \frac{1 + \alpha_s(T_s - T_r)}{1 + \alpha_t(T_t - T_r)}$$

$$\approx \frac{F_t(p,T)}{F_s(p,T)} \cdot \left\{1 + \alpha_s(T_s - T_r)\right\} . \left\{1 - \alpha_t(T_t - T_r)\right\}$$

$$= \frac{F_t(p,T)}{F_s(p,T)} . f$$

where f is the temperature correction factor.

The sensitivity factor for uncertainty in correction factor f due to the type B uncertainty of the thermometers is given by:

$$\frac{\partial f}{\partial T_r} = \alpha_t - \alpha_s$$

A.7 The uncertainty in the calculated distortion coefficient - (spreadsheet 5)

Since

$$\lambda = \frac{1}{E}\left(2\nu + \frac{R^2}{R'^2 - R^2}\right)$$

$$\frac{\partial \lambda}{\partial E} = -\frac{1}{E^2}\left(2\nu + \frac{R^2}{R'^2 - R^2}\right)$$

$$\frac{\partial \lambda}{\partial \nu} = \frac{2}{E}$$

$$\frac{\partial \lambda}{\partial R} = \frac{2}{E} \cdot \frac{R.R'^2}{(R'^2 - R^2)^2}$$

and

$$\frac{\partial \lambda}{\partial R'} = -\frac{2}{E} \cdot \frac{R^2.R'}{(R'^2 - R^2)^2}$$

B EQUATIONS FOR FITTING A STRAIGHT LINE TO A SET OF PAIRS OF EXPERIMENTAL OBSERVATIONS

A convenient way of finding the best equation to characterise a set of experimental points is to use the method developed by Gauss. Only the case of a linear equation (straight line graph) will be considered here.

Suppose the independent variable is denoted by **x** and the dependent variable by **y**. In the context of this procedure **x** is usually the pressure **p**, while **y** is either the effective area of the testee at the reference temperature, $\mathbf{A_t(p,T_r)}$, or the ratio of the effective area of the testee to that of the standard at the pressure **p** and the reference temperature.

Suppose that in the general case the best fit straight line is given by the equation:

$$y = a + b.x$$

where **b** is the gradient and **a** is the intercept of the line with the **y** axis. The method of Gauss finds the equation of the straight line which has the lowest value for the sum of the squares of the differences between the experimental values of **y** and the values calculated, using that equation and the experimental values of **x**. In the version given below, all experimental observations have been given the same weighting.

Suppose that there are **n** pairs of experimental points. Then the gradient **b** of the best-fit straight line is given by:

$$b = \frac{n.\Sigma(x.y) - \Sigma\, x.\Sigma\, y}{n.\Sigma(x^2) - (\Sigma\, x)^2}$$

where the summations are for the **n** pairs of experimental points.

The intercept **a** of the best-fit straight line with the **y** axis is given by:

$$a = \frac{1}{n}\left[\Sigma\, y - \Sigma\, x \left(\frac{n.\Sigma\,(x.y) - \Sigma\, x.\ \Sigma\, y}{n.\Sigma\,(x^2) - (\Sigma\, x)^2}\right)\right]$$

$$= \frac{1}{n}\left[\, \Sigma y - b.\Sigma x \,\right]$$

The variance V of the residuals is given by:

$$V = \frac{1}{n(n-2)}\left[n.\Sigma x^2 - (\Sigma x)^2 - \frac{[n.\Sigma(x.y) - \Sigma x.\Sigma y]^2}{n.\Sigma(x^2) - (\Sigma x)^2} \right]$$

V_a, the variance of a is given by:

$$V_a = V\left[\frac{\Sigma\,(x^2)}{n.\Sigma\,(x^2) - (\Sigma\,x)^2}\right]$$

while V_b, the variance of **b** is given by:

$$V_b = V\left[\frac{n}{n.\Sigma\,(x^2) - (\Sigma\,x)^2}\right]$$

In the context of this procedure, the effective area extrapolated to zero applied pressure will be derived from **a**, the distortion coefficient will be derived from **b** and their standard uncertainties from V_a and V_b respectively.

C EXAMPLE OF UNCERTAINTY BUDGET FOR OIL OPERATED PRESSURE BALANCE CALIBRATED BY CROSS-FLOATING

1. BASIC DATA

1A. TESTED PRESSURE BALANCE (TESTEE)

Type of pressure balance tested	Budenberg K281, steel p/c, cast iron weights, brass weight-carrier
Approximate Ao of tested instrument	8.06E-05 m^2
Approx distortion coefficient of test inst	4.06 ppm/MPa
Maximum operating pressure	21.00 MPa
Minimum operating pressure	1.00 MPa

1B. STANDARD PRESSURE BALANCE

Type of standard pressure balance	Ruska 2481, size B3, WC p/c, stainless st weights, stainless st wt-carrier
Approximate Ao of standard	8.40E-05 m^2
Approx distortion coefficient of standard	-1.40 ppm/MPa
Maximum operating pressure	28.00 MPa
Minimum operating pressure	1.40 MPa

1C. CALIBRATION CONDITIONS

Reference temperature for Ao	20.00 degC
Maximum temperature difference from reference	2.00 degC
Maximum pressure of calibration	21.00 MPa
Minimum pressure of calibration	1.40 MPa
Total testee load at max calibration pressure	172.51 kg
Total testee load at min calibration pressure	11.50 kg
Total load on standard at max cal pressure	179.82 kg
Total load on standard at min cal pressure	11.99 kg
Value of g at calibration laboratory	9.81 m/s^2

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2B. UNCERTAINTY IN FORCE COMPONENT  INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS   UNCERTAINTY IN ADDITIONAL FORCE
PROPORTIONAL TO PRESSURE ABOVE MIN -----------------------------------------------    -------------------------------
     1           2         3        4        5        6         7        8          9         10          11
Influence    Influence   Approx  Rect dist Uncert at Uncert at Uncert at  Equiv     Coefft    Std devn    Variance
quantity       units      value  limits +/-  3 sigma  2.5 sigma  2 sigma  std devn  N/unit/Pa   N/Pa     (N/Pa)^2
----------------------------------- -----------------------------------------------    -------------------------------
Additional mass    kg/Pa 8.21E-06                              1.64E-11  8.21E-12   9.81E+00  8.06E-11    6.49E-21
Dens added-mass   kg/m^3 7.00E+03  3.50E+02                              2.02E+02   1.97E-12  3.99E-10    1.59E-19
----------------------------------- -----------------------------------------------    -------------------------------
Force uncertainty per    4.07E-10 N/Pa  (k=1)                                                             1.66E-19
additional Pa above min      5.05 ppm                 Force per additional pascal  8.06E-05  N

2C. UNCERTAINTY IN FORCE COMPONENT  INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS   REL. UNCERTAINTY IN TOTAL  RCE
    PROPORTIONAL TO TOTAL PRESSURE -----------------------------------------------    -------------------------------
     1           2         3        4        5        6         7        8          9         10          11
Influence    Influence   Approx  Rect dist Uncert at Uncert at Uncert at  Equiv     Coefft    Relative    Relative
quantity       units      value  limits +/-  3 sigma  2.5 sigma  2 sigma  std devn  rel/unit  std devn    variance
----------------------------------- ---------------------------------------------    -------------------------------
Temp of testee P/C     K 3.00E+02  2.00E-01                              1.15E-01   2.30E-05  2.66E-06   7.05E-12
Thermom type B uncert  K 0.00E+00                              5.00E-02  2.50E-02   2.30E-05  5.75E-07   3.31E-13
Area exp coeff testee /K 2.30E-05  1.50E-06                              8.66E-07   2.00E+00  1.73E-06   3.00E-12
Tilt of testee       rad 7.31E-04  7.31E-04                              4.22E-04   7.26E-04  2.67E-07   7.14E-14
----------------------------------- -----------------------------------------------    -------------------------------
Relative uncertainty     3.23E-06 (k=1)                                                                  1.05E-11
     in force                3.23 ppm
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3. TYPE B UNCERTAINTIES IN DOWNWARD FORCES ACTING ON P/C OF STANDARD

3A. UNCERTAINTY IN FORCE COMPONENT  INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS   UNCERTAINTIES IN DOWNWARD FORCES
   AT MINIMUM CALIBRATION PRESSURE ---------------------------------------------------  ----------------------------------
      1            2          3         4          5          6         7        8         9          10          11
Influence     Influence    Approx   Rect dist Uncert at Uncert at Uncert at   Equiv     Coefft      Force     Variance
quantity          units     value  limits +/-  3 sigma  2.5 sigma  2 sigma  std devn    N/unit    std dev/N    (N^2)
---------------------------------  ---------------------------------------------------  ----------------------------------
Mass of piston          kg 7.00E-03                                1.40E-08  7.00E-09  9.81E+00  6.87E-08   4.72E-15
Density of piston kg/m^3 1.50E+04  7.50E+02                                  4.33E+02  3.66E-10  1.59E-07   2.52E-14
Mass of wt carrier      kg 1.18E+01                                2.40E-08  1.20E-08  9.81E+00  1.18E-07   1.39E-14
Dens wt carrier   kg/m^3 7.80E+03  3.90E+02                                  2.25E+02  2.28E-06  5.14E-04   2.64E-07
Mass min added load     kg 1.81E-01                                3.62E-07  1.81E-07  9.81E+00  1.78E-06   3.15E-12
Dens min add load kg/m^3 7.80E+03  3.90E+02                                  2.25E+02  3.50E-08  7.88E-06   6.21E-11
Buoyancy vol piston    m^3 0.00E+00  7.00E-09                                  4.04E-09  8.67E+03  3.51E-05   1.23E-09
Datum height position    m 0.00E+00  2.00E-04                                  1.15E-04  7.29E-01  8.42E-05   7.08E-09
Reference pressure      Pa 0.00E+00  0.00E+00                                  0.00E+00  8.40E-05  0.00E+00   0.00E+00
Poss additional forces N 0.00E+00  1.00E-04                                  5.77E-05  1.00E+00  5.77E-05   3.33E-09
---------------------------------  ---------------------------------------------------  ----------------------------------
Force uncertainty at     5.25E-04 N  (k=1)                                                                    2.76E-07
min calibration pressure    4.47 ppm        Force at minimum calibration pressure      1.18E+02  N
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3B. UNCERTAINTY IN FORCE COMPONENT  INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS   UNCERTAINTY IN ADDITIONAL FORCE
PROPORTIONAL TO PRESSURE ABOVE MIN ----------------------------------------------------  -------------------------------
    1           2        3        4        5          6          7         8          9          10          11
Influence   Influence  Approx  Rect dist Uncert at Uncert at  Uncert at   Equiv      Coefft    Std devn    Variance
quantity      units     value  limits +/- 3 sigma  2.5 sigma   2 sigma   std devn   N/unit/Pa    N/Pa      (N/Pa)^2
---------------------------------- ---------------------------------------------- ---------  -------------------------------
Mass / additional Pa  kg 8.56E-06                              1.71E-11   8.56E-12   9.81E+00   8.40E-11    7.06E-21
Dens added-mass   kg/m^3 7.80E+03  3.90E+02                               2.25E+02   1.66E-12   3.73E-10    1.39E-19
---------------------------------- --------------------------------------------------------  -------------------------------
Force uncertainty per    3.82E-10 N/Pa  (k=1)                                                                  1.46E-19
additional Pa above min      4.55 ppm                   Force per additional pascal  8.40E-05 .

3C. UNCERTAINTY IN FORCE COMPONENT  INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS   REL. UNCERTAINTY IN TOTAL FORCE
    PROPORTIONAL TO TOTAL PRESSURE ----------------------------------------------------  -------------------------------
    1           2        3        4        5          6          7         8          9          10          11
Influence   Influence  Approx  Rect dist Uncert at Uncert at  Uncert at   Equiv      Coefft    Relative    Relative
quantity      units     value  limits +/- 3 sigma  2.5 sigma   2 sigma   std devn   rel/unit   std devn    variance
---------------------------------- ---------------- ---------------------- ---------  -------------------------------
Temp of standard P/C   K 3.00E+02  2.00E-01                               1.15E-01   1.04E-05   1.20E-06    1.44E-12
Thermom type B uncert  K 0.00E+00                              5.00E-02   2.50E-02   1.04E-05   2.60E-07    6.76E-14
Area exp coeff std    /K 1.04E-05  1.50E-06                               8.66E-07   2.00E+00   1.73E-06    3.00E-12
Tilt of standard     rad 7.31E-04  7.31E-04                               4.22E-04   7.29E-04   1.67E-07    7.14E-14
---------------------------------- --------------------------------------------------------  -------------------------------
Relative uncertainty     2.14E-06 (k=1)                                                                        4.58E-12
   in force                  2.14 ppm
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4. TYPE B UNCERTAINIES IN FORCES DUE TO INFLUENCES AFFECTING BOTH P/Cs

4A. UNCERT IN FORCE DIFFERENCE AT MINIMUM CALIBRATION PRESSURE			INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS					UNCERTAINTY IN FORCE DIFFERENCE		
1 Influence quantity	2 Influence units	3 Approx value	4 Rect dist limits +/-	5 Uncert at 3 sigma	6 Uncert at 2.5 sigma	7 Uncert at 2 sigma	8 Equiv std devn	9 Coefft N/unit	10 Force diff std dev/N	11 Variance (N^2)
Density of air	kg/m^3	1.20E+00	5.00E-02				2.89E-02	6.93E-05	2.00E-06	4.00E-12
Density of oil	kg/m^3	8.85E+02	1.00E+01				5.77E+00	5.42E-06	3.13E-05	9.81E-10
Fluid head correction	m	1.20E-01	2.00E-04				1.15E-04	2.97E-02	3.43E-06	1.17E-11
Surface tension	N/m	3.15E-02	3.15E-03				1.82E-03	6.67E-04	1.21E-06	1.47E-12
g	m/s^2	9.81E+00	2.00E-05				1.15E-05	4.88E-01	5.63E-06	3.17E-11
Uncertainty in force difference at min calibration pressure		3.21E-05 N (k=1)								1.03E-09

4B. UNCERT IN FORCE DIFFERENCE PROP TO PRESS ABOVE MINIMUM			INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS					UNCERTAINTY IN FORCE DIFFERENCE		
1 Influence quantity	2 Influence units	3 Approx value	4 Rect dist limits +/-	5 Uncert at 3 sigma	6 Uncert at 2.5 sigma	7 Uncert at 2 sigma	8 Equiv std devn	9 Coefft N/Pa/unit	10 Std devn N/Pa	11 Variance (N/Pa)^2
Density of air	kg/m^3	1.20E+00	5.00E-02				2.89E-02	7.57E-11	2.19E-12	4.78E-24
g	m/s^2	9.81E+00	2.00E-05				1.15E-05	3.48E-07	4.02E-12	1.62E-23
Uncert in force diff per additional Pa above min		4.58E-12 N/Pa (k=1)								2.09E-23

5. TYPE B UNCERTAINTY IN CALCULATED DISTORTION COEFFICIENT OF STANDARD

			INFLUENCE UNCERTAINTIES IN QUOTED UNITS & LEVELS					UNCERTAINTY IN DISTORTION COEFFT		
1 Influence quantity	2 Influence units	3 Approx value	4 Rect dist limits +/-	5 Uncert at 3 sigma	6 Uncert at 2.5 sigma	7 Uncert at 2 sigma	8 Equiv std devn	9 Coefft Pa^-1/unit	10 Dist coefft SD/Pa^-1	11 Variance (N^2)
Elastic modulus	N/m^2	5.83E+11	6.00E+10				3.46E+10	1.79E-24	6.20E-14	3.85E-27
Poisson's ratio		2.30E-01	2.00E-02				1.15E-02	3.43E-12	3.96E-14	1.57E-27
Cylinder outer radius	m	1.44E-02	1.00E-04				5.77E-05	1.13E-10	6.53E-15	4.26E-29
Cylinder inner radius	m	5.18E-03	2.50E-07				1.44E-07	4.07E-11	5.87E-18	3.45E-35
Uncert in calculated		7.39E-14	/Pa (k=1)							5.46E-27
distortion coefficient		0.074	ppm/MPa (k=1)			Calculated dist coeff		1.04E+00	ppm/MPa	

6. SUMMATION OF UNCERTAINTY COMPONENTS

1	2	3	4	5	6	7	8	9	10
Pressure (MPa)	Relative Uncert Ft ex 2A&B	Relative Uncert Ft ex 2C	Relative Uncert Fs ex 3A&B	Relative Uncert Fs ex 3C	Relative Uncert R ex2&3,AB&C	Relative Uncert R ex 4A&B	Relative Type A uncert	Relative uncert Ap std	Relative total uncert k=1
1.40	3.54E-06	3.23E-06	4.47E-06	2.14E-06	6.89E-06	2.84E-07	4.50E-06	8.80E-06	1.21E-05
3.36	3.29E-06	3.23E-06	3.24E-06	2.14E-06	6.03E-06	1.23E-07	3.60E-06	8.80E-06	1.13E-05
5.32	3.84E-06	3.23E-06	3.55E-06	2.14E-06	6.51E-06	8.57E-08	2.90E-06	8.80E-06	1.13E-05
7.28	4.13E-06	3.23E-06	3.78E-06	2.14E-06	6.81E-06	7.14E-08	2.40E-06	8.80E-06	1.14E-05
9.24	4.32E-06	3.23E-06	3.92E-06	2.14E-06	7.00E-06	6.46E-08	2.10E-06	8.80E-06	1.14E-05
11.20	4.44E-06	3.23E-06	4.02E-06	2.14E-06	7.14E-06	6.11E-08	2.00E-06	8.80E-06	1.15E-05
13.16	4.53E-06	3.23E-06	4.10E-06	2.14E-06	7.23E-06	5.91E-08	2.10E-06	8.80E-06	1.16E-05
15.12	4.59E-06	3.23E-06	4.15E-06	2.14E-06	7.31E-06	5.79E-08	2.40E-06	8.80E-06	1.17E-05
17.28	4.65E-06	3.23E-06	4.20E-06	2.14E-06	7.37E-06	5.70E-08	2.90E-06	8.80E-06	1.18E-05
19.04	4.68E-06	3.23E-06	4.23E-06	2.14E-06	7.41E-06	5.66E-08	3.60E-06	8.80E-06	1.21E-05
21.00	4.72E-06	3.23E-06	4.26E-06	2.14E-06	7.45E-06	5.63E-08	4.50E-06	8.80E-06	1.24E-05

7. ANALYSIS OF RELATIVE VARIANCES BY SOURCE

PRESSURE	1.4 MPa		11.2 MPa		21.0 MPa	
SOURCE	Variance	% total	Variance	% total	Variance	% total
Ap, standard	7.74E-11	53.3	7.74E-11	58.5	7.74E-11	50.6
Other Type B	4.76E-11	32.8	5.09E-11	38.5	5.54E-11	36.2
Type A	2.03E-11	13.9	4.00E-12	3.0	2.03E-11	13.2