

# Recommended Methods for Determining Young's Modulus from Tensile Tests

## Summary

This note is a summary of the work carried out at NPL to measure Young's modulus using mechanical methods. Recommendations for strain measurement techniques and data analysis are given in order to reduce the uncertainty in the measurements.

Current standards for tensile testing are ambiguous regarding the analysis of data and the calculation of Young's modulus. NPL has been involved in a number of intercomparison exercises to measure the tensile properties of metal matrix composites (MMC). The uncertainties in the Young's modulus from the first two intercomparison exercises on a SiC particulate reinforced MMC was  $\pm 12-14\%$ , which is unacceptably high. From this work, various recommendations were suggested to improve the accuracy of the modulus measurements and these were incorporated into a draft test method [1]. This new test method has been validated via two further intercomparisons, resulting in a reduction in the uncertainty in Young's modulus to  $\pm 4\%$ .

Analysis of the data showed that double-sided strain measurement methods and computer analysis of the data gave the most consistent and reliable results. Tests at NPL which used strain gauges on both sides of the testpiece and analysis of the tangent and secant moduli to calculate an accurate value for Young's modulus showed typical uncertainties of  $\pm 1\%$ .

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## Introduction

The work described in this Measurement Note has been carried out on MMC, but the points and issues covered are valid for a wider range of materials.

One of the driving forces behind the interest in MMC is their high specific strength and stiffness and this focused attention on accurate modulus measurement. In particular, accurate measurement of Young's modulus in MMC is important for several reasons:

- a. Improvements in specific stiffness are an important driver in increasing the use of MMC over conventional materials. An accurate knowledge of the engineering value of Young's modulus is vital for preliminary design studies.
- b. Proof stress measurements require a prior knowledge of the Young's modulus. If the material of interest has a high work hardening rate in the early stage of yield then inaccuracies in the Young's modulus can give significant inaccuracies in proof stress.
- c. MMC have low proportional limits because of internal residual stresses. It is important to be able to measure the proportional limit accurately and to assess the extent of yield at low strains. An accurate value of Young's modulus is required to obtain reliable values for the proportional limit.
- d. Accurate measurements of Young's modulus are required to give good fits to the constitutive equations for the stress-strain data, and for finite element calculations.

## Results From Intercomparison Exercises

Results from the first intercomparison exercise showed unacceptably high uncertainties in the modulus measurement, and further work was necessary to improve the accuracy of the measurements.

The figure opposite shows the scatter in the modulus values from an exercise to measure the properties of a SiC whisker reinforced MMC [2]. These data are from tests on different testpieces and the material variability will contribute to some of the uncertainty. To overcome this, a separate exercise was organised [3] in which the same testpiece was used (within the elastic regime) by all participants. Unfortunately, similar levels of uncertainty were measured which indicated that the problem lay with the test method and data analysis. Further analysis of the data showed that there was a clear relationship between the accuracy and reliability of the results, the strain measurement method used, and also how the data were analysed. These are covered in later sections.

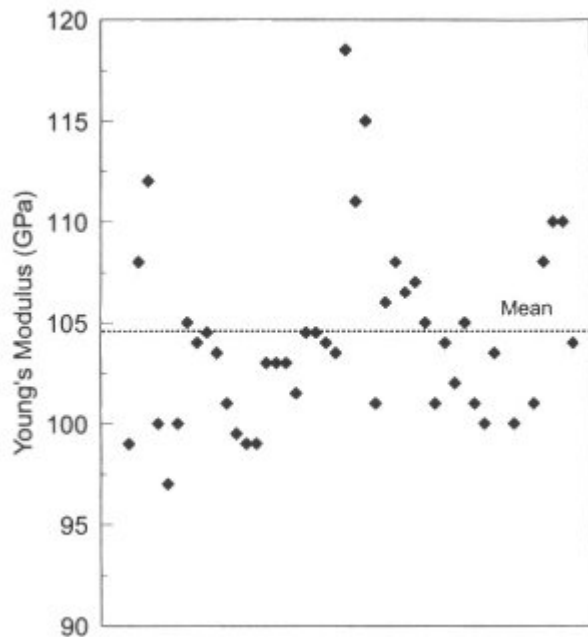


Fig 1. Typical scatter in modulus measurements from an intercomparison exercise

## Standards For Modulus Measurement

Young's modulus has traditionally been measured from the linear elastic region of a uniaxial tensile stress-strain curve, and is defined as the ratio of stress to strain during elastic loading. Traditionally, the modulus was determined 'by eye' from a straight line drawn on the stress-strain curve; more recently automatic testing machines using computer control and data acquisition use some form of curve fitting to get a best fit to the data. With all the standards at present, there is little guidance on which modulus parameter should be used, and aspects of strain measurement are covered only briefly. ASTM E111 defines the tangent and chordal modulus methods and advocates their use for non-linear materials; the secant modulus method is not recommended. EN10002:Part 1 gives no formal definition for modulus.

Based on the results from two intercomparison exercises and extensive in-house testing at NPL, a new test procedure has been developed [1] for measuring the tensile properties of MMC, with a considerable emphasis on the measurement of Young's modulus.

## Strain Measurement Methods

The preferred method of strain measurement is to use a double sided data system, either:

- a. Double sided extensometry
- b. Strain gauges bonded to each side of the testpiece.

For a Class 1 extensometer, the maximum permissible total uncertainty is  $\pm 1\%$  or  $3 \mu\text{m}$ , whichever is the greater, which can lead to significant errors at low strains. Typically, for a 25 mm gauge length and 0.1% strain, the error in modulus can be as high as  $\pm 12\%$  for a Grade 'C' extensometer. The strain error increases as the strain decreases due to the  $\pm 3 \mu\text{m}$  lower limit.

Strain gauges are only suitable for measuring the full tensile properties if the testpiece failure strains are less than about 3%. The resolution of the strain gauge reading depends on the gauge factor and instrumentation gain. For the data acquisition system used at NPL, generally for modulus measurements, a high system gain is chosen. Typically this gives a resolution of  $\pm 1\text{-}2 \mu\text{E}$  but the maximum strain that can be measured is limited to 0.5% (5000  $\mu\text{E}$ ). If larger strains are to be measured then a compromise must be reached between the maximum strain and the measurement resolution required.

Strain gauge installations may be susceptible also to other uncertainties which are difficult to quantify. The instrumentation itself can be calibrated by using a shunt resistor, but the individual gauge installation on the testpiece itself cannot be calibrated easily. Errors can arise due to misalignment of the gauge, poor gauge installation and bonding, temperature effects, Wheatstone bridge nonlinearities and transverse sensitivity. All are important factors but are difficult to quantify. However, for modulus measurements at low strain levels, strain gauges are preferred and uncertainties of  $\pm 1\%$  should be readily achievable. It is vital however, that the gauges should be applied to both sides of the testpiece and averaged to take account of out-of-plane bending.

## Data Analysis Methods

The method used to analyse the data can also have an important effect on the calculated modulus value. Several methods can be used, including graphical techniques and computer-based analyses which use a chordal method. NPL has developed an analysis which uses the tangent and secant moduli to obtain a best fit for Young's modulus.

Three alternative methods are outlined below.

### a) Graphical Method

The load and strains (single or averaged) are plotted on a chart recorder, and a straight line is drawn parallel to the initial portion of the load/strain curve. The slope of this line, (when the load has been divided by the testpiece cross-section to convert to values of stress) is the Young's modulus. To reduce errors, the scale should be chosen so that the curve occupies a significant proportion of the paper and the angle between the plot and the strain axis should be  $45^\circ \pm 2^\circ$ . For angles above  $70^\circ$ , the uncertainties in the measured values for modulus can be as high as 5-10%.

### b) Chordal Method

This method is often used when machine dedicated software is available to calculate the modulus values. It is strongly recommended that the software is validated/calibrated by the use of reference testpieces of known stiffness.

Usually, two values of stress are chosen on the stress-strain curve to mark the lower and upper limits of a chordal modulus. Then either a straight line is drawn between the two points or a straight line regression fit is made to the experimental data between these two chosen values of stress using a validated software procedure. The slope of this line corresponds to the Young's modulus value.

Examination of the data from the intercomparisons shows that there is a wide range of values chosen for the upper and lower limits which can contribute to scatter in the calculations. These limits are material dependent and should be quoted in any report of the Young's modulus value.

### c) Tangent/Secant method

NPL has developed a software package for collecting and analysing stress-strain data which has been validated by analysing and regenerating mathematically derived input data. The NPL software uses the tangent and secant moduli, calculated at each datapoint, to produce the best fit for modulus. Ideally, for a good fit to the linear part of the curve, the tangent and secant moduli should coincide. The analysis of the secant and tangent moduli data is a very sensitive method for checking whether the value chosen is a good fit to the stress-strain curve.

A typical stress-strain curve, with the Young's modulus fit and 0.2% proof stress value is shown in [Fig. 2](#), together with the corresponding tangent/secant moduli plot in [Fig. 3](#). The value for Young's modulus is calculated automatically where these two curves are flattest and coincide. The full procedure for this method is given in [Ref. 4](#).

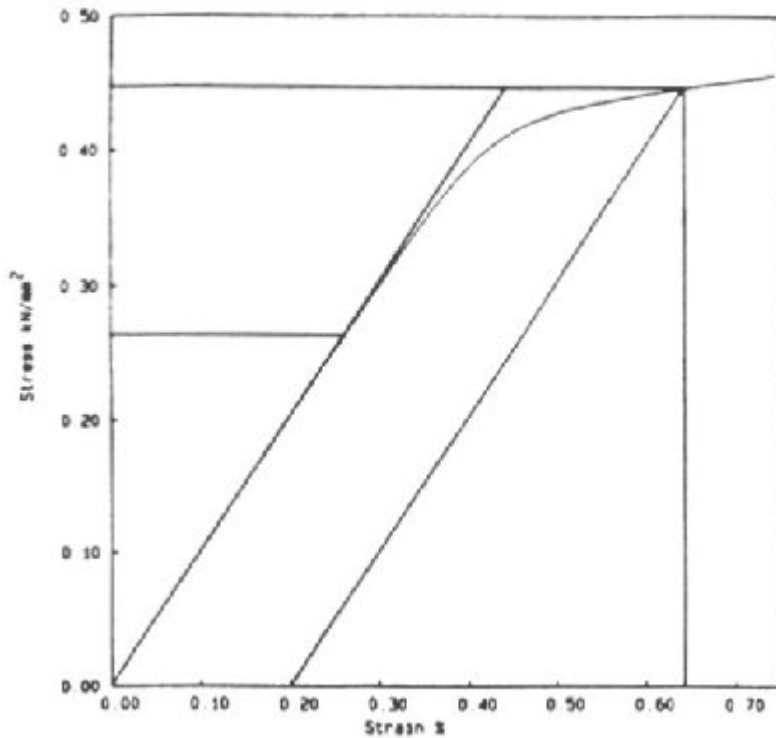


Fig. 2: Typical stress-strain curve for a particulate reinforced MMC

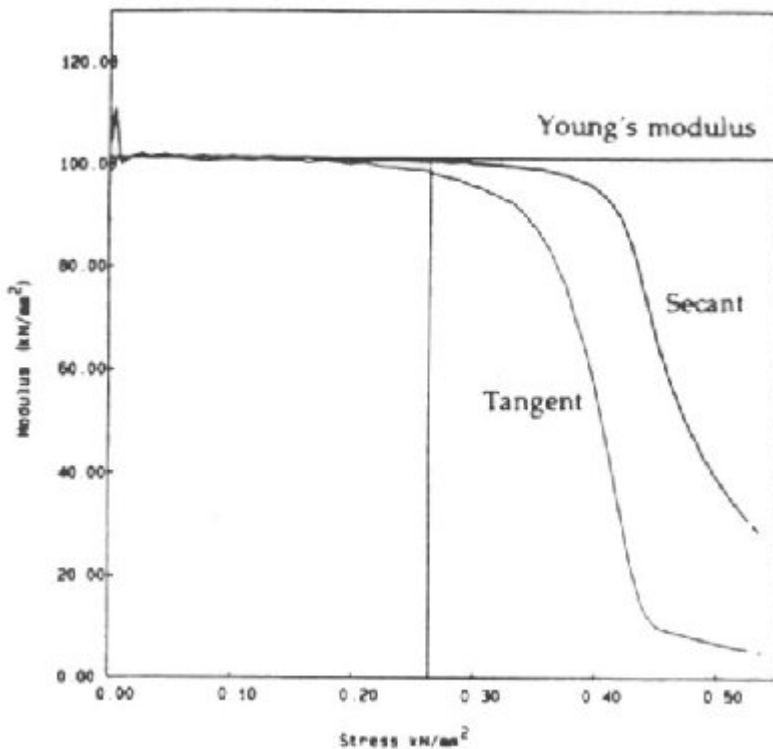


Fig 3: Corresponding tangent/secant moduli plot and Young's modulus fit from NPL analysis package.

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## References

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