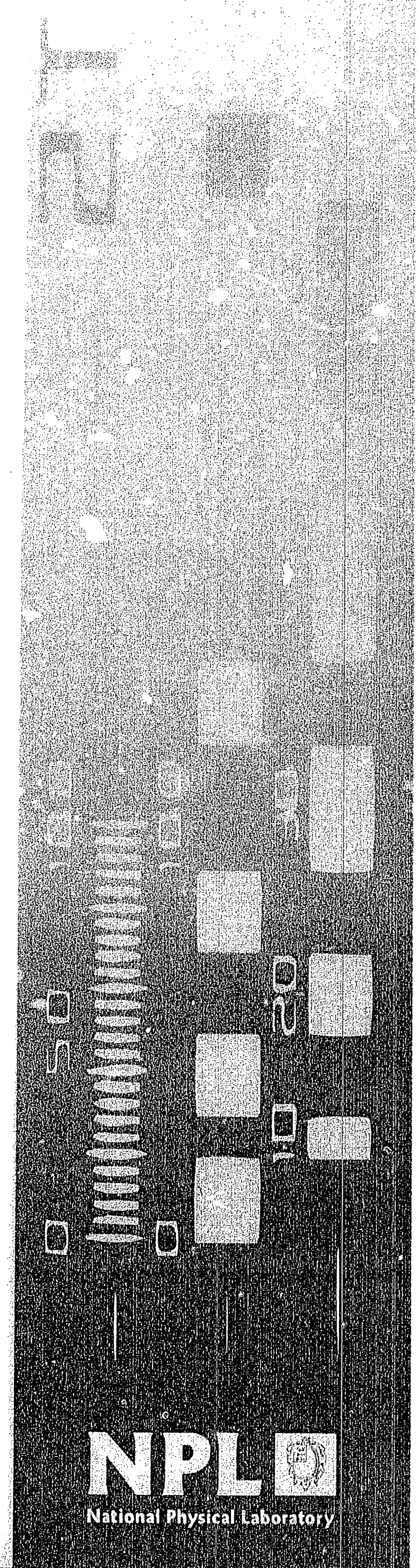
REPORT

A Comparison Of Hydrostatic Weighing Methods Used To Determine The Density Of Solid Artefacts

At the
National Physical Laboratory (NPL), UK
and
Institut National de Metrologie (BNM-INM), FR



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A Comparison Of Hydrostatic Weighing Methods Used To Determine The Density Of Solid Artefacts At The National Physical Laboratory (NPL), UK And Institut National de Metrologie (BNM-INM), FR

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ABSTRACT

The density of solid artefacts can be measured by using hydrostatic weighing techniques. This report describes a comparison of the performance of hydrostatic weighing apparatus and techniques used at the National Physical Laboratory, UK and the Institut National de Metrologie, France.

Each laboratory independently determined the density of four artefacts. The results and associated measurement uncertainties are compared and discussed. This work was undertaken as EUROMET Project N° 373.

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1 INTRODUCTION

This document reports on the results of a bi-lateral comparison between the National Physical Laboratory (NPL), UK and the Institut National de Métrologie (BNM-INM), FR of the ability to measure the density of solid artefacts by hydrostatic weighing (ie deducing the density from a knowledge of the mass of an artefact and its apparent mass when suspended in a liquid of known density).

Four artefacts were chosen for measurement, three of nominal mass 1000 g of differing densities, each measured using the same apparatus and one of nominal mass 100 g to be measured on a different apparatus.

2 DETAILS OF THE ARTEFACTS

2.1 ARTEFACT SUPPLIED BY NPL

<u>1000 g stainless steel cylinder A1</u>

Two solid stainless steel cylinders were purchased by NPL from Oertling, (UK) in December 1984 to act as transfer standards in a density intercomparison coordinated by the Bureau International des Poids et Mesures (BIPM). They are 'right' cylinders (ie of nominally equal height and diameter, and were manufactured from a single billet of Immaculate V[(1)] stainless steel with an approximate density of 7 890 kg m^{-3}. Their surface finish is comparable to that of a stainless steel mass standard. To identify the cylinders, one was engraved with 'A1' on its top face and the other 'A2'. Cylinder A1 was used in this comparison and has been designated as 1000_{A1} in this report.

2.2 ARTEFACTS SUPPLIED BY BNM-INM

<u>100 g Alacrite cylinder n° 16</u>

A right cylinder of XSH Alacrite[(2)] manufactured by BNM in 1983 from a billet provided by Albert et Duval (France) with an approximate density of 9 135 kg m^{-3}. The cylinder has been engraved with '16' on its side and has been designated as 100_{16} in this report.

<u>1000 g Alacrite cylinder n° 9</u>

A right cylinder of XSH Alacrite manufactured by BNM from a billet provided by Albert et Duval (France) with an approximate density of 9 148 kg m^{-3}. The cylinder has been engraved with '09' on its side and has been designated as 1000_{09} in this report. The surface finish of these two artefacts is comparable to that of an Alacrite mass standard.

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1 composition of Immaculate V, 52.78% Fe 23.5% Cr 21.5% Ni 1.3% Si 0.12% C 0.8% Mn

2 composition of Alacrite, 55% Co 20% Cr 15% W 10% Ni

1000 g Platinum-Iridium cylinder n° 44

A right cylinder of Platinum-Iridium[3], manufactured by BNM-INM from an old mass standard re-cast with an additional amount of Platinum-Iridium by Comptoir Lyon Allemand, with an approximate density of 21 502 kg m^{-3}. The cylinder has been engraved with '44' on its side and has been designated as 1000_{44} in this report. Its surface presents many scratches.

3 OUTLINE OF METHOD

The basic measurement procedure used at NPL for the measurement of solid density is to weigh an artefact in air to determine its true mass and then weigh the artefact in water to determine its volume and hence its density. An iterative process is used to perform these calculations. Having calculated a value for an artefact's volume from the hydrostatic weighings, the new volume is used to correct the air buoyancy calculations made at the time of the weighings in air, thus giving a more accurate value for the mass. This in turn is used to give a more accurate value of density from the hydrostatic weighings.

The formula for calculating the density is given by:

$$density,\ \rho = \frac{\rho_W}{(1 - M_W / M)}$$

where ρ_W is the density of the water at the time of weighing
M_W is the apparent mass of an artefact in water
M is the true mass of an artefact

Appendix 1 gives the derivation of this equation.

The procedure used at BNM-INM differs from that described above in that the apparent mass in air of an artefact is determined, not the true mass, thus avoiding the need to perform an iterative calculation.

4 WEIGHING IN AIR

Each laboratory independently determined the true mass for each of the artefacts[4] using appropriate balances and mass standards. Table 1 reports the results of these measurements together with the associated uncertainties.

3 composition of Platinum-Iridium, 90% Pt 10% Ir

4 with the exception that BNM-INM used a mass value for artefact 1000_{Al} supplied by NPL

5 HYDROSTATIC WEIGHING

5.1 DESCRIPTION OF THE APPARATUS

5.1.1 1 kg apparatus

GENERAL

The basic apparatus required for high-accuracy density measurements on 1 kg artefacts consists of a single-pan balance mounted over a temperature controlled water bath. A glass vessel is placed within the bath and into this pure water is added (see 5.2). An artefact can thus be suspended directly from the balance into the pure water and its apparent mass in the water determined.

The internal weights of the balance are not used other than as a counterpoise: all comparisons are made against stainless steel standards of known mass and density which are placed on the balance pan.

A thermometer, dew-point hygrometer and pressure gauge are used to measure the ambient conditions at the time of weighing in order that the air density can be determined and buoyancy corrections applied.

NPL APPARATUS

Figure 1 is a schematic diagram of the NPL 1 kg density apparatus. The balance is a Mettler H315 with a readability of 0.1 mg, mounted over a Tamson TXVMB70 temperature controlled bath. The pure water within the glass vessel is stirred using a peristaltic pump and a platinum resistance thermometer is used to measure the water temperature in close proximity to the artefact.

Figure 2 shows the arrangement of the weighing and support cradles together with the PRT and pump-tubing. All components of the cradles which come into contact with the artefact under test are manufactured from Nylon.

The suspension wire is formed using stainless steel wire with a diameter of 0.5 mm and is coated with platinum-black[5] to ensure good wetting of the suspension wire at the water/air interface.

BNM-INM APPARATUS

Figure 4 is a schematic diagram of the BNM-INM 1 kg density apparatus. The balance is a Mettler H315 with a readability of 0.1 mg, mounted over a Lauda RH 20 temperature controlled bath. The water in the outer temperature controlled jacket is stirred by a pump incorporated within the bath. A thermistor probe is used to measure the water temperature in close proximity to the artefact.

The suspension wire is formed using stainless steel wire with a diameter of 0.35 mm and is not treated.

5 "platinum-black" is the term used to describe a coating of Platinum/Lead soot deposited on the surface of the suspension wire by electrolysis

5.1.2 100 g apparatus

GENERAL

The basic apparatus used for the density measurements of smaller artefacts consists of a single-pan balance which has been modified so that an artefact may be suspended from the balance into a glass vessel containing pure water.

The internal weights of the balance are not used other than as a counterpoise: all comparisons are made against stainless steel weights of known mass and density which are placed on the balance pan.

A thermometer, dew-point hygrometer and pressure gauge are used to measure the ambient conditions at the time of weighing in order that the air-density can be determined and buoyancy corrections applied. A thermistor probe is used to measure the water temperature in close proximity to an artefact.

NPL APPARATUS

Figure 3 is a schematic diagram of the NPL 100 g density apparatus which was used to perform the hydrostatic weighing the 100 g artefact. The balance is a Mettler AT201 top-loading balance with a readability of 0.01 mg, modified with a Mettler density determination kit ME-210250. Unlike the 1 kg apparatus, there is no means by which the water temperature can be controlled.

The suspension wire is formed using stainless steel wire with a diameter of 0.5 mm and was coated with platinum-black.

BNM-INM APPARATUS

Figure 5 is a schematic diagram of the BNM-INM 100 g density apparatus. The balance is a Mettler HL 52 with a readability of 0.01 mg, mounted over a Lauda RH 20 temperature controlled bath. The water in the [illegible] temperature controlled jacket is stirred by a pump incorporated within the bath.

The suspension wire is formed using stainless steel wire with a diameter of 0.35 mm and is not treated.

5.2 PREPARATION OF THE WATER

NPL

Tap water was de-ionised and then distilled into five-litre glass flasks. The water was then poured vigorously into the glass vessel to saturate it with air. Both the flasks and glass vessel had previously been acid cleaned and rinsed with de-ionised water to avoid contaminating the water. A conductivity meter was used to check the purity of the water after the preparations had been completed and the value found to be in the region of 1 to 2 μS.

A fresh supply of water was prepared for each artefact and then its series of measurement runs performed (see 5.3.1). Artefacts 1000_{09} and 1000_{A1} each had three series of measurement runs performed on them and for each series a fresh supply of water was prepared.

BNM-INM

Tap water was de-ionised and then bi-distilled (using a PB 15 quartex infrared distiller) into glass flasks. After this treatment, the water is considered to be completely de-gassed. To correct for gas absorption at the time of the measurements, the number of days that have elapsed since the preparation of the water was introduced in the formula given by Bowman and Schoonover [1] and the correction applied. Both the flasks and vessel had been previously cleaned with distilled water.

A fresh supply of water was prepared for each set of hydrostatic weighings performed on an artefact (see 5.3.2).

5.3 WEIGHING PROCEDURE

5.3.1 NPL

1 kg density apparatus

The density of each artefact was determined through a series of measurement 'runs' where a single run consisted of five weighings made in the following sequence:

(a) weighing cradle + mass standards on balance pan
(b) weighing cradle + artefact
(c) weighing cradle + mass standards on balance pan
(d) weighing cradle + artefact
(e) weighing cradle + mass standards on balance pan

Each weighing involved recording five balance readings, arresting and releasing the balance between each one. The air temperature within the balance case, the pressure and the humidity were recorded during weighings (b) and (d).

From each measurement run, two values of the apparent mass of the artefact were calculated, after applying air buoyancy corrections to the value of the mass standards placed on the balance pan. The first value was based upon the balance reading recorded at (b) minus the mean of (a) and (c) and the second on (d) minus the mean of (c) and (e).

Measurement runs were made at three distinct temperatures (nominally 19 °C, 20 °C and 21 °C) to enable the thermal coefficient of cubic expansion to be determined for each artefact (see 6.2). Two runs were performed at each temperature making a total of six, yielding 12 measurements of the density. A fresh supply of water was used for each artefact.

Artefacts 1000_{A1} and 1000_{09} each had a further two series of measurement runs performed on them at a nominal temperature of 20 °C. A fresh supply of water was used for each series.

Immediately prior to a measurement run, a check weight (nominally 0.02 g) was added and removed from the balance pan to ensure systematic wetting of the suspension wire.

Tables A2.2 to A2.8, given in Appendix 2, give all the measurements recorded throughout each of the measurement runs, for all three artefacts, together with the calculated densities for each run.

Table 2 reports the final value of the density of each artefact, corrected to 20 °C, together with its estimated uncertainty. These values have been calculated by taking the mean of the densities calculated from each measurement run performed on an artefact.

100 g density apparatus

The weighing procedure was identical to that described for the 1 kg apparatus except that there is no means by which to control the water temperature. The density measurements of the Alacrite artefact 100_{16} have been corrected to 20 °C assuming its thermal coefficient of cubic expansion is the same as that measured for the artefact 1000_{09} on the 1 kg apparatus.

Six measurement runs were performed on this artefact. Table A2.1, given in Appendix 2, reports all the measurements recorded throughout each measurement run together with the calculated densities.

5.3.2 BNM-INM

1 kg density apparatus

The density of each artefact was determined by performing several sets of measurements where a set consisted of 10 measurements of an artefact's density. Each measurement was obtained by performing 3 weighings made in the following sequence:

(a) support + mass standards on balance pan
(b) support + artefact
(c) support + mass standards on balance pan

Each weighing consisted of recording a single balance reading. The air temperature within the balance case, the pressure and humidity readings were recorded during weighings (a), (b) and (c).

For each measurement, the apparent mass of the artefact was calculated after applying air buoyancy corrections to the value of the mass standards on the balance pan. The apparent mass was calculated from the balance reading recorded at (b) minus the mean of (a) and (c).

Eight sets of measurements were performed on the artefact 1000_{A1} (giving a total of 80 measurements of the density) and six sets on each of the artefacts 1000_{09} and 1000_{44}. A fresh supply of water was prepared for each set of measurements.

All the weighings were performed in water at a temperature of nominally 20 °C: the density measurements were corrected to their values at exactly 20 °C by using a previously determined value of the thermal coefficient of cubic expansion for each artefact. Table 2 gives the values of the thermal coefficient of cubic expansion used for each artefact.

Tables A3.10 to A3.29, given in Appendix 3, give all the readings recorded throughout each set of measurements, for all three artefacts, together with the calculated densities.

Table 2 reports the final value of the density of each artefact, calculated at 20 °C, together with its estimated uncertainty. These values have been calculated by taking the mean of the densities calculated from each set of measurements performed on an artefact.

100 g density apparatus

The weighing procedure was identical to that described for the 1 kg apparatus. Nine sets of measurements were performed on artefact 100_{16} , giving a total of 90 measurements of the density. Tables A3.1 to A3.9, given in Appendix 3, give all the readings recorded throughout each set of measurements together with the calculated densities.

6 CALCULATIONS

6.1 AIR AND WATER DENSITY

In the two laboratories, the air density was calculated from the measured parameters of pressure, temperature and dew-point using the equation recommended by BIPM[2].

NPL

For each hydrostatic weighing, the water density was calculated from its temperature at the time of weighing using the formula given by Bettin and Spieweck[3]. This formula is given for water that is de-gassed (ie air free), has an isotopic concentration defined by Standard Mean Ocean Water (ie SMOW) and is under one atmosphere of pressure. The following corrections are applied to take account of the conditions encountered at NPL.

(a) The water used at NPL is assumed to be saturated with air and so a correction, calculated using the formula given by Bignell[4], is applied to the calculated water density.

(b) The water density is corrected for the variation in ambient pressure from one atmosphere (ie 1013.25 mbar). This corresponds to a correction of + 0.000 05 kg m^{-3} per mbar increase in pressure and has been calculated according to the compressibility data given by Kell [5].

(c) Isotopic content of the water: from analysis of the isotopic content of the water at NPL[(6)] a correction of - 0.002 08 kg m^{-3} is applied to the calculated density.

(d) Depth of immersion: during the measurements made on the 1 kg apparatus the artefacts were immersed to a depth of approximately 20 cm. This resulted in a correction of + 0.000 9 kg m^{-3} being applied which has been calculated according to the compressibility data given by Kell. The depth of immersion on the 100 g apparatus is approximately 3 cm and resulted in a correction of + 0.000 1 kg m^{-3} being applied to the water density.

6 A sample of tap water was sent to BIPM in 1986 for analysis of its isotopic content

BNM-INM

For each hydrostatic weighing, the water density was calculated from its temperature at the time of weighing using the formula given by Watanabe [6]. This formula is given for water that is de-gassed, has an isotopic composition defined by SMOW and is under one atmosphere of pressure. The following corrections are applied to take account of the conditions encountered at BNM-INM:

(a) Immediately after bi-distillation the water is considered as de-gassed. To correct for gas absorption at the time of the measurements, the number of days that have elapsed since the preparation of the water was introduced in the formula given by Bowman and Schoonover and the correction applied.

(b) The depth of immersion of an artefact was approximately 30 cm and the correction was calculated according to the formula of Bowman and Schoonver.

6.2 THERMAL COEFFICIENT OF CUBIC EXPANSION

NPL determined the thermal coefficient of cubic expansion for each artefact measured on the 1 kg apparatus by performing the hydrostatic weighings at three distinct temperatures, nominally 19 °C, 20 °C and 21 °C. A regression analysis was performed on this data and the thermal coefficient of cubic expansion calculated.

BNM-INM used previously determined values of the thermal coefficient of cubic expansion for each of the artefacts they supplied: they used the NPL measured value for the artefact 1000_{Al}.

Table 2 gives a summary of the measured densities of each artefact along with the calculated thermal coefficients of cubic expansion.

7 ANALYSIS OF RESULTS

Table 2 reports the final value of the density of each of the artefacts together with the estimated uncertainty in these measurements, based on a standard uncertainty multiplied by a coverage factor k=2. From these results, it is clear that there is a systematic difference between the densities calculated by NPL and BNM-INM, with the NPL values always being lower. This information is summarised in the table below:

Artefact	Nominal density kg m^{-3}	Difference (NPL - INM) kg m^{-3}	NPL uncertainty kg m^{-3}	BNM-INM uncertainty kg m^{-3}
100_{16}	9 135	- 0.28	0.43	0.50
1000_{A1}	7 889	- 0.18	0.12	0.17
1000_{09}	9 148	- 0.20	0.17	0.080
1000_{44}	21 502	- 0.63	0.76	0.59

For the artefacts 1000_{A1} and 1000_{09}, the magnitude of the difference exceeds the maximum reported uncertainty. This is best demonstrated in Graph 1 which shows the density values reported by both laboratories together with bars representing the magnitude of the uncertainties. The measured difference appears to be approximately proportional to an artefact's density.

The source (or sources) of the systematic difference must originate within the hydrostatic weighings and not those performed in air as the difference in the true mass values determined by each laboratory would cause a difference of less than 0.01 kg m^{-3} in the density of an artefact.

Possible factors which could give rise to a systematic difference in the hydrostatic weighings are now discussed.

Water density

As described in 6.1, the general method by which the water density is calculated is to use a formula which gives the density as a function of temperature and is based on water that is air-free, has an isotopic concentration of Standard Mean Ocean Water and is under one atmosphere of pressure. Corrections are then applied to the density to allow for the conditions encountered at each laboratory.

Formulae

At NPL, the formula given by Bettin and Spieweck is used to calculate the water density at the temperature of the weighing, whilst at BNM-INM, the formula given by Watanabe is used. The NPL water densities have been re-calculated using the Watanabe formula and the artefact's densities re-calculated. These new values are given in the following table together with the difference between these values and that obtained by BNM-INM.

Artefact	NPL Bettin+Spieweck kg m^{-3}	NPL$_W$ Watanabe kg m^{-3}	BNM-INM kg m^{-3}	Difference (NPL$_W$ - INM) kg m^{-3}
100_{16}	9 134.94	9 135.00	9 135.22	- 0.22
1000_{A1}	7 889.81	7 889.86	7 889.99	- 0.13
1000_{09}	9 148.20	9 148.26	9 148.40	- 0.14
1000_{44}	21 502.29	21502.42	21502.92	- 0.50

Thus, for each artefact, the magnitude of the difference has been significantly reduced. This improvement can be seen in Graph 2 which shows the density values calculated using the Watanabe formula.

Gas content

As already stated, the formulae of Bettin and Spieweck and Watanabe apply to water that has been de-gassed (ie air free). At NPL, the water is assumed to be saturated with air, having been poured vigorously into the glass vessel before the hydrostatic weighings are performed. To correct the water density, the correction given by Bignell is applied (see 6.1). BNM-INM de-gas their water by bi-distillation and then apply a correction for gas re-absorption using the Bowman and Schoonover formula. The magnitude of this correction is approximately 0.002 5 kg m^{-3} between water that is saturated and that which is air-free. Even if the correction were not applied to the NPL water, the error arising in the measurement of the density of the artefact 1000_{A1} would be approximately +0.02 kg m^{-3}. Thus, the measurement and correction of the gas content of the water does not cause a significant difference in measured density.

Isotopic concentration, ambient pressure and depth of immersion

The corrections to the water density arising from the variation in ambient pressure, the isotopic concentration and the depth of immersion of an artefact (see 6.1) are all very small in magnitude and a significant error in the measurement of one or all of these parameters would not give rise to the difference observed.

Preparation of the water

At NPL, the artefacts 1000_{09} and 1000_{A1} each had three series of measurement runs performed on them and for each series, a fresh supply of water was prepared. As described in 5.2, the preparation of the water involves two stages: tap water is first de-ionised and then it is distilled. The de-ionisation of the water is achieved by trickling tap water through a cartridge which contains a resin which removes foreign particles and inorganic salts from the water.

For artefact 1000_{09}, two different cartridges were used to de-ionise the tap water, one supplied by Elgastat Ltd and the other by Kinetico. The table below gives the mean density calculated from each measurement series together with the corresponding standard deviation and the supplier of the de-ionising cartridge used for each series.

Artefact 1000_{09}				
Date	**Table**	**Density at 20 °C** kg m^{-3}	σ_{n-1} kg m^{-3}	**Supplier**
31:08:94	A2.5	9 148.256	0.023	Kinetico
14:09:94	A2.6	9 148.187	0.019	Elgastat
21:09:94	A2.7	9 148.168	0.031	Kinetico

The reproducibility of the measured densities is poor and the mean value of these three results is 9 148.204 kg m^{-3} with a standard deviation (σ_{n-1}) of 0.046 kg m . However, the poor reproducibility is not due to which cartridge is used de-ionise the water: the spread of the mean densities is greatest between the measurement series performed using the Kinetico cartridge. The same apparatus was used to distil the water for all the artefacts.

Much better agreement was obtained between the three series of measurement runs performed on the artefact 1000_{A1} which is summarised in the table below. The cartridge supplied by Kinetico was used for all three series of measurement runs.

Artefact 1000_{A1}				
Date	**Table**	**Density at 20 °C** kg m^{-3}	σ_{n-1} kg m^{-3}	**Supplier**
05:04:94	A2.2	7 889.801	0.018	Kinetico
19:04:95	A2.3	7 889.802	0.015	Kinetico
05:05:95	A2.4	7 889.814	0.033	Kinetico

The mean density of these three measurement runs is 7 889.806 kg m^{-3} with a standard deviation (σ_{n-1}) of 0.007 2 kg m^{-3}. Furthermore, these results are in good agreement with the densities reported in the CIPM intercomparison performed in 1986[7], which are :

NPL value 7 889.812 kg m^{-3}

Mean value of participating laboratories(7) 7 889.837 kg m^{-3}

NOTE: In order to compare the values given above with those in the table, approximately 0.02 kg $^{-3}$ must be subtracted from the above values as the water density was calculated using the formula given by Bigg[8] in the CIPM intercomparison.

7 Bureau International des Poids et Mesures (BIPM), Istituto di Metrologia "G. Colonnetti", Italy (IMGC), National Measurement Laboratory of CSIRO, Australia (NML), National Physical Laboratory, U.K. (NPL), Physikalisch-Technische Bundesanstalt, Germany (PTB).

Density formulae

Appendix 1 contains a derivation of the formula used by NPL to calculate the density of an artefact. This appendix also gives the formula used by BNM-INM and a proof that the two formulae are identical.

8 UNCERTAINTIES OF MEASUREMENT

NPL

Tables 3 to 6 give a summary of the uncertainty budget for each artefact. Each table gives the estimated uncertainty in the main measured parameters of air density, water density, true mass and apparent mass in water. All the contributions have been estimated at the k=1 level and summed by taking the Root Sum of the Squares.

BNM-INM

Tables 7 to 10 give a summary of the uncertainty budget for each artefact.

Table 2 reports the final value of the density of each artefact together with the estimated uncertainty, quoted at the k=2 level.

9 SUMMARY

A systematic difference between the density values measured by NPL and BNM-INM has been identified, which originates within the hydrostatic weighings and associated calculations. For the artefacts 1000_{A1} and 1000_{09}, the magnitude of this difference exceeds the maximum reported uncertainty and in one case exceeds the combined uncertainty.

The NPL water densities have been re-calculated using the thermal dilatation formula used by BNM-INM and new values for the artefacts' density obtained. This lead to a significant reduction in the magnitude of the systematic difference. For the artefacts 1000_{A1} and 1000_{09}, the difference was reduced by approximately 30% and is now less than the maximum reported uncertainty in both cases. No other significant contribution could be identified.

In December 1996, NPL is to participate in the EUROMET project 339 to determine the density of ceramic spheres. This will provide NPL with a good opportunity to further compare their ability to measure the density of solid artefacts.

10 ACKNOWLEDGEMENTS

The authors wish to thank Dr P.Pinot of BNM-INM, and Mr D.R.Armitage and Mr S.Davidson of NPL for their advice and assistance. The work of NPL on this intercomparison was funded by the Department of Trade and Industry, National Measurement System Policy Unit, under the National Measurement System Program for Mass, 1993-1996.

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Figure 1: NPL 1 kg density apparatus

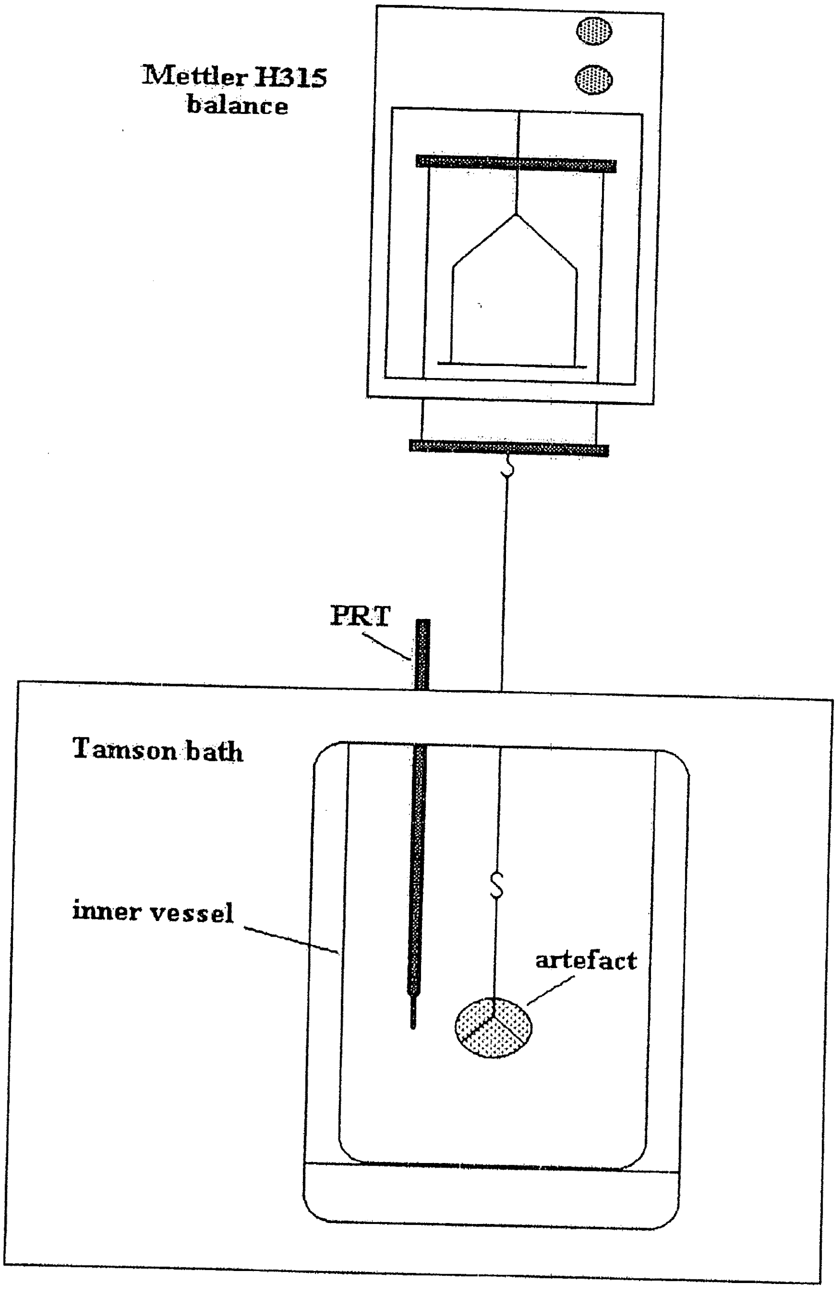


Figure 2: Artefact exchanger mechanism

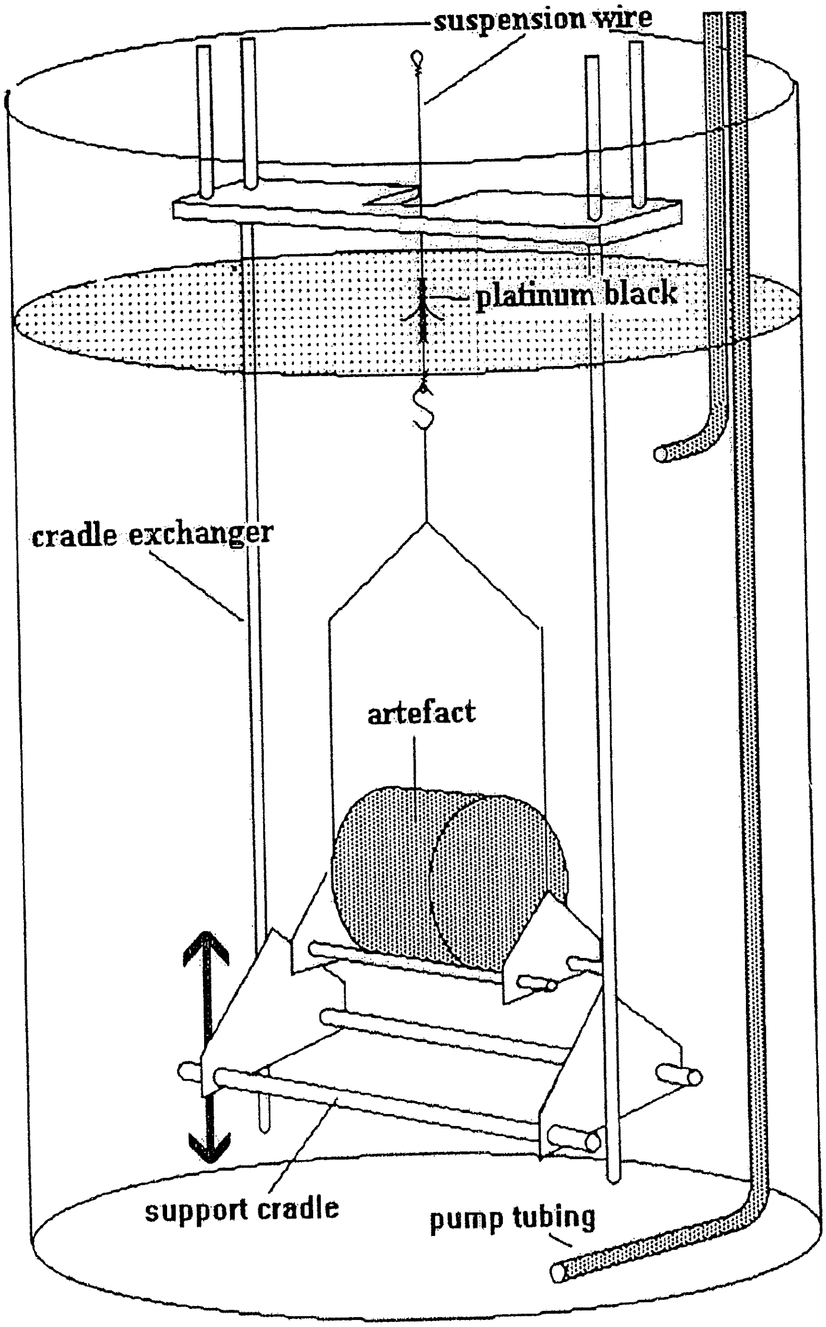


Figure 3: NPL 100 g density apparatus

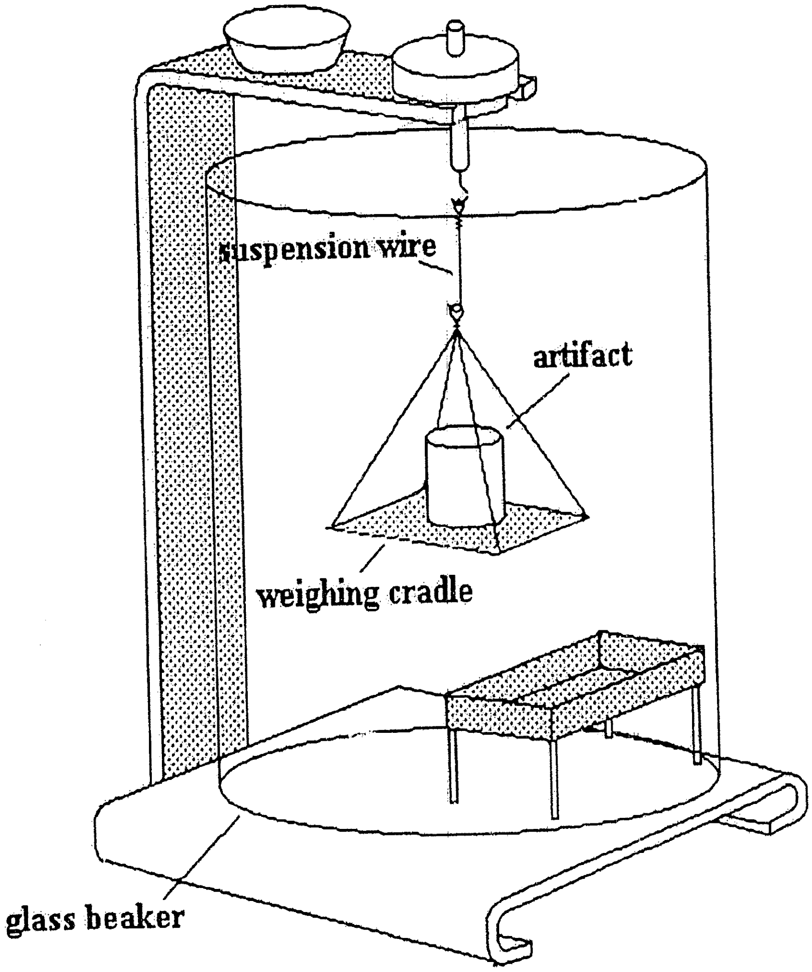


Figure 4 : BNM-INM 1 kg density apparatus

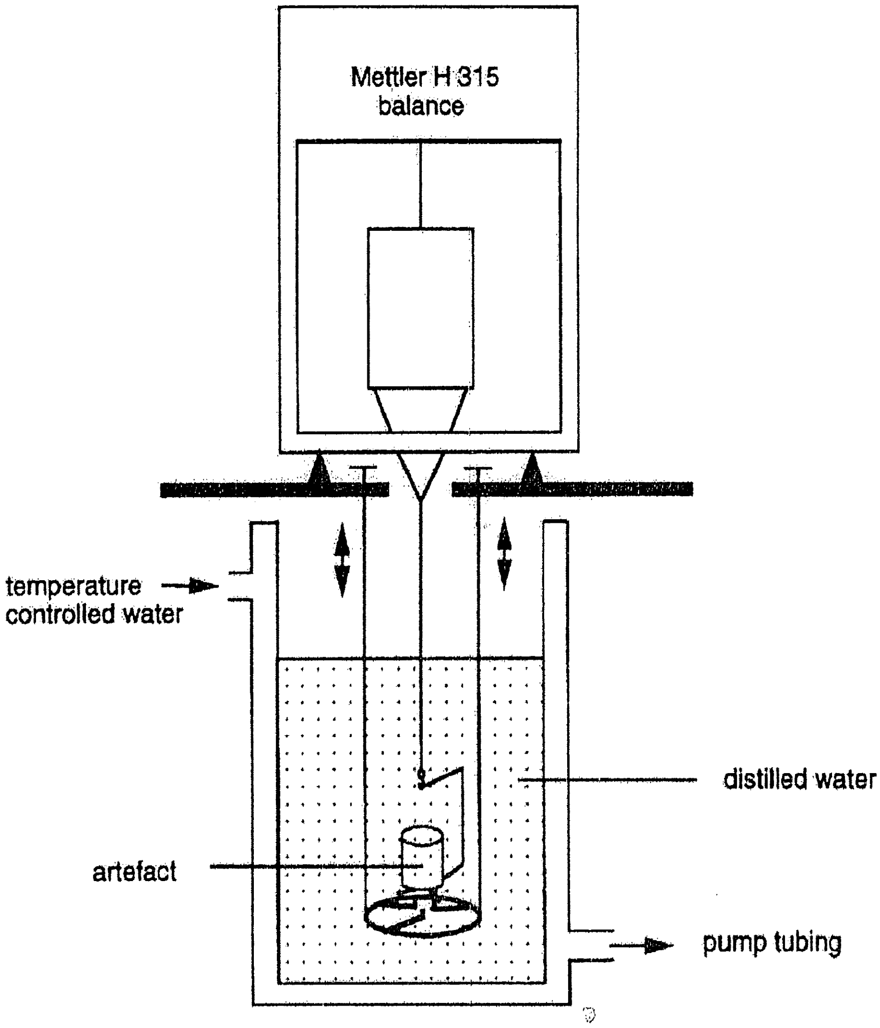


Figure 5 : BNM-INM 100 g density apparatus

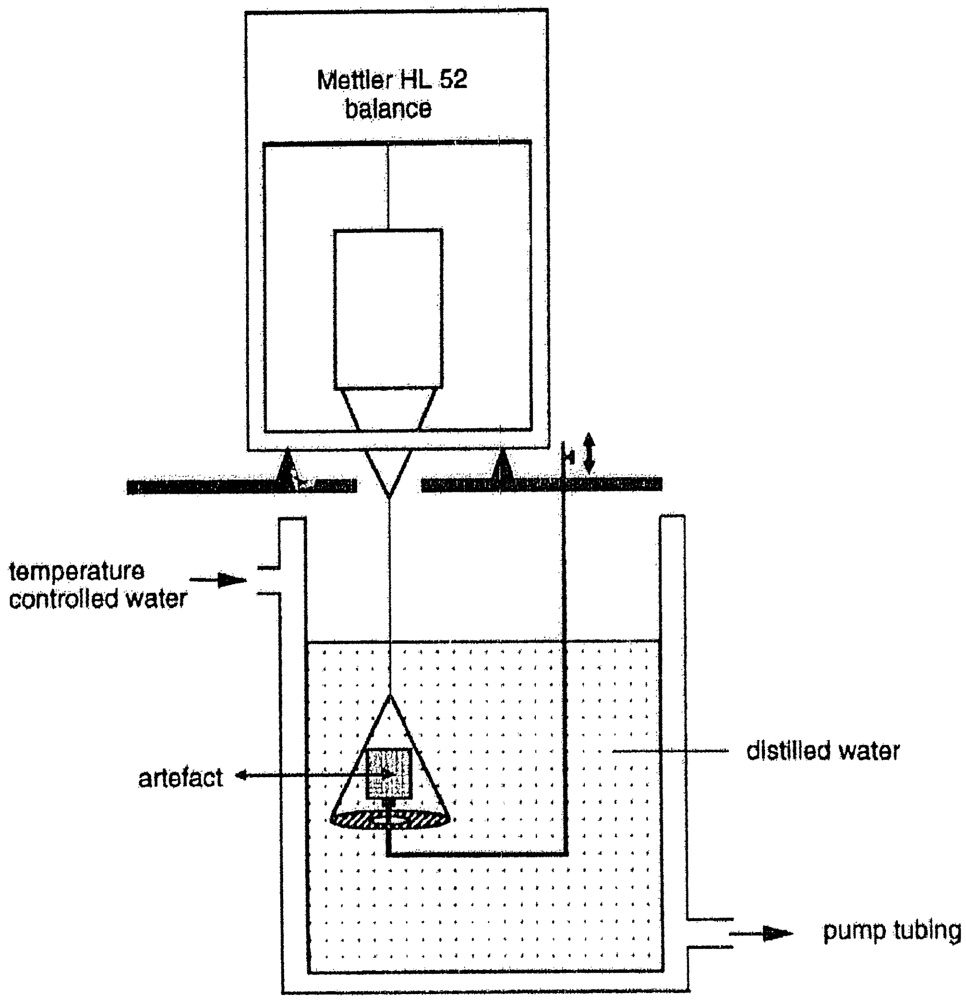


Table 1
TRUE MASS VALUES OF THE ARTEFACTS

artefact	NPL measurements			BNM-INM measurements		
	measurement date	mass value g	uncertainty µg	measurement date	mass value g	uncertainty µg
100_{16}	October 94	100.000 566	± 25	1980	100.000 557	± 35
1000_{A1}	March 94	999.557 636	± 200	-	999.557 636*	-
1000_{9}	July 94	1 000.001 761	± 200	April 94	1 000.001 811	± 12
1000_{44}	July 94	1 000.000 923	± 200	Feb 95	1 000.000 878	± 8

* the apparent mass in air of the artefact, M_a = 999.405 616 g, calculated using an air density of 1.2 kg m^{-3}, has been supplied by NPL

Table 2
DENSITY VALUES OF THE ARTEFACTS USING DIFFERENT DILATATION FORMULAE

artefact	NPL measurements			BNM-INM measurements			measured difference (NPL-INM)
	density at 20 °C kg m^{-3}	uncertainty kg m^{-3}	thermal coeff of cubic exp °C^{-1}	density at 20 °C kg m^{-3}	uncertainty kg m^{-3}	thermal coeff of cubic exp °C^{-1}	kg m^{-3}
100_{16}	9 134.94	0.43	0.000 038	9 135.22	0.50	0.000 039 3	- 0.28
1000_{A1}	7 889.81	0.12	0.000 046	7 889.99	0.17	0.000 046	- 0.18
1000_{9}	9 148.20	0.17	0.000 038	9 148.40	0.080	0.000 039 3	- 0.20
1000_{44}	21 502.29	0.76	0.000 026	21 502.92	0.59	0.000 026 0	- 0.63

Table 3

NPL UNCERTAINTY BUDGET - 100_{16}

Measured parameter	Type A uncertainties		Type B uncertainties	
	Actual	Contribution kg m^{-3}	Actual	Contribution kg m^{-3}
air density	-	-	± 0.000 55 kg m^{-3}	± 0.005
water density	-	-	± 0.014 kg m^{-3}	± 0.128
apparent mass	-	-	± 0.000 071 g	± 0.059
true mass	-	-	± 0.000 013 g	± 0.010
std deviation	-	± 0.160	-	-
Total RSS		± 0.160		± 0.141
Combined (k=1)				**± 0.213 kg m^{-3}**

Table 4

NPL UNCERTAINTY BUDGET - 1000_{A1}

Measured parameter	Type A uncertainties		Type B uncertainties	
	Actual	Contribution kg m^{-3}	Actual	Contribution kg m^{-3}
air density	-	-	± 0.000 84 kg m^{-3}	± 0.006
water density	-	-	± 0.004 5 kg m^{-3}	± 0.036
apparent mass	-	-	± 0.000 71 g	± 0.044
true mass	-	-	± 0.000 10 g	± 0.005
std deviation	-	± 0.007	-	-
Total RSS		± 0.007		± 0.058
Combined (k=1)				**± 0.116 kg m^{-3}**

Table 5 NPL UNCERTAINTY BUDGET - 1000_{09}				
Measured parameter	**Type A uncertainties**		**Type B uncertainties**	
	Actual	**Contribution kg m^{-3}**	**Actual**	**Contribution kg m^{-3}**
air density	-	-	± 0.000 84 kg m^{-3}	± 0.008
water density	-	-	± 0.004 5 kg m^{-3}	± 0.041
apparent mass	-	-	± 0.000 71 g	± 0.059
true mass	-	-	± 0.000 10 g	± 0.007
std deviation	-	± 0.046	-	-
Total RSS		± 0.046		± 0.073
Combined (k=1)				**± 0.086 kg m^{-3}**

Table 6 NPL UNCERTAINTY BUDGET - 1000_{44}				
Measured parameter	**Type A uncertainties**		**Type B uncertainties**	
	Actual	**Contribution kg m^{-3}**	**Actual**	**Contribution kg m^{-3}**
air density	-	-	± 0.000 84 kg m^{-3}	± 0.048
water density	-	-	± 0.004 5 kg m^{-3}	± 0.096
apparent mass	-	-	± 0.000 71 g	± 0.328
true mass	-	-	± 0.000 10 g	± 0.045
std deviation	-	± 0.151	-	-
Total RSS		± 0.151		± 0.348
Combined (k=1)				**± 0.379 kg m^{-3}**

Table 7

BNM-INM UNCERTAINTY BUDGET - 1000_{09}

Measured parameter	Uncertainties (kg m^{-3}) (type A method)	Uncertainties (kg m^{-3}) (type B method)
water density	0.002	0.023
apparent mass in water	0.010	0.010
temperature of water	0.001	0.004
standard deviation	0.030	-
Total RSS	0.032	0.025
combined uncertainties (k=1)	0.040	

Table 8

BNM-INM UNCERTAINTY BUDGET - 1000_{44}

Measured parameter	Uncertainties (kg m^{-3}) type A method)	Uncertainties (kg m^{-3}) (type B method)
water density	0.002	0.054
apparent mass in water	0.023	0.023
temperature of water	0.001	0.009
standard deviation	0.290	-
Total RSS	0.290	0.059
combined uncertainties (k=1)	0.296	

Table 9

BNM-INM UNCERTAINTY BUDGET - 100_{16}

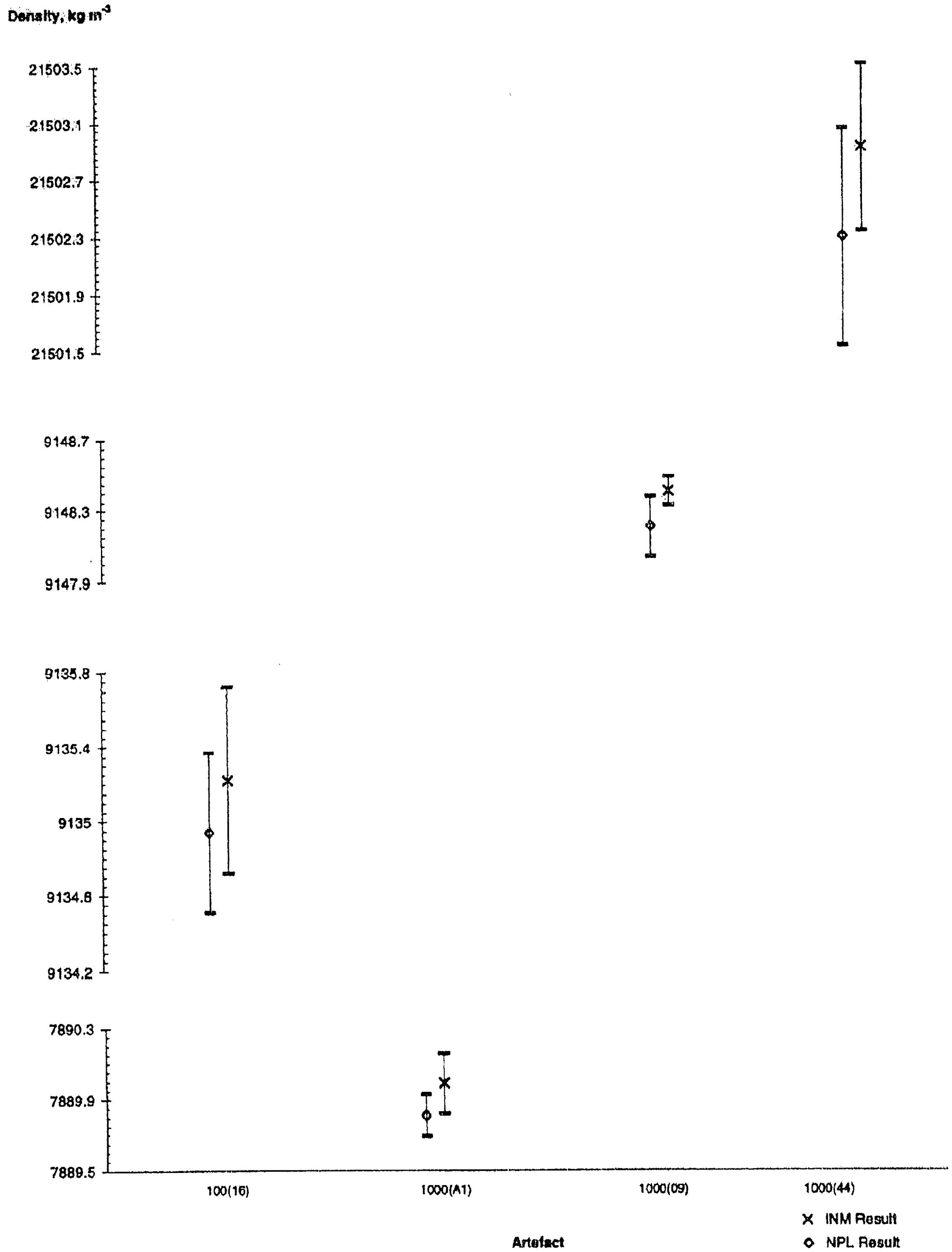
Measured parameter	Uncertainties (kg m^{-3}) (type A method)	Uncertainties (kg m^{-3}) (type B method)
water density	0.002	0.023
apparent mass in water	0.013	0.013
temperature of water	0.001	0.003
standard deviation	0.250	-
Total RSS	0.250	0.026
combined uncertainties (k=1)	0.252	

Table 10

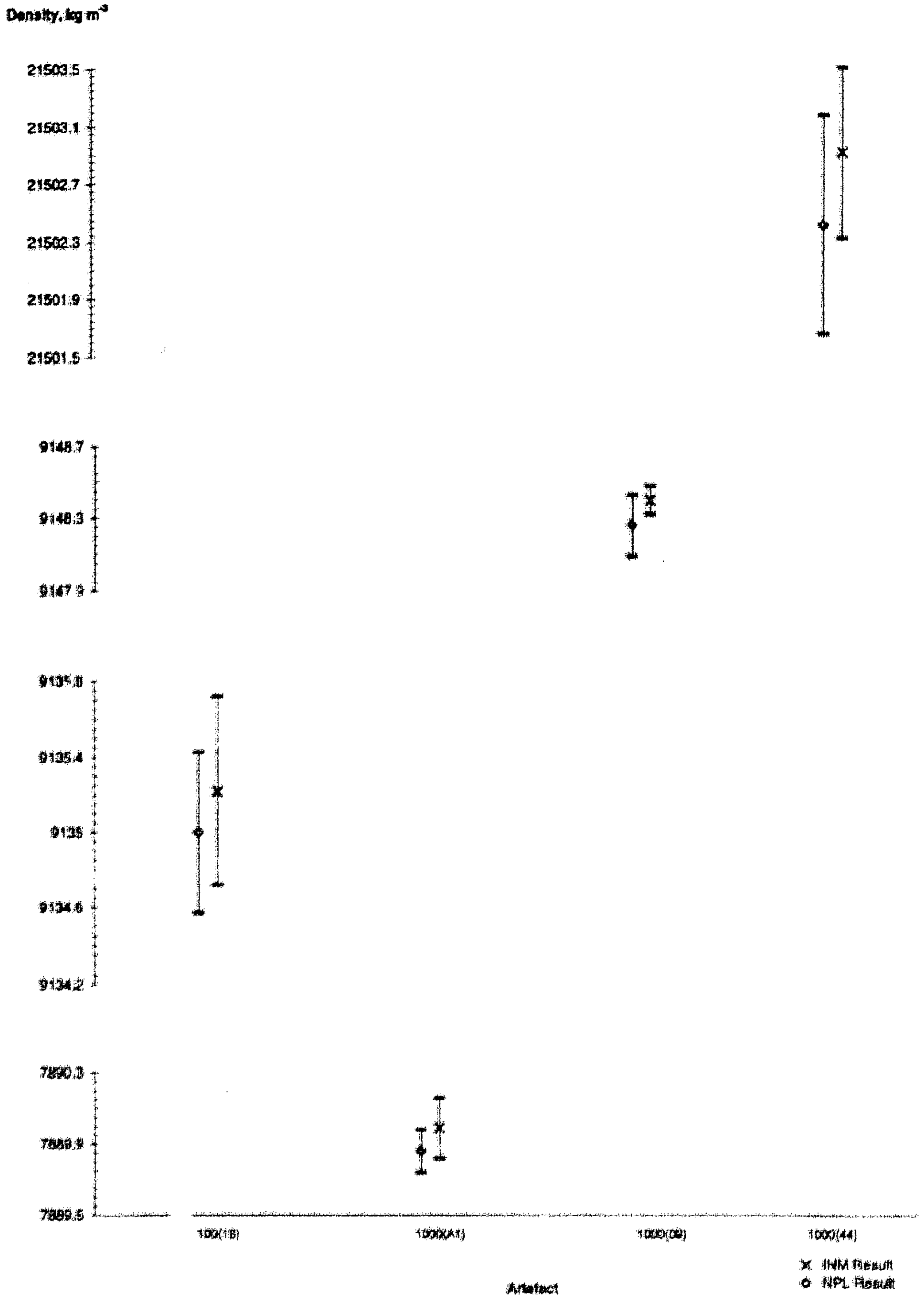
BNM-INM UNCERTAINTY BUDGET - 1000_{A1}

Measured parameter	Uncertainties (kg m^{-3}) (type A method)	Uncertainties (kg m^{-3}) (type B method)
water density	0.002	0.020
apparent mass in water	0.009	0.009
temperature of water	0.001	0.003
standard deviation	0.080	-
Total RSS	0.080	0.022
combined uncertainties (k=1)	0.083	

Graph 1: NPL (Bettin and Spelweck) v BNM-INM (Watanabe)



Graph 2: NPL (Watanabe) v BNM-INM (Watanabe)



APPENDIX 1: Derivation of the density formula

NPL FORMULA

The density of an artefact can be determined by measuring its apparent mass when suspended in water. The density is then calculated as:

$$\rho = \frac{\rho_W}{(1 - M_W / M)} \quad kg\ m^{-3} \ldots\ldots\ldots\ldots(1)$$

where ρ is the density of the artefact at temperature t °C in kg m^{-3}
ρ_W is the density of the water at temperature t °C in kg m^{-3}
M_W is the apparent mass of the artefact in the water at temperature t °C in kg
M is the true mass of the artefact in kg

DERIVATION

The density of an artefact is equal to its true mass divided by its volume (V)

$$\rho = \frac{M}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The volume of the artefact is given by the mass of the water (W) displaced by the artefact, divided by the density of the water

$$V = \frac{W}{\rho_W} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The mass of the water displaced by the artefact is equal to the difference between the true mass of the artefact and its apparent mass in the water. Hence (3) may be written as

$$V = \frac{M - M_W}{\rho_W} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Substituting (4) in equation (2) the expression for calculating the density becomes

$$\rho = \frac{M}{M - M_W} * \rho_W \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

This expression can be re-arranged to give the formula at (1).

APPARENT MASS

The apparent mass of an artefact in water is determined by comparing balance indications when the artefact is suspended from the balance with those with standards placed on the balance pan. However, the standards on the pan experience an upthrust acting on them due to the surrounding air. Thus, the apparent mass of an artefact in the water is given by the true mass value of the standards (M_S) less this buoyancy effect and can be calculated as:

$$M_W = M_S * (1 - \rho_A / \rho_S) \quad kgm^{-3} \text{.......(6)}$$

where ρ_A is the density of the air at the time of the weighing in kg m^{-3}
ρ_S is the density of the standards in kg m^{-3}

NOTE: If "conventional values" for the standards have been used then ρ_S is equal to 8000 kg m^{-3}.

BNM-INM FORMULA

BNM-INM calculate the density of an artefact using the formula:

$$\rho_{20} = \frac{M_A*\rho_W*(1 - \alpha*(t_A - 20)) - M_W*\rho_A*(1 - \alpha*(t_W - 20))}{(M_A - M_W)*(1 - \alpha*(t_A - 20))*(1 - \alpha*(t_W - 20))} \text{......(7)}$$

where ρ_{20} is the density of the artefact at 20 °C in kg m^{-3}
ρ_A is the density of the air during the weighings in air in kg m^{-3}
ρ_W is the density of the water at temperature t_W in kg m^{-3}
M_A is the apparent mass of the artefact in air in kg
M_W is the apparent mass of the artefact in water in kg
t_A is the temperature of the air during the weighings in air in °C
t_W is the temperature of the water in °C
α is the thermal coefficient of cubic expansion of the artefact in °C^{-1}

The NPL and BNM-INM formulae can be shown to be identical as follows:

Using the BNM-INM notation, the NPL equation (5) can be written as

$$M_W = M*(1 - \frac{\rho_W}{\rho_{t_W}})$$

giving $$M = \frac{M_W}{(1 - \frac{\rho_W}{\rho_{t_W}})} \quad ......(8)$$

where ρ_{t_W} is the density of an artefact at t_W, the temperature of the water

The apparent mass in air of an artefact is given by:

$$M_A = M*(1 - \frac{\rho_A}{\rho_{t_A}})$$

giving $$M = \frac{M_A}{(1 - \frac{\rho_A}{\rho_{t_A}})} \quad ......(9)$$

where ρ_{t_A} is the density of an artefact at t_A, the air temperature during the weighings in air

Equating (8) and (9) we have

$$\frac{M_A}{(1 - \frac{\rho_A}{\rho_{t_A}})} = \frac{M_W}{(1 - \frac{\rho_W}{\rho_{t_W}})}$$

$$M_A*(1 - \frac{\rho_W}{\rho_{t_W}}) = M_W*(1 - \frac{\rho_A}{\rho_{t_A}}) \quad ....(10)$$

Let $$\rho_{t_A} = \rho_{20}*(1 - \alpha\ (t_A - 20)) = \rho_{20}*constant\ K_A \quad ......(11)$$

$$\rho_{t_W} = \rho_{20}*(1 - \alpha\ (t_W - 20)) = \rho_{20}*constant\ K_W \quad ......(12)$$

where α is the thermal coefficient of cubic expansion of the artefact
ρ_{20} is the density of the artefact at 20 °C

Substituting for (11) and (12) into equation (10) we get

$$M_A*(1 - \frac{\rho_W}{\rho_{20}*K_W}) = M_W*(1 - \frac{\rho_A}{\rho_{20}*K_A})$$

$$M_A - \frac{M_A*\rho_W}{\rho_{20}*K_W} = M_W - \frac{M_W*\rho_A}{\rho_{20}*K_A}$$

Multiplying both sides by ρ_{20}, we get

$$\rho_{20}*M_A - \frac{M_A*\rho_W}{K_W} = \rho_{20}*M_W - \frac{M_W*\rho_A}{K_A}$$

$$\rho_{20}*(M_A - M_W) = \frac{M_A*\rho_W}{K_W} - \frac{M_W*\rho_A}{K_A}$$

$$\rho_{20} = \frac{M_A*\rho_W*K_A - M_W*\rho_A*K_W}{(M_A - M_W)*K_W*K_A} \quad \text{......(13)}$$

Substituting for K_A and K_W in equation (13), this is now identical to equation (7) and therefore the NPL and BNM-INM formuale are identical.

APPENDIX 2: Measurements made during NPL hydrostatic weighings

Table A2.1

Hydrostatic weighings on 100_{16}

Mass: 100.000 566 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density kg m^{-3}	apparent mass g	water temp t °C	water density kg m^{-3}	ratio of densities at t °C	density at 20 °C kg m^{-3}
1	23:12:94	1035.54	19.61	3.0	1.2292	89.070 274	18.20	998.5526	0.109 302 12	9135.087
		1035.50	19.68	3.1	1.2288	89.070 334	18.21	998.5508	0.109 301 52	9135.123
2	23:12:94	1035.10	19.52	3.0	1.2291	89.070 489	18.24	998.5450	0.109 299 97	9135.211
		1035.06	19.76	3.0	1.2280	89.070 367	18.25	998.5433	0.109 301 20	9135.094
3	23:12:94	1034.67	19.34	3.1	1.2293	89.070 259	18.24	998.5450	0.109 302 27	9135.019
		1034.74	19.16	3.3	1.2301	89.070 222	18.26	998.5413	0.109 302 64	9134.960
4	23:12:94	1034.78	19.19	3.3	1.2300	89.070 334	18.28	998.5376	0.109 301 52	9135.028
		1034.72	19.20	3.4	1.2299	89.070 296	18.28	998.5376	0.109 301 90	9134.995
5	23:12:94	1034.71	19.21	3.4	1.2298	89.069 941	18.26	998.5413	0.109 305 45	9134.725
		1034.73	19.22	3.5	1.2298	89.070 077	18.29	998.5358	0.109 304 10	9134.797
6	23:12:94	1034.62	19.23	3.5	1.2296	89.070 037	18.29	998.5358	0.109 304 49	9134.764
		1034.52	19.23	3.4	1.2295	89.070 078	18.30	998.5338	0.109 304 08	9134.786

Mean density at 20 °C = 9134.950 kg m^{-3}
σ_{n-1} = 0.160 kg m^{-3}

Table A2.2

Hydrostatic weighings on 1000_{A1}

Mass: 999.557 636 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density kg m^{-3}	apparent mass g	water temp t °C	water density kg m^{-3}	ratio of densities at t °C	density at 20°C kg m^{-3}
1	05:04:94	1005.41	20.82	5.5	1.1878	873.080 10	19.195	998.3611	0.126 533 51	7889.804
		1005.48	21.12	6.7	1.1863	873.080 09	19.188	998.3624	0.126 533 53	7889.811
2	06:04:94	1009.94	20.40	5.2	1.1949	873.079 71	19.164	998.3672	0.126 533 90	7889.816
		1010.01	20.82	6.3	1.1930	873.079 63	19.164	998.3672	0.126 533 98	7889.810
3	07:04:94	998.31	20.35	6.1	1.1811	873.098 06	20.081	998.1816	0.126 515 54	7889.826
		998.14	20.97	7.4	1.1780	873.098 38	20.080	998.1818	0.126 515 22	7889.846
4	07:04:94	998.08	22.42	7.4	1.1721	873.098 45	20.134	998.1706	0.126 515 15	7889.777
		998.17	22.78	8.4	1.1705	873.098 71	20.136	998.1702	0.126 514 89	7889.793
5	08:04:94	1005.98	22.02	3.9	1.1840	873.118 30	21.066	997.9724	0.126 495 29	7889.786
		1004.83	22.52	5.4	1.1803	873.118 75	21.052	997.9754	0.126 494 85	7889.836
6	08:04:94	1000.04	22.52	5.8	1.1745	873.118 280	21.047	997.9764	0.126 495 32	7889.811
		998.51	22.83	7.6	1.1709	873.118 372	21.047	997.9764	0.126 495 22	7889.817

Mean density at 20 °C = 7889.811 kg m^{-3}
σ_{n-1} = 0.020 kg m^{-3}

Table A2.3

Hydrostatic weighings on 1000_{A1}

Mass: 999.557 667 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density $kg\ m^{-3}$	apparent mass g	water temp t °C	water density $kg\ m^{-3}$	ratio of densities at t °C	density at 20°C $kg\ m^{-3}$
1	19:04:95	1003.62	20.77	3.0	1.1865	873.099 60	20.154	998.1649	0.126 514 03	7889.812
		1003.56	20.99	2.9	1.1856	873.099 18	20.164	998.1628	0.126 514 45	7889.773
2	19:04:95	1003.10	20.28	1.5	1.1882	873.099 45	20.158	998.1640	0.126 514 18	7889.797
		1003.06	20.64	2.1	1.1866	873.099 43	20.154	998.1649	0.126 514 20	7889.801
3	19:04:95	1002.86	20.49	2.0	1.1870	873.099 71	20.166	998.1623	0.126 513 91	7889.803
		1002.83	20.85	2.4	1.1854	873.099 79	20.170	998.1615	0.126 513 84	7889.803
4	19:04:95	1002.80	20.34	1.4	1.1876	873.100 08	20.167	998.1621	0.126 513 55	7889.825
		1002.92	20.69	1.8	1.1863	873.099 81	20.166	998.1623	0.126 513 82	7889.809

Mean density at 20 °C = 7889.803 kg m-3
σ_{n-1} = 0.015 $kg\ m^{-3}$

Table A2.4

Hydrostatic weighings on 1000_{A1}

Mass: 999.557 667 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density kg m^{-3}	apparent mass g	water temp t °C	water density kg m^{-3}	ratio of densities at t °C	density at 20°C kg m^{-3}
1	02:05:95	1024.47	20.59	9.6	1.2100	873.100 18	20.194	998.1576	0.126 513 44	7889.805
		1024.44	20.79	9.6	1.2092	873.099 58	20.204	998.1555	0.126 514 05	7889.755
2	02:05:95	1023.98	19.85	10.7	1.2121	873.100 99	20.218	998.1526	0.126 512 63	7889.825
		1023.98	20.26	10.6	1.2104	873.100 44	20.220	998.1521	0.126 513 19	7889.787
3	02:05:95	1023.38	19.66	8.2	1.2130	873.099 97	20.170	998.1625	0.126 513 66	7889.822
		1023.44	20.16	8.7	1.2109	873.100 42	20.173	998.1619	0.126 513 21	7889.846
4	02:05:95	1023.20	19.61	11.1	1.2120	873.100 78	20.186	998.1592	0.126 512 85	7889.852
		1023.16	20.20	10.9	1.2096	873.100 79	20.199	998.1565	0.126 512 84	7889.836

Mean density at 20 °C = 7889.816 kg m^{-3}
σ_{n-1} = 0.033 kg m^{-3}

Table A2.5

Hydrostatic weighings on 1000_9

Mass: 1000.001 761 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density $kg\ m^{-3}$	apparent mass g	water temp t °C	water density $kg\ m^{-3}$	ratio of densities at t °C	density at 20°C $kg\ m^{-3}$
1	31:08:94	1010.82	21.04	13.0	1.1906	890.873 97	19.213	998.3563	0.109 127 60	9148.249
		1010.56	21.20	12.8	1.1898	890.874 32	19.216	998.3557	0.109 127 24	9148.274
2	31:08:94	1008.38	20.61	14.3	1.1889	890.874 12	19.221	998.3546	0.109 127 44	9148.249
		1008.53	20.87	13.8	1.1883	890.874 21	19.224	998.3540	0.109 127 36	9148.252
3	01:09:94	1007.98	20.51	9.1	1.1909	890.891 31	20.176	998.1605	0.109 110 26	9148.243
		1008.10	20.43	12.9	1.1899	890.891 72	20.166	998.1626	0.109 109 85	9148.293
4	01:09:94	1010.22	20.38	14.4	1.1920	890.891 49	20.190	998.1577	0.109 110 08	9148.237
		1010.33	20.33	11.1	1.1937	890.891 43	20.181	998.1596	0.109 110 14	9148.246
5	02:09:94	1018.56	20.21	9.1	1.2047	890.907 60	21.044	997.9764	0.109 093 97	9148.223
		1018.53	20.38	12.7	1.2026	890.908 21	21.029	997.9796	0.109 093 36	9148.299
6	02:09:94	1018.12	20.20	7.6	1.2048	890.908 72	21.072	997.9703	0.109 092 85	9148.271
		1018.06	20.38	12.0	1.2023	890.908 20	21.060	997.9729	0.109 093 37	9148.247

Mean density at 20 °C = 9148.257 $kg\ m^{-3}$
σ_{n-1} = 0.023 $kg\ m^{-3}$

Table A2.6

Hydrostatic weighings on 1000_9

Mass: 1000.001 761 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density kg m^{-3}	apparent mass g	water temp t °C	water density kg m^{-3}	ratio of densities at t °C	density at 20°C kg m^{-3}
1	14:09:94	997.15	20.65	9.5	1.1774	890.889 07	20.107	998.1743	0.109 112 50	9148.158
		997.18	20.69	12.3	1.1762	890.889 47	20.101	998.1756	0.109 112 10	9148.201
2	14:09:94	995.17	20.43	13.7	1.1742	890.890 36	20.130	998.1694	0.109 111 21	9148.230
		994.96	20.57	9.2	1.1752	890.889 50	20.122	998.1711	0.109 112 07	9148.170
3	15:09:94	995.48	20.83	9.8	1.1746	890.889 53	20.118	998.1720	0.109 112 04	9148.178
		995.82	80.82	11.5	1.1744	890.889 47	20.112	998.1732	0.109 112 10	9148.183
4	15:09:94	998.66	21.07	11.0	1.1769	890.890 03	20.141	998.1673	0.109 111 54	9148.186
		998.77	21.17	11.1	1.1766	890.890 11	20.134	998.1688	0.109 111 46	9148.204
5	16:09:94	1009.82	20.81	9.1	1.1919	890.889 36	20.112	998.1739	0.109 112 21	9148.180
		1009.99	20.81	9.5	1.1920	890.889 48	20.108	998.1748	0.109 112 09	9148.197
6	16:09:94	1012.22	20.47	6.3	1.1970	890.889 18	20.110	998.1745	0.109 112 39	9148.170
		1012.35	20.50	9.5	1.1960	890.889 32	20.102	998.1761	0.109 112 25	9148.194

Mean density at 20 °C = 9148.188 kg m^{-3}
σ_{n-1} = 0.019 kg m^{-3}

Table A2.7

Hydrostatic weighings on 1000_9

Mass: 1000.001 761 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density kg m^{-3}	apparent mass g	water temp t °C	water density kg m^{-3}	ratio of densities at t °C	density at 20°C kg m^{-3}
1	21:09:94	1014.30	20.19	8.3	1.2000	890.888 85	20.122	998.1721	0.109 112 72	9148.124
		1014.38	20.33	12.9	1.1978	890.889 22	20.110	998.1746	0.109 112 35	9148.174
2	22:09:94	1020.93	20.33	12.4	1.2058	890.889 63	20.112	998.1745	0.109 111 94	9148.208
		1020.98	20.52	8.5	1.2065	890.889 19	20.108	998.1753	0.109 112 38	9148.178
3	23:09:94	1020.43	20.33	11.1	1.2057	890.889 24	20.132	998.1703	0.109 112 33	9148.144
		1020.45	20.57	12.4	1.5042	890.889 57	20.122	998.1724	0.109 112 00	9148.187

Mean density at 20 °C =9148.169 kg m^{-3}
σ_{n-1} = 0.030 kg m^{-3}

Table A2.8

Hydrostatic weighings on 1000_{44}

Mass: 1000.000 851 g

run	date	pressure mbar	air temp °C	dewpoint °C	air density kg m^{-3}	apparent mass g	water temp t °C	water density kg m^{-3}	ratio of densities at t °C	density at 20 °C kg m^{-3}
1	27:10:94	1003.82	20.43	7.2	1.1870	953.570 87	19.158	998.3668	0.046 429 94	21 502.18
		1003.95	20.48	7.4	1.1868	953.570 99	19.156	998.3673	0.046 429 82	21 502.25
2	28:10:94	1008.20	20.43	7.8	1.1920	953.571 76	19.154	998.3679	0.046 429 06	21 502.61
		1008.27	20.58	8.2	1.1913	953.571 07	19.158	998.3671	0.046 429 74	21 502.28
3	31:10:94	996.58	21.41	10.6	1.1733	953.579 00	20.120	998.1716	0.046 421 82	21 502.28
		996.61	21.55	12.3	1.1721	953.579 43	20.122	998.1712	0.046 421 38	21 502.47
4	31:10:94	996.53	21.34	9.9	1.1737	953.579 43	20.140	998.1674	0.046 421 38	21 502.40
		996.57	21.53	12.5	1.1720	953.578 63	20.142	998.1670	0.046 422 18	21 502.02
5	01:11:94	1015.72	21.31	9.3	1.1968	953.587 74	21.140	997.9553	0.046 413 07	21 502.24
		1015.86	21.45	7.2	1.1971	953.588 12	21.136	997.9562	0.046 412 70	21 502.43
6	01:11:94	1017.16	21.24	7.9	1.1993	953.588 14	21.160	997.9510	0.046 412 67	21 502.34
		1017.14	21.47	9.5	1.1978	953.588 10	21.153	997.9526	0.046 412 71	21 502.35

Mean density at 20 °C = 21 502.32 kg m^{-3}
σ_{n-1} = 0.15 kg m^{-3}

APPENDIX 3: Measurements taken during INM hydrostatic weighings

Table A3.1

Artefact 100_{16}, data set 1

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	24.128 24.143	20.037	1.1847 1.1846	-0.00061 -0.01551	998.1996	9135.714
2	24.153 24.172	20.037	1.1845 1.1844	-0.00050 -0.01590	998.1994	9135.155
3	24.177 24.182	20.037	1.1844 1.1844	-0.00005 -0.01594	998.1994	9135.059
4	24.190 24.193	20.038	1.1843 1.1843	-0.00035 -0.01587	998.1992	9135.328
5	24.164 24.160	20.039	1.1845 1.1845	-0.00056 -0.01578	998.1990	9135.384
6	24.181 24.182	20.037	1.1844 1.1844	-0.00031 -0.01603	998.1991	9135.076
7	24.184 24.179	20.040	1.1844 1.1844	-0.00032 -0.01577	998.1988	9135.262
8	24.174 24.195	20.040	1.1845 1.1844	-0.00024 -0.01614	998.1988	9135.145
9	24.202 24.197	20.043	1.1844 1.1844	-0.00078 -0.01617	998.1986	9135.323
10	24.196 24.180	20.043	1.1844 1.1845	-0.00073 -0.01583	998.1985	9135.283
	24.128		1.1846	-0.00001		

mean density at 20 °C = 9135.273 kg m^{-3}
standard deviation = 0.37 kg m^{-3}

Table A3.2

Artefact 100_{16}, data set 2

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.075 22.172	19.955 19.949	1.1934 1.1929	0.00075 -0.01478	998.2166	9135.280
2	22.273 22.356	19.959 19.960	1.1924 1.1920	0.00076 -0.01514	998.2154	9134.981
3	22.427 22.495	19.962 19.960	1.1916 1.1913	0.00074 -0.01495	998.2149	9135.535
4	22.579 22.651	19.967 19.954	1.1909 1.1906	-0.00018 -0.01480	998.2139	9135.956
5	22.737 22.809	19.982 19.983	1.1902 1.1898	0.00003 -0.0156	998.2106	9134.916
6	22.870 22.931	19.986 19.992	1.1896 1.1893	0.00066 -0.01567	998.2092	9134.630
7	22.972 23.015	19.993 19.997	1.1891 1.1889	0.00056 -0.01556	998.2079	9135.143
8	23.057 23.095	19.999 20.002	1.1887 1.1885	-0.00036 -0.01556	998.2069	9135.485
9	23.128 23.166	20.004 20.006	1.1884 1.1881	-0.00027 -0.01577	998.2060	9135.222
10	23.198 23.240	20.008 20.012	1.1880 1.1878	-0.00016 -0.01575	998.2046	9135.014
	22.075	20.018	1.1876	0.00025		

mean density at 20 °C = 9135.216 kg m^{-3}
standard deviation = 0.37 kg m^{-3}

Table A3.3

Artefact 100_{16}, data set 3

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.305 23.359	20.061 20.063	1.1702 1.1699	0.00000 -0.01520	998.1935	9135.691
2	23.407 23.452	20.064 20.064	1.1697 1.1695	-0.00028 -0.01534	998.1931	9135.391
3	23.534 23.582	20.065 20.066	1.1692 1.1689	0.00044 -0.01570	998.1927	9135.024
4	23.620 23.653	20.068 20.069	1.1687 1.1685	-0.00012 -0.01538	998.1922	9135.622
5	23.687 23.726	20.069 20.070	1.1683 1.1681	-0.00035 -0.01551	998.1920	9135.397
6	23.762 23.798	20.071 20.071	1.1680 1.1677	0.00016 -0.01552	998.1917	9135.302
7	23.842 23.871	20.073 20.074	1.1675 1.1675	-0.00014 -0.01543	998.1911	9135.443
8	23.918 23.944	20.075 20.075	1.1672 1.1671	0.00000 -0.01558	998.1908	9135.347
9	23.972 24.007	20.076 20.078	1.1671 1.1667	-0.00021 -0.01553	998.1905	9135.431
10	24.031 24.060	20.077 20.077	1.1666 1.1663	-0.00010 -0.01575	998.1905	9135.266
	23.305	20.078	1.1662	-0.00025		

mean density at 20 °C =9135.391 kg m^{-3}
standard deviation = 0.18 kg m^{-3}

Table A3.4 Artefact 100_{16}, data set 4						
series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.064 23.113	20.029 20.043	1.1935 1.1933	-0.00007 -0.01527	998.1987	9135.424
2	23.148 23.201	20.053 20.050	1.1930 1.1927	-0.00007 -0.01477	998.1975	9135.716
3	23.230 23.260	20.040 20.054	1.1924 1.1923	0.00022 -0.01451	998.1969	9135.712
4	23.296 23.333	20.057 20.061	1.1920 1.1917	0.00046 -0.01448	998.1951	9135.631
5	23.362 23.402	20.060 20.063	1.1915 1.1913	0.00045 -0.01510	998.1944	9135.232
6	23.462 23.490	20.064 20.061	1.1910 1.1908	0.00017 -0.01555	998.1943	9134.888
7	23.528 23.574	20.064 20.068	1.1906 1.1904	0.00038 -0.01486	998.1934	9135.407
8	23.611 23.643	20.070 20.074	1.1902 1.1900	0.00030 -0.01555	998.1925	9134.784
9	23.678 23.704	20.072 20.074	1.1899 1.1898	0.00048 -0.01548	998.1922	9134.722
10	23.739 23.775	20.073 20.075	1.1896 1.1894	0.00059 -0.01578	998.1918	9134.462
	23.064	20.077	1.1891	0.00050		

mean density at 20 °C = 9135.198 kg m^{-3}
standard deviation = 0.42 kg m^{-3}

Table A3.5

Artefact 100_{16}, data set 5

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.916 23.038	20.116 20.111	1.1806 1.1800	0.00004 -0.01492	998.1836	9135.561
2	23.112 23.175	20.110 20.105	1.1796 1.1792	0.00018 -0.01464	998.1850	9135.646
3	23.235 23.291	20.102 20.105	1.1788 1.1785	0.00044 -0.01483	998.1850	9135.149
4	23.348 23.409	20.111 20.113	1.1783 1.1779	0.00100 -0.01466	998.1835	9135.113
5	23.455 23.498	20.115 20.115	1.1776 1.1774	0.00085 -0.01440	998.1830	9135.109
6	23.538 23.590	20.116 20.116	1.1773 1.1770	0.00153 -0.01468	998.1826	9134.917
7	23.642 23.685	20.120 20.120	1.1767 1.1765	0.00075 -0.01483	998.1819	9135.067
8	23.751 23.794	20.122 20.123	1.1763 1.1759	0.00087 -0.01435	998.1813	9135.418
9	23.844 23.884	20.126 20.126	1.1756 1.1754	0.00087 -0.01461	998.1807	9135.242
10	23.918 23.949	20.127 20.127	1.1753 1.1751	0.00070 -0.01475	998.1805	9135.097
	22.916	20.128	1.1749	0.00094		

mean density at 20 °C = 9135.232 kg m^{-3}
standard deviation = 0.21 kg m^{-3}

Table A3.6

Artefact 100_{16}, data set 6

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.290 22.505	20.083 20.087	1.1762 1.1799	0.00017 -0.01441	998.189	9135.727
2	22.600 22.672	20.088 20.087	1.1794 1.1787	0.00080 -0.0141	998.1903	9135.729
3	22.736 22.793	20.064 20.068	1.1784 1.1782	0.00080 -0.01459	998.1917	9135.394
4	22.857 22.914	20.086 20.090	1.1780 1.1778	0.00066 -0.01424	998.1885	9135.691
5	22.971 23.033	20.089 20.079	1.1775 1.1773	0.00074 -0.01479	998.1890	9135.176
6	23.103 23.158	20.090 20.091	1.1769 1.1767	0.00081 -0.01460	998.1878	9135.273
7	23.218 23.272	20.093 20.094	1.1763 1.1758	0.00088 -0.01477	998.1872	9135.003
8	23.335 23.389	20.097 20.096	1.1767 1.1764	0.00111 -0.01476	998.1867	9135.057
9	23.440 23.498	20.097 20.093	1.1761 1.1746	0.00076 -0.01472	998.1865	9135.063
10	23.577 23.627	20.103 20.103	1.1742 1.1740	0.00120 -0.01497	998.1855	9134.704
	23.290	20.102	1.1738	0.00112		

mean density at 20 °C = 9135.282 kg m^{-3}
standard deviation = 0.33 kg m^{-3}

Table A3.7

Artefact 100_{16}, data set 7

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.162 21.233	20.028 20.026	1.1951 1.1947	0.00003 -0.01548	998.2017	9135.154
2	21.348 21.405	20.025 20.024	1.1942 1.1939	0.0001 -0.0155	998.2021	9135.097
3	21.472 21.526	20.023 20.021	1.1937 1.1935	0.00015 -0.01594	998.2027	9134.720
4	21.590 21.658	20.019 20.019	1.1931 1.1929	0.00014 -0.01584	998.2032	9134.816
5	21.742 21.823	20.018 20.017	1.1926 1.1923	0.00014 -0.01563	998.2033	9134.947
6	21.905 21.987	20.019 20.017	1.1920 1.1917	0.00026 -0.01540	998.2033	9135.070
7	22.061 22.133	20.019 20.019	1.1913 1.1911	0.00032 -0.01533	998.2031	9135.011
8	22.221 22.267	20.020 20.021	1.1906 1.1904	0.00055 -0.01519	998.2027	9134.958
9	22.344 22.422	20.023 20.024	1.1901 1.1897	0.00073 -0.01517	998.2022	9135.001
10	22.481 22.523	20.024 20.025	1.1894 1.1893	0.00049 -0.01502	998.2018	9135.424
	21.162	20.028	1.1891	0.00002		

mean density at 20 °C = 9135.020 kg m^{-3}
standard deviation = 0.19 kg m^{-3}

Table A3.8

Artefact 100_{16}, data set 8

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.083 22.174	20.059 20.061	1.1903 1.1900	0.00007 -0.01532	998.1947	9135.258
2	22.285 22.359	20.060 20.062	1.1895 1.1892	0.0001 -0.01525	998.1944	9135.295
3	22.444 22.507	20.061 20.061	1.1888 1.1885	0.00013 -0.01517	998.1944	9135.338
4	22.587 22.661	20.061 20.063	1.1883 1.1881	0.00017 -0.0152	998.1942	9135.303
5	22.747 22.815	20.063 20.064	1.1877 1.8750	0.00016 -0.01525	998.1938	0135.234
6	22.881 22.936	20.065 20.064	1.1872 1.1869	0.00024 -0.01535	998.1937	9135.141
7	22.983 23.025	20.064 20.064	1.1868 1.1867	0.00019 -0.01529	998.1937	9135.256
8	23.067 23.104	20.065 20.067	1.1865 1.1863	0.00009 -0.01544	998.1933	9134.934
9	23.136 23.174	20.068 20.067	1.1862 1.1860	0.00066 -0.01521	998.1932	9135.111
10	23.213 23.249	20.067 20.068	1.1858 1.1857	0.00013 -0.01529	998.1929	9135.237
	22.083	20.070	1.1855	0.0002		

mean density at 20 °C = 9135.211 kg m^{-3}
standard deviation = 0.12 kg m^{-3}

Table A3.9

Artefact 100_{16}, data set 9

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.319 21.419	20.045 20.044	1.1927 1.1923	0.00001 -0.01546	998.1979	9135.192
2	21.490 21.589	20.045 20.043	1.1920 1.1916	0.00005 -0.01538	998.1981	9135.303
3	21.668 21.727	20.044 20.042	1.1913 1.1910	-0.00008 -0.01526	998.1983	9135.444
4	21.801 21.879	20.043 20.042	1.1907 1.1906	-0.00005 -0.0153	998.1984	9135.319
5	21.970 22.038	20.042 20.041	1.1902 1.1899	0.00015 -0.01542	998.1986	9135.136
6	22.091 22.138	20.042 20.043	1.1897 1.1894	0.00018 -0.01556	998.1985	9134.980
7	22.186 22.229	20.042 20.041	1.1893 1.1892	0.00025 -0.01554	998.1985	9135.008
8	22.271 22.297	20.043 20.040	1.1890 1.1889	0.00016 -0.0155	998.1987	9135.070
9	22.336 22.367	20.041 20.041	1.1887 1.1886	0.00019 -0.015478	998.1987	9135.104
10	22.408 22.460	20.041 20.039	1.1884 1.1882	0.00013 -0.01531	998.1989	9135.299
	21.319	20.040	1.1881	0.00005		

mean density at 20 °C = 9135.186 kg m^{-3}
standard deviation = 0.15 kg m^{-3}

Table A3.10

Artefact 1000_{A1}, data set 1

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	24.802 24.830	20.065 20.067	1.1832 1.1832	0.7362 0.9450	998.1936	7889.889
2	24.866 24.893	20.066 20.066	1.1830 1.1829	0.7375 0.9450	998.1936	7889.844
3	24.902 24.927	20.065 20.066	1.1828 1.1827	0.7377 0.9454	998.1936	7889.865
4	24.947 24.966	20.066 20.066	1.1824 1.1822	0.7377 0.9455	998.1936	7889.895
5	24.979 25.007	20.065 20.065	1.1822 1.1821	0.7370 0.9449	998.1936	7889.884
6	25.026 25.058	20.067 20.066	1.1820 1.1812	0.7369 0.9446	998.1936	7889.884
7	25.003 25.028	20.065 20.064	1.1821 1.1819	0.7364 0.9446	998.1939	7889.889
8	25.053 25.082	20.064 20.065	1.1819 1.1818	0.7368 0.9447	998.1939	7889.894
9	25.087 25.104	20.064 20.064	1.1816 1.1815	0.7365 0.9446	998.1940	7889.899
10	25.119 25.128	20.064 20.064	1.1815 1.1814	0.7365 0.9443	998.1940	7889.880
	24.802	20.064	1.1813	0.7365		

mean density at 20 °C = 7889.882 kg m^{-3}
standard deviation = 0.02 kg m^{-3}

Table A3.11

Artefact 1000_{A1}, data set 2

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.854 23.898	20.021 20.021	1.1824 1.1822	0.7344 0.9417	998.2029	7889.911
2	23.950 23.954	20.021 20.021	1.1819 1.1820	0.7342 0.9424	998.2028	7889.965
3	23.989 24.008	20.021 20.021	1.1818 1.1829	0.7341 0.9435	998.2027	7990.020
4	24.028 24.054	20.021 20.021	1.1816 1.1815	0.7346 0.9429	998.2027	7889.955
5	24.076 24.091	20.021 20.022	1.1814 1.1812	0.7351 0.9431	998.2026	7889.93
6	24.106 24.132	20.022 20.022	1.1812 1.1810	0.7358 0.9436	998.2025	7889.947
7	24.151 24.173	20.022 20.022	1.1809 1.1808	0.7356 0.9434	998.2025	7889.948
8	24.179 24.200	20.022 20.021	1.1809 1.1809	0.7354 0.9434	998.2025	7889.957
9	24.206 24.227	20.023 20.023	1.1808 1.1808	0.7353 0.9433	998.2024	7889.950
10	24.213 24.240	20.023 20.024	1.1808 1.1807	0.7354 0.9433	998.2023	7889.953
	23.854	20.023	1.1808	0.7352		

mean density at 20 °C = 7889.954 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.12

Artefact 1000_{A1}, data set 3

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.388 21.466	20.002 19.991	1.2093 1.2088	0.7321 0.9438	998.2087	7890.061
2	21.553 21.608	19.991 19.967	1.2084 1.2082	0.7321 0.9444	998.2117	7890.111
3	21.705 21.777	19.982 19.981	1.2078 1.2074	0.7325 0.9446	998.2113	7890.088
4	21.858 21.921	19.983 19.956	1.2069 1.2066	0.7333 0.9453	998.2132	7890.099
5	22.014 22.077	19.980 19.974	1.2061 1.2058	0.7341 0.9453	998.2122	7890.056
6	22.273 22.330	19.981 19.982	1.2050 1.2047	0.7346 0.9448	998.2111	7890.046
7	22.395 22.446	19.987 19.989	1.2040 1.2042	0.7334 0.9436	998.2096	7889.995
8	22.517 22.563	19.995 19.997	1.2038 1.2036	0.7336 0.9444	998.2081	7890.023
9	22.647 22.696	20.002 20.002	1.2032 1.2029	0.7339 0.9442	998.2069	7890.000
10	22.761 22.804	20.007 20.009	1.2026 1.2025	0.7338 0.9437	998.2059	7889.971
	21.388	20.010	1.2021	0.7337		

mean density at 20 °C = 7890.045 kg m^{-3}
standard deviation = 0.04 kg m^{-3}

Table A3.13

Artefact 1000_{A1}, data set 4

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.395 21.474	19.952 19.953	1.1989 1.1985	0.7307 0.9416	998.2171	7890.079
2	21.591 21.678	19.952 19.946	1.1980 1.1975	0.7323 0.9409	998.2182	7890.030
3	21.776 21.857	19.943 19.941	1.1970 1.1967	0.7313 0.9423	998.2197	7890.114
4	22.005 22.067	19.936 19.933	1.9594 1.1956	0.7329 0.9436	998.2205	7890.135
5	22.160 22.227	19.939 19.937	1.1955 1.9485	0.7336 0.9443	998.2201	7890.118
6	22.315 22.390	19.937 19.937	1.1943 1.1940	0.7349 0.9448	998.2201	7890.138
7	22.449 22.488	19.938 19.945	1.1937 1.1935	0.7341 0.9446	998.2190	7890.132
8	22.647 22.695	19.946 19.949	1.1927 1.1925	0.7346 0.9445	998.2179	7890.089
9	22.798 22.849	19.950 19.952	1.1919 1.1917	0.7352 0.9448	998.2169	7890.087
10	22.925 22.992	19.958 19.958	1.1914 1.1911	0.7352 0.9442	998.2155	7890.047
	21.395	19.965	1.1907	0.7351		

mean density at 20 °C = 7890.097 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.14

Artefact 1000_{A1}, data set 5

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.958 21.981	20.010 20.012	1.7326 1.1731	0.7339 0.9412	998.2039	7889.944
2	22.005 22.032	20.013 20.017	1.1730 1.1729	0.7346 0.9418	998.2033	7889.972
3	22.066 22.092	20.015 20.013	1.1727 1.1727	0.7341 0.9415	998.2035	7890.010
4	22.133 22.150	20.013 20.015	1.1724 1.1723	0.7329 0.9411	998.2033	7889.942
5	22.182 22.189	20.017 20.018	1.1721 1.1720	0.7355 0.9416	998.2027	7889.915
6	22.246 22.275	20.018 20.017	1.1717 1.1715	0.7347 0.9394	998.2028	7889.874
7	22.310 22.363	20.017 20.010	1.1714 1.1711	0.7325 0.9408	998.2037	7889.988
8	22.374 22.404	20.013 20.017	1.1711 1.1710	0.7341 0.9411	998.2030	7889.940
9	22.426 22.443	20.019 20.019	1.1710 1.1710	0.7345 0.9410	998.2024	7889.930
10	22.429 22.454	20.020 20.019	1.1711 1.1710	0.7341 0.9410	998.2027	7889.945
	21.958	20.015	1.1708	0.7341		

mean density at 20 °C = 7889.946 kg m^{-3}
standard deviation = 0.04 kg m^{-3}

Table A3.15

Artefact 1000_{A1}, data set 6

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.766 23.756	20.041 20.042	1.1669 1.1671	0.7328 0.9388	998.1979	7889.898
2	23.752 23.747	20.041 20.042	1.1672 1.1672	0.7322 0.9386	998.1978	7889.944
3	23.741 23.717	20.042 20.042	1.1671 1.1672	0.7309 0.9385	998.1978	7889.928
4	23.719 23.724	20.041 20.042	1.1672 1.1672	0.7325 0.9383	998.1978	7889.856
5	23.717 23.682	20.042 20.042	1.1673 1.1676	0.7328 0.9379	998.1977	7889.844
6	23.686 23.682	20.042 20.042	1.1676 1.1676	0.7320 0.9368	998.1978	7889.810
7	23.652 23.592	20.042 20.041	1.1678 1.1682	0.7317 0.9381	998.1979	7889.908
8	23.604 23.597	20.041 20.041	1.1681 1.1682	0.7314 0.9378	998.1980	7889.886
9	23.568 23.593	20.041 20.042	1.1684 1.1683	0.7318 0.9377	998.1980	7889.895
10	23.581 23.568	20.039 20.040	1.1683 1.1685	0.7309 0.9376	998.1983	7889.924
	23.766	20.039	1.1684	0.7307		

mean density at 20 °C = 7889.889 kg m^{-3}
standard deviation = 0.04 kg m^{-3}

Table A3.16

Artefact 1000_{A1}, data set 7

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.855 21.966	19.940 19.936	1.1735 1.1731	0.7367 0.9443	998.2195	7890.064
2	22.036 22.139	19.934 19.926	1.1727 1.1721	0.77375 0.9448	998.2209	7890.092
3	22.210 22.306	19.929 19.929	1.1716 1.1712	0.7373 0.9442	998.2212	7890.062
4	22.354 22.431	19.926 19.927	1.1709 1.1705	0.7375 0.9445	998.2210	7890.077
5	22.492 22.564	19.934 19.931	1.1702 1.1699	0.7375 0.9438	998.2198	7890.026
6	22.614 22.681	19.939 19.938	1.1698 1.1695	0.7376 0.9438	998.2190	7890.029
7	22.728 22.787	19.939 19.943	1.1692 1.1691	0.7373 0.9435	998.2182	7890.017
8	22.832 22.877	19.945 19.946	1.1689 1.1686	0.7373 0.9435	998.2170	7890.021
9	22.913 22.957	19.953 19.952	1.1684 1.1682	0.7370 0.9430	998.2158	7889.987
10	22.992 23.028	19.957 19.957	1.1679 1.1677	0.7372 0.9429	998.2152	7889.973
	21.855	19.957	1.1676	0.7372		

mean density at 20 °C = 7890.035 kg m^{-3}
standard deviation = 0.04 kg m^{-3}

Table A3.17

Artefact 1000_{A1}, data set 8

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.607 21.622	20.003 20.000	1.1947 1.1946	0.7349 0.9448	998.2072	7890.008
2	21.660 21.734	19.996 19.984	1.1944 1.1942	0.7354 0.9456	998.2089	7890.039
3	21.785 21.864	19.990 19.996	1.1941 1.1938	0.7359 0.9462	998.2081	7890.030
4	21.928 22.015	19.995 19.988	1.1935 1.1930	0.7368 0.9470	998.2093	7890.046
5	22.099 22.175	19.985 19.980	1.1927 1.1922	0.7374 0.9474	998.2107	7890.045
6	22.246 22.317	19.983 19.965	1.1919 1.1916	0.7381 0.9481	998.2125	7890.067
7	22.367 22.420	19.974 19.975	1.1914 1.1912	0.7386 0.9481	998.2118	7890.049
8	22.470 22.521	19.983 19.973	1.1910 1.1908	0.7386 0.9482	998.2112	7890.045
9	22.558 22.603	19.985 19.977	1.1905 1.1903	0.7389 0.9480	998.2106	7890.015
10	22.635 22.676	19.988 19.983	1.1901 1.1899	0.7391 0.9483	998.2103	7890.016
	21.607	19.984	1.1897	0.7395		

mean density at 20 °C = 7890.036 kg m^{-3}
standard deviation = 0.015 kg m^{-3}

Table A3.18

Artefact 1000_{09}, data set 1

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.548 22.626	19.887 19.891	1.2051 1.2046	0.7354 0.7382	998.2303	9148.395
2	22.668 22.737	19.894 19.897	1.2044 1.2041	0.7358 0.7379	998.2290	9148.360
3	22.777 22.852	19.900 19.902	1.2039 1.2035	0.7355 0.7381	998.2280	9148.378
4	22.885 22.944	19.904 19.907	1.2033 1.2030	0.7357 0.7368	998.2269	9148.337
5	22.981 23.036	19.910 19.911	1.2028 1.2026	0.7338 0.7368	998.2260	9148.409
6	23.055 23.111	19.913 19.914	1.2025 1.2022	0.7339 0.7371	998.2253	9148.432
7	23.125 23.189	19.916 19.912	1.2021 1.2018	0.7338 0.7368	998.2253	9148.414
8	23.194 23.246	19.916 19.918	1.2017 1.2015	0.7338 0.7372	998.2245	9148.441
9	23.276 23.322	19.921 19.923	1.2013 1.2012	0.7339 0.7369	998.2234	9148.414
10	23.343 23.390	19.928 19.929	1.2011 1.2009	0.7337 0.7371	998.2225	9148.426
	22.548	19.928	1.2008	0.7339		

mean density at 20 °C = 9148.401 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.19

Artefact 1000_{09}, data set 2

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.384 23.425	19.987 19.987	1.1824 1.1822	0.7333 0.7359	998.2097	9148.404
2	23.486 23.515	19.987 19.987	1.1818 1.1817	0.7338 0.7359	998.2098	9148.408
3	23.561 23.595	19.985 19.985	1.1815 1.1813	0.7333 0.7358	998.2099	9148.396
4	23.638 23.684	19.986 19.985	1.1811 1.1807	0.7340 0.7360	998.2098	9148.386
5	23.727 23.760	19.987 19.986	1.1806 1.1805	0.7340 0.7361	998.2097	9148.376
6	23.825 23.843	19.987 19.988	1.1801 1.1801	0.7345 0.7365	998.2096	9148.399
7	23.875 23.903	19.987 19.988	1.1800 1.1798	0.7343 0.7363	998.2095	9148.379
8	23.942 23.967	19.988 19.989	1.1796 1.1795	0.7346 0.7351	998.2093	9148.334
9	24.003 24.024	19.988 19.991	1.1794 1.1792	0.7330 0.7355	998.2091	9148.427
10	24.046 24.053	19.990 19.991	1.1791 1.1791	0.7332 0.7361	998.2089	9148.431
	23.384	19.991	1.1790	0.7341		

mean density at 20 °C = 9148.394 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.20

Artefact 1000$_{09}$, data set 3

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.240 22.144	19.988 19.989	1.1944 1.1948	0.7293 0.7327	998.2098	9148.392
2	22.174 22.112	19.986 19.984	1.1947 1.1950	0.7293 0.7323	998.2103	9148.396
3	22.087 21.995	19.985 19.982	1.1940 1.1955	0.7285 0.7309	998.2107	9148.345
4	21.999 21.966	19.983 19.981	1.1955 1.1957	0.7277 0.7311	998.2111	9148.393
5	21.950 21.863	19.980 19.981	1.1957 1.1961	0.7277 0.7308	998.2116	9148.373
6	21861 21.857	19.977 19.974	1.1960 1.1961	0.7276 0.7309	998.2123	9148.386
7	21.844 21.870	19.976 19.975	1.1961 1.1960	0.7277 0.7311	998.2124	9148.400
8	21.838 21.825	19.974 19.970	1.1961 1.1961	0.7277 0.7310	998.2131	9148.375
9	21.828 21.785	19.971 19.970	1.1961 1.1963	0.7282 0.7310	998.2134	9148.376
10	21.794 21.794	19.970 19.967	1.1962 1.1962	0.7277 0.7315	998.2138	9148.417
	22.240	19.968	1.1962	0.7283		

mean density at 20 °C = 9148.385 kg m^{-3}
standard deviation = 0.02 kg m^{-3}

Table A3.21

Artefact 1000_{09}, data set 4

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.062 22.109	19.934 19.934	1.1824 1.1820	0.7324 0.7347	998.2199	9148.463
2	22.151 22.223	19.940 19.937	1.1820 1.1820	0.7327 0.7349	998.2194	9148.410
3	22.275 22.335	19.938 19.932	1.1818 1.1816	0.7340 0.7359	998.2195	9148.393
4	22.382 22.436	19.944 19.943	1.1814 1.1813	0.7352 0.7370	998.2182	9148.399
5	22.485 22.539	19.945 19.945	1.1811 1.1807	0.7359 0.7377	998.2180	9148.405
6	22.581 22.632	19.945 19.946	1.1806 1.1806	0.7365 0.7382	998.2181	9148.417
7	22.672 22.724	19.943 19.946	1.1802 1.1801	0.7367 0.7364	998.2182	9148.329
8	22.762 22.800	19.944 19.943	1.1799 1.1797	0.7351 0.7370	998.2182	9148.424
9	22.835 22.874	19.945 19.947	1.1796 1.1793	0.7357 0.7375	998.2178	9148.440
10	22.899 22.943	19.946 19.947	1.1792 1.1791	0.7357 0.7373	998.2177	9148.420
	22.063	19.948	1.1790	0.7358		

mean density at 20 °C = 9148.410 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.22

Artefact 1000_{09}, data set 5

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.168 23.191	19.956 19.958	1.1816 1.1814	0.9561 0.9579	998.2157	9148.413
2	23.227 23.239	19.955 19.958	1.1812 1.1809	0.9560 0.9576	998.2156	9148.390
3	23.262 23.277	19.958 19.958	1.1807 1.1806	0.9561 0.9571	998.2153	9148.378
4	23.289 23.299	19.960 19.958	1.1806 1.1803	0.9553 0.9570	998.2154	9148.389
5	23.316 23.331	19.956 19.958	1.1802 1.1799	0.9557 0.9572	998.2153	9148.412
6	23.345 23.362	19.961 19.959	1.1799 1.1796	0.9552 0.9570	998.2150	9148.412
7	23.382 23.397	19.960 19.960	1.1796 1.1795	0.9553 0.9570	998.2150	9148.422
8	23.407 23.415	19.959 19.963	1.1795 1.1795	0.9550 0.9570	998.2148	9148.425
9	23.413 23.415	19.962 19.964	1.1794 1.1793	0.9552 0.9566	998.2145	9148.349
10	23.414 23.417	19.961 19.962	1.1791 1.1791	0.9560 0.9561	998.2146	9148.352
	23.168	19.963	1.1790	0.9542		

mean density at 20 °C = 9148.394 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.23

Artefact 1000_{09}, data set 6

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.140 21.191	19.956 19.954	1.1851 1.1848	0.9532 0.9560	998.2160	9148.387
2	21.247 21.327	19.953 19.952	1.1846 1.1842	0.9551 0.9570	998.2163	9148.381
3	21.383 21.454	19.954 19.955	1.1839 1.1837	0.9555 0.9572	998.2162	9148.385
4	21.513 21.579	19.952 19.953	1.1833 1.1830	0.9555 0.9578	998.2164	9148.442
5	21.629 21.698	19.952 19.952	1.1828 1.1826	0.9555 0.9580	998.2167	9148.453
6	21.749 21.810	19.948 19.950	1.1823 1.1819	0.9558 0.9581	998.2172	9148.436
7	21.860 21.924	19.948 19.952	1.1817 1.1814	0.9563 0.9583	998.2170	9148.434
8	21.986 22.025	19.950 19.944	1.1812 1.1810	0.9563 0.9581	998.2177	9148.418
9	22.074 22.112	19.944 19.946	1.1808 1.1807	0.9565 0.9582	998.2177	9148.387
10	22.162 22.203	19.948 19.948	1.1804 1.1801	0.9573 0.9586	998.2171	9148.391
	21.140	19.951	1.1800	0.9572		

mean density at 20 °C = 9148.411 kg m^{-3}
standard deviation = 0.03 kg m^{-3}

Table A3.24

Artefact 1000_{44}, data set 1

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.833 22.884	19.951 19.952	1.1764 1.1760	0.8539 0.5519	998.2166	21502.38
2	22.935 22.987	19.952 19.952	1.1758 1.1756	0.8538 0.5525	998.2162	21502.58
3	23.032 23.081	19.955 19.954	1.1754 1.1752	0.8543 0.5529	998.2159	21502.54
4	23.123 23.171	19.956 19.958	1.1750 1.1748	0.8548 0.5530	998.2153	21502.44
5	23.206 23.245	19.959 19.959	1.1746 1.1745	0.8550 0.5532	998.2152	21502.50
6	23.277 23.321	19.958 19.959	1.1743 1.1742	0.8550 0.5531	998.2152	21502.51
7	23.337 23.372	19.958 19.959	1.1741 1.1739	0.8548 0.5531	998.2150	21502.52
8	23.423 23.468	19.960 19.960	1.1736 1.1733	0.8550 0.5524	998.2148	21502.43
9	23.507 23.540	19.960 19.961	1.1730 1.1729	0.8539 0.5521	998.2146	21502.42
10	23.558 23.585	19.962 19.963	1.1728 1.1727	0.8545 0.5524	998.2143	21502.35
	22.833	19.963	1.1727	0.8548		

mean density at 20 °C = 21 502.47 kg m^{-3}
standard deviation = 0.07 kg m^{-3}

Table A3.25

Artefact 1000_{44}, data set 2

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.930	20.015	1.1754	0.8549		
	23.970	20.015	1.1753	0.5550	998.2036	21502.90
2	23.982	20.015	1.1753	0.8552		
	24.011	20.015	1.1752	0.5554	998.2036	21502.90
3	24.033	20.015	1.1750	0.8556		
	24.058	20.016	1.1749	0.5556	998.2036	21502.90
4	24.081	20.015	1.1748	0.8558		
	24.108	20.016	1.1747	0.5554	998.2034	21502.77
5	24.128	20.017	1.1747	0.8558		
	24.139	20.017	1.1746	0.5557	998.2033	21502.84
6	24.158	20.017	1.1745	0.8561		
	24.189	20.017	1.1743	0.5555	998.2033	21502.64
7	24.203	20.017	1.1742	0.8563		
	24.218	20.017	1.1742	0.5557	998.2032	21502.72
8	24.237	20.017	1.1741	0.8562		
	24.259	20.017	1.1740	0.5557	998.2032	21502.75
9	24.256	20.018	1.1740	0.8562		
	24.263	20.018	1.1741	0.5556	998.2029	21502.72
10	24.294	20.020	1.1739	0.8561		
	24.327	20.019	1.1738	0.5555	998.2028	21502.68
	23.930	20.018	1.1738	0.8562		

mean density at 20 °C = 21 502.78 kg m^{-3}
standard deviation = 0.09 kg m^{-3}

Table A3.26

Artefact 1000_{44}, data set 3

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.030 23.074	19.983 19.990	1.1685 1.1683	0.6329 0.3327	998.2087	21503.20
2	23.125 23.160	19.994 19.995	1.1681 1.1680	0.6332 0.3334	998.2075	21503.29
3	23.210 23.237	19.995 19.995	1.1677 1.1676	0.6339 0.3334	998.2074	21503.09
4	23.285 23.307	19.995 19.998	1.1674 1.1673	0.6341 0.3338	998.2072	21503.15
5	23.338 23.362	19.995 19.997	1.1672 1.1670	0.6345 0.3324	998.2070	21502.44
6	23.394 23.409	19.998 19.998	1.1669 1.1668	0.6344 0.3326	998.2067	21502.58
7	23.438 23.450	19.999 19.999	1.1666 1.1665	0.6343 0.3331	998.2065	21502.83
8	23.491 23.483	19.999 20.001	1.1663 1.1663	0.6344 0.3338	998.2063	21503.06
9	23.516 23.542	20.000 20.001	1.1662 1.1662	0.6347 0.3337	998.2062	21502.90
10	23.567 23.566	20.002 20.002	1.1661 1.1661	0.6349 0.3341	998.2060	21503.02
	23.030	20.002	1.1660	0.6350		

mean density at 20 °C = 21 502.96 kg m^{-3}
standard deviation = 0.28 kg m^{-3}

Table A3.27

Artefact 1000_{44}, data set 4

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	22.612 22.698	19.932 19.925	1.1810 1.1804	0.8554 0.5559	998.2214	21503.24
2	22.762 22.834	19.930 19.929	1.1802 1.1799	0.8554 0.5565	998.2214	21503.29
3	22.883 22.955	19.928 19.926	1.1797 1.1795	0.8565 0.5572	998.2221	21503.33
4	22.988 23.041	19.923 19.927	1.1794 1.1793	0.8568 0.5576	998.2220	21503.24
5	23.077 23.171	19.928 19.927	1.1791 1.1785	0.8577 0.5577	998.2218	21503.08
6	23.210 23.258	19.925 19.930	1.1785 1.1781	0.8578 0.5580	998.2218	21503.18
7	23.299 23.354	19.926 19.927	1.1778 1.1775	0.8580 0.5584	998.2219	21503.30
8	23.392 23.430	19.926 19.927	1.1773 1.1771	0.8582 0.5582	998.2218	21503.13
9	23.458 23.492	19.927 19.928	1.1769 1.1768	0.8584 0.5582	998.2215	21503.19
10	23.523 23.550	19.930 19.930	1.1766 1.1765	0.8580 0.5580	998.2211	21503.15
	22.612	19.930	1.1763	0.8582		

mean density at 20 °C = 21 503.21 kg m^{-3}
standard deviation = 0.09 kg m^{-3}

Table A3.28

Artefact 1000_{44}, data set 5

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	21.615 21.688	19.917 19.919	1.2002 1.1999	0.8534 0.5557	998.2241	21502.96
2	21.737 21.817	19.920 19.920	1.1998 1.1995	0.8544 0.5568	998.2237	21503.14
3	21.865 21.934	19.922 19.921	1.1992 1.1990	0.8549 0.5571	998.2237	21503.09
4	21.986 22.057	19.920 19.925	1.1986 1.1983	0.8553 0.5576	998.2233	21503.14
5	22.110 22.168	19.923 19.923	1.1981 1.1979	0.8558 0.5582	998.2232	21503.23
6	22.217 22.284	19.923 19.924	1.1977 1.1973	0.8562 0.5579	998.2230	21502.92
7	22.326 22.390	19.925 19.926	1.1972 1.1970	0.8566 0.5582	928.2226	21503.12
8	22.422 22.474	19.927 19.925	1.1968 1.1965	0.8560 0.5583	998.2227	21503.16
9	22.512 22.564	19.925 19.928	1.1964 1.1962	0.8567 0.5585	998.2222	21503.08
10	22.604 22.664	19.930 19.930	1.1960 1.1957	0.8568 0.5584	998.2160	21502.97
	21.615	19.932	1.1956	0.857		

mean density at 20 °C = 21 503.08 kg m^{-3}
standard deviation = 0.09 kg m^{-3}

Table A3.29

Artefact 1000_{44}, data set 6

series	air temp °C	water temp °C	air density kg m^{-3}	balance indication g	water density kg m^{-3}	density at 20 °C kg m^{-3}
1	23.229 23.254	19.988 19.988	1.1857 1.1856	0.8536 0.5545	998.2095	21502.77
2	23.275 23.295	19.988 19.986	1.1855 1.8541	0.8544 0.5557	998.2098	21503.13
3	23.323 23.343	19.986 19.988	1.1853 1.1851	0.8545 0.5563	998.2099	21503.27
4	23.368 23.295	19.985 19.989	1.1849 1.1848	0.8551 0.5559	998.2096	21502.84
5	23.408 23.426	19.988 19.990	1.1846 1.8457	0.8556 0.5569	998.2092	21503.16
6	23.451 23.470	19.990 19.988	1.1845 1.1844	0.8557 0.5569	998.2091	21503.03
7	23.502 23.531	19.991 19.992	1.1842 1.8409	0.8562 0.5581	998.2088	21503.32
8	23.561 23.582	19.990 19.991	1.1839 1.1839	0.8569 0.5581	998.2089	21503.12
9	23.602 23.622	19.991 19.991	1.1837 1.1836	0.8571 0.5571	998.2088	21502.67
10	23.643 23.657	19.991 19.993	1.1836 1.1836	0.8569 0.5573	998.2085	21502.92
	23.229	19.994	1.1835	0.8564		

mean density at 20 °C =21 503.02 kg m^{-3}
standard deviation = 0.21 kg m^{-3}