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REPORT

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by

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SUMMARY

It is shown how energy balance principles can be used to predict the steady state delamination associated with transverse cracks in a cross-ply laminate, taking full account of the effects of triaxial loading and of thermal residual stresses. Preliminary results for a fixed value of the fracture energy for delamination, in the absence of thermal residual stresses, inydicates that higher delamination stresses are predicted for laminates having higher stiffness. It is shown that steady state delamination is possible only for certain ranges of the stress ratios defining the triaxial loading conditions. The effects of varying the laminate lay-up, the fracture energy and the stress-free temperature of the laminate are investigated.

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1. Introduction

O'Brien [1] used energy balance methods to develop a simple analytical formula to predict steady state delamination induced by transverse cracks in cross-ply laminates. His result applies only to delamination growth for which steady state conditions prevail in the region of stress transfer associated with the delaminations. Also O'Brien's analysis applies only to plane stress conditions (i.e. transverse stresses are always zero for uniaxial loading) and assumes that thermal residual stresses may be ignored. Nairn and Hu [2] carried out a variational analysis for uniaxial loading that predicts both the initiation and growth of delaminations from transverse cracks in cross-ply laminates, taking account of transverse crack interactions and thermal residual stresses. They present results that are consistent with O'Brien's results when there are no transverse crack interactions, and when thermal residual stresses are ignored. Takeda and Ogihara [3] later performed the corresponding (more approximate) analysis based on shear lag theory rather than on variational principles.

The objective of this note is to revisit the steady state delamination problem for cross-ply laminates tackled by O'Brien by including in the analysis a treatment of both multi-axial loading and of thermal residual stresses, in a way that is consistent with the approach used by McCartney [4,5] when analysing transverse crack formation in laminates for the case when delamination does not occur.

2. Geometry

Consider a 0°-90°-0° cross-ply laminate subject to biaxial loading such that the effective applied stress in the axial direction is σ and that in the transverse direction is σ_t. In addition the laminate is subject to a uniform through-thickness pressure p. Symmetry is assumed about the mid-plane of the laminate. As shown in Fig.1 the half-thickness of the inner 90° ply (to be referred to as ply 1) is h_1 and the thickness of the outer 0° plies (to be referred to as ply 2) is h_2 such that the total laminate thickness is 2h where $h = h_1 + h_2$. An isolated transverse crack is assumed to have formed in the 90° ply and two delaminations of equal length have grown in the interface between the 0° and 90° plies. Referring to Fig.1, Region I represents unit length of undamaged laminate that is not affected by the stress transfer that occurs in the region containing the delamination crack tips. Region II represents unit length of the laminate where delamination has already occurred but is not affected by the transverse crack, nor is influenced by stress transfer that occurs in the region containing the delamination crack tips. Clearly the delaminations are assumed to have grown a distance from the transverse crack that is large enough for the effects of the transverse crack to be negligible with the result that the following analysis cannot be used to predict the initiation of delamination growth, but can be used to investigate the stress needed for steady state delamination growth.

A set of rectangular Cartesian coordinates (x, y, z) is selected so that the mid-plane of the laminate is defined by x = 0, the interface by $x = h_1$ and the outer surface by x = h. The transverse crack lies in the plane y = 0 and the z-axis is directed in the transverse direction (normal to the page of Fig.1).

3. Equilibrium equations

The laminate is assumed to be in equilibrium, so that the following equations must be satisfied for i = 1, 2

$$\frac{\partial\sigma^i_{xx}}{\partial x} + \frac{\partial\sigma^i_{xy}}{\partial y} + \frac{\partial\sigma^i_{xz}}{\partial z} = 0\ , \qquad (1)$$

$$\frac{\partial\sigma^i_{xy}}{\partial x} + \frac{\partial\sigma^i_{yy}}{\partial y} + \frac{\partial\sigma^i_{yz}}{\partial z} = 0\ , \qquad (2)$$

$$\frac{\partial\sigma^i_{xz}}{\partial x} + \frac{\partial\sigma^i_{yz}}{\partial y} + \frac{\partial\sigma^i_{zz}}{\partial z} = 0\ , \qquad (3)$$

where σ^i_{xx}, σ^i_{xy}, σ^i_{xz}, σ^i_{yy}, σ^i_{yz} and σ^i_{zz} are the components of the stress tensor for the i^{th} ply.

4. Strain-displacement relations

The displacements in the x, y and z directions are denoted by u_i, v_i and w_i respectively, so that the strains can be calculated as follows for i = 1, 2

$$\varepsilon^i_{xx} = \frac{du_i}{dx}\ , \quad \varepsilon^i_{xy} = \varepsilon^i_{yx} = \frac{1}{2}\left(\frac{du_i}{dy} + \frac{dv_i}{dx}\right), \qquad (4)$$

$$\varepsilon^i_{yy} = \frac{dv_i}{dy}\ , \quad \varepsilon^i_{yz} = \varepsilon^i_{zy} = \frac{1}{2}\left(\frac{dv_i}{dz} + \frac{dw_i}{dy}\right), \qquad (5)$$

$$\varepsilon^i_{zz} = \frac{dw_i}{dz}\ , \quad \varepsilon^i_{zx} = \varepsilon^i_{xz} = \frac{1}{2}\left(\frac{dw_i}{dx} + \frac{du_i}{dz}\right). \qquad (6)$$

5. Stress-strain-temperature relations

For a cross-ply laminate the stress-strain-temperature relations are assumed to be of the following linear form, for i = 1, 2

$$\varepsilon^i_{xx} = \frac{1}{E^i_t}\sigma^i_{xx} - \frac{\nu^i_a}{E^i_A}\sigma^i_{yy} - \frac{\nu^i_t}{E^i_T}\sigma^i_{zz} + \alpha^i_t\Delta T\ , \qquad (7)$$

$$\varepsilon^i_{yy} = -\frac{\nu^i_a}{E^i_A}\sigma^i_{xx} + \frac{1}{E^i_A}\sigma^i_{yy} - \frac{\nu^i_A}{E^i_A}\sigma^i_{zz} + \alpha^i_A\Delta T\ , \qquad (8)$$

$$\varepsilon^i_{zz} = -\frac{\nu^i_t}{E^i_T}\sigma^i_{xx} - \frac{\nu^i_A}{E^i_A}\sigma^i_{yy} + \frac{1}{E^i_T}\sigma^i_{zz} + \alpha^i_T \Delta T \,, \tag{9}$$

$$\varepsilon^i_{xy} = \frac{\sigma^i_{xy}}{2\mu^i_a}\,, \quad \varepsilon^i_{xz} = \frac{\sigma^i_{xz}}{2\mu^i_t}\,, \quad \varepsilon^i_{yz} = \frac{\sigma^i_{yz}}{2\mu^i_A}\,, \tag{10}$$

where ΔT is the temperature difference, defined by $\Delta T = T - T_0$ where T is the current temperature of the material, and T_0 is the "manufacturing" temperature at which the strain is zero and the material is everywhere stress-free, with no internal or imposed external stresses and displacements. The parameters E, ν, μ and α denote the Young's modulus, Poisson's ratio, shear modulus and thermal expansion coefficient respectively. The thermoelastic constants E, ν, μ and α are allowed to have different values in each ply of the laminate. The superscript i attached to the thermoelastic constants indicates the number of the ply to which they refer. Each ply has been assumed to be orthotropic so that twelve thermoelastic constants are required to characterise linear behaviour. The upper case subscripts A and T are attached to axial and transverse thermoelastic constants to denote that they refer to in-plane stresses and deformation while the corresponding lower case subscripts denote thermoelastic constants that involve out-of-plane stresses and deformations. The 0° and 90° plies of the laminate are assumed to be made of the same material, but it should be noted that the form of the stress/strain relations (7-10) has assumed that the axial direction of each ply is oriented in the y-direction (independent of the fibre direction) so that for the 90° ply

$$E^1_A = E_T\,, \quad E^1_T = E_A\,, \quad E^1_t = E_t\,,$$

$$\nu^1_A = \nu_A \frac{E_T}{E_A}\,, \quad \nu^1_a = \nu_t\,, \quad \nu^1_t = \nu_a\,,$$

$$\mu^1_A = \mu_A\,, \quad \mu^1_a = \mu_t\,, \quad \mu^1_t = \mu_a\,,$$

$$\alpha^1_A = \alpha_T\,, \quad \alpha^1_T = \alpha_A\,, \quad \alpha^1_t = \alpha_t\,,$$

where E_A, E_T, E_t, ν_A, ν_a, ν_t, μ_A, μ_a, μ_t, α_A, α_T and α_t are the twelve independent thermoelastic constants of the plies of the composite. For the 0° plies the following identifications must be made

$$E^2_A = E_A\,, \quad E^2_T = E_T\,, \quad E^2_t = E_t\,,$$

$$\nu^2_A = \nu_A\,, \quad \nu^2_a = \nu_a\,, \quad \nu^2_t = \nu_t\,,$$

$$\mu^2_A = \mu_A\,, \quad \mu^2_a = \mu_a\,, \quad \mu^2_t = \mu_t\,,$$

$$\alpha^2_A = \alpha_A\,, \quad \alpha^2_T = \alpha_T\,, \quad \alpha^2_t = \alpha_t\,.$$

6. Thermoelastic constants for undamaged laminates

Consider an undamaged cross-ply laminate subject to biaxial in-plane and through-thickness loading such that the uniform longitudinal strain is ε, and the uniform transverse strain is

ε^*. The through-thickness strain in the i[th] ply is uniform and is denoted by $\hat{\varepsilon}_i$, i = 1, 2. It is assumed that the external surfaces of the laminate are subject to a uniform applied pressure p, and that the through-thickness stress $\sigma_{xx} = -p$ throughout the laminate. It then follows that

$$\sigma^i_{xx} \equiv -p\,, \quad \sigma^i_{xy} \equiv \sigma^i_{xz} \equiv \sigma^i_{yz} \equiv 0\,, \quad i = 1, 2\,. \tag{11}$$

It is emphasised that the model is invalid if p < 0, as the delaminated plies may separate and be unable to transmit stress across the interface, exhibiting a through-thickness displacement discontinuity that is not to be modelled.

Denoting the longitudinal and transverse stresses experienced by the i[th] ply to be σ_i and σ_i^* respectively, equations (7-9) may be written, for i = 1, 2

$$\hat{\varepsilon}_i = -\frac{p}{E^i_t} - \frac{\nu^i_a}{E^i_A}\sigma_i - \frac{\nu^i_t}{E^i_T}\sigma^*_i + \alpha^i_t \Delta T\,, \tag{12}$$

$$\varepsilon = \frac{\nu^i_a}{E^i_A}p + \frac{1}{E^i_A}\sigma_i - \frac{\nu^i_A}{E^i_A}\sigma^*_i + \alpha^i_A \Delta T\,, \tag{13}$$

$$\varepsilon^* = \frac{\nu^i_t}{E^i_T}p - \frac{\nu^i_A}{E^i_A}\sigma_i + \frac{1}{E^i_T}\sigma^*_i + \alpha^i_T \Delta T\,. \tag{14}$$

As σ and σ_t are, respectively, the corresponding effective axial and transverse stresses experienced by the laminate as a whole it is deduced from a consideration of mechanical equilibrium that

$$\sigma = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \sigma_i\right), \quad \sigma_t = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \sigma^*_i\right). \tag{15}$$

Let $\hat{\varepsilon}$ denote the effective through-thickness strain of the laminate defined by

$$\hat{\varepsilon} = \frac{1}{h}\sum_{i=1}^{2} h_i \hat{\varepsilon}_i\,. \tag{16}$$

Inverting equations (13) and (14), to obtain the ply stresses in terms of ε and ε^*, leads to

$$\sigma_i = \hat{E}^i_A\left(\varepsilon + \nu^i_A\frac{E^i_T}{E^i_A}\varepsilon^* - \frac{\hat{\nu}^i_a}{E^i_A}p - \hat{\alpha}^i_A \Delta T\right), \tag{17}$$

$$\sigma_i^* = \hat{E}_T^i \left(\nu_A^i \varepsilon + \varepsilon^* - \frac{\hat{\nu}_t^i}{E_T^i} p - \hat{\alpha}_T^i \Delta T \right) , \tag{18}$$

where

$$\frac{1}{\hat{E}_A^i} = \frac{1}{E_A^i}\left(1 - (\nu_A^i)^2 \frac{E_T^i}{E_A^i} \right), \quad \hat{\nu}_a^i = \nu_a^i + \nu_t^i \nu_A^i , \quad \hat{\alpha}_A^i = \alpha_A^i + \nu_A^i \frac{E_T^i}{E_A^i} \alpha_T^i , \tag{19}$$

$$\frac{1}{\hat{E}_T^i} = \frac{1}{E_T^i}\left(1 - (\nu_A^i)^2 \frac{E_T^i}{E_A^i} \right), \quad \hat{\nu}_t^i = \nu_t^i + \nu_a^i \nu_A^i \frac{E_T^i}{E_A^i} , \quad \hat{\alpha}_T^i = \alpha_T^i + \nu_A^i \alpha_A^i . \tag{20}$$

Substituting these expressions for σ_i and σ^*_i into (15) enables the effective stresses to be expressed in terms of the effective strains as follows

$$\sigma = A\varepsilon + B\varepsilon^* - Fp - P\Delta T , \tag{21}$$

$$\sigma_t = B\varepsilon + C\varepsilon^* - Gp - Q\Delta T , \tag{22}$$

where

$$A = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \hat{E}_A^i \right), \quad B = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \nu_A^i \hat{E}_T^i \right), \quad C = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \hat{E}_T^i \right), \tag{23}$$

$$F = \frac{1}{h}\sum_{i=1}^{2} h_i \hat{E}_A^i \frac{\hat{\nu}_a^i}{E_A^i} , \quad G = \frac{1}{h}\sum_{i=1}^{2} h_i \hat{E}_T^i \frac{\hat{\nu}_t^i}{E_T^i} , \tag{24}$$

$$P = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \hat{E}_A^i \hat{\alpha}_A^i \right), \quad Q = \frac{1}{h}\left(\sum_{i=1}^{2} h_i \hat{E}_T^i \hat{\alpha}_T^i \right). \tag{25}$$

On making ε and ε^* the subjects of the equations (21) and (22), so that they are of the same form as equations (13) and (14), it can be shown that

$$\varepsilon = \frac{\nu_a^c}{E_A^c} p + \frac{1}{E_A^c} \sigma - \frac{\nu_A^c}{E_A^c} \sigma_t + \alpha_A^c \Delta T , \tag{26}$$

$$\varepsilon^* = \frac{\nu_t^c}{E_T^c} p - \frac{\nu_A^c}{E_A^c} \sigma + \frac{1}{E_T^c} \sigma_t + \alpha_T^c \Delta T \,, \tag{27}$$

where

$$E_A^c = A - \frac{B^2}{C} \,, \qquad E_T^c = C - \frac{B^2}{A} \,, \qquad \nu_A^c = \frac{B}{C} \,, \tag{28}$$

$$\nu_a^c = F - \nu_A^c G \,, \qquad \nu_t^c = G - \nu_A^c \frac{E_T^c}{E_A^c} F \,, \tag{29}$$

$$\alpha_A^c = \frac{P - \nu_A^c Q}{E_A^c} \,, \qquad \alpha_T^c = \frac{1}{E_T^c} \left(Q - \nu_A^c \frac{E_T^c}{E_A^c} P \right) . \tag{30}$$

The superscript c is used to denote that the thermoelastic constant refers to the undamaged composite laminate as a whole. It can be shown that

$$A = \frac{E_A^c}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,, \quad B = \frac{\nu_A^c E_T^c}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,, \quad C = \frac{E_T^c}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,, \tag{31}$$

$$F = \frac{\nu_a^c + \nu_t^c \nu_A^c}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,, \qquad G = \frac{\nu_t^c + \nu_a^c \nu_A^c E_T^c / E_A^c}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,, \tag{32}$$

$$P = \frac{E_A^c \alpha_A^c + \nu_A^c E_T^c \alpha_T^c}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,, \qquad Q = \frac{E_T^c \left[\alpha_T^c + \nu_A^c \alpha_A^c \right]}{1 - (\nu_A^c)^2 E_T^c / E_A^c} \,. \tag{33}$$

On substituting (26) and (27) into (17) and (18) expressions for σ_i and σ_i^* can be derived in terms of the parameters σ, σ_t, p and ΔT. On using (12) and (16) the following expression for the effective through-thickness strain is obtained

$$\hat{\varepsilon} = \frac{1}{h} \sum_{i=1}^{2} h_i \hat{\varepsilon}_i \equiv - \frac{p}{E_t^c} - \frac{\nu_a^c}{E_A^c} \sigma - \frac{\nu_t^c}{E_T^c} \sigma_t + \alpha_t^c \Delta T \,, \tag{34}$$

where the through-thickness transverse modulus is given by

$$\frac{1}{E_t^c} = \frac{1}{h}\sum_{i=1}^{2} h_i \left[\frac{1}{E_t^i} + \frac{\hat{E}_A^i}{E_A^i} \left\{ \left(\frac{\nu_a^c}{E_A^c} - \frac{\nu_a^i}{E_A^i} \right) \hat{\nu}_a^i + \left(\frac{\nu_t^c}{E_T^c} - \frac{\nu_t^i}{E_T^i} \right) \hat{\nu}_t^i \right\} \right], \tag{35}$$

and where the through-thickness thermal expansion coefficient is given by

$$\alpha_t^c = \frac{1}{h}\sum_{i=1}^{2} h_i \left[\alpha_t^i - \frac{\hat{E}_A^i}{E_A^i} \left(\hat{\nu}_a^i \alpha_A^c + \hat{\nu}_t^i \alpha_T^c - \nu_a^i \hat{\alpha}_A^i - \nu_t^i \hat{\alpha}_T^i \right) \right]. \tag{36}$$

Thus all the required thermo-elastic constants for the undamaged laminate have now been determined.

7. Calculation of the Gibbs free energy

When written in tensor form the Gibbs free energy per unit volume for a homogeneous medium may be written

$$g = -\frac{1}{2} S_{ijkl}\, \sigma_{ij}\, \sigma_{kl} - \sigma_{ij}\, \alpha_{ij}\, \Delta T + g_o(\Delta T)\,, \tag{37}$$

where S_{ijkl} are the elastic constants and α_{ij} are the thermal expansion coefficients. On using (11) the relation (37) may be written in terms of the thermoelastic constants as follows

$$\begin{aligned} g = &-\frac{1}{2}\frac{\sigma^2}{E_A} - \frac{1}{2}\frac{\sigma_t^2}{E_T} - \frac{1}{2}\frac{p^2}{E_t} + \frac{\nu_A}{E_A}\sigma\sigma_t - \frac{\nu_a}{E_A}\sigma p - \frac{\nu_t}{E_T}\sigma_t p \\ &- \left[\sigma\alpha_A + \sigma_t\alpha_T - p\,\alpha_t\right]\Delta T + g_o(\Delta T)\,. \end{aligned} \tag{38}$$

It should be noted that the stress-strain relations of the type (26), (27) and (34) are recovered by differentiation as follows

$$\varepsilon = -\frac{\partial g}{\partial \sigma}\,, \quad \varepsilon^* = -\frac{\partial g}{\partial \sigma_t}\,, \quad \hat{\varepsilon} = \frac{\partial g}{\partial p}\,. \tag{39}$$

8. Criterion for delamination

Steady state delamination growth, under fixed applied effective stresses and temperature, is governed by the energy balance equation $\Delta G + \Delta\Gamma = 0$, where ΔG is the change of Gibbs free energy for a finite increment of delamination growth during which an amount $\Delta\Gamma$ of energy is absorbed at the crack tips. For the case under discussion the energy balance equation may be written

$$G^{I} - G^{II} = 4\gamma , \tag{40}$$

where G^I is the Gibbs free energy in unit length of the uncracked Region I, and where it is assumed that unit width of laminate is being considered. Similarly G^{II} is the Gibbs free energy in unit length of Region II where delamination has occurred. The fracture energy for delamination is denoted by γ and the factor four appears because four fracture surfaces are generated when the two delamination cracks grow. The superscripts I and II are used rather than subscripts in order to avoid confusion with the mode I and mode II strain energy release rates that are normally written G_I and G_{II} respectively.

9. Gibbs free energy for Region I

In Region I of the cross-ply laminate the analysis of section 6 is valid. It follows from (38) that the Gibbs free energy in this region having volume 2h is given by

$$G^{I} = 2h\left[-\frac{1}{2}\frac{\sigma^2}{E_A^c} - \frac{1}{2}\frac{\sigma_t^2}{E_T^c} - \frac{1}{2}\frac{p^2}{E_t^c} + \frac{\nu_A^c}{E_A^c}\sigma\sigma_t - \frac{\nu_a^c}{E_A^c}\sigma p \right. \\ \left. - \frac{\nu_t^c}{E_T^c}\sigma_t p - \left(\sigma\alpha_A^c + \sigma_t\alpha_T^c - p\alpha_t^c\right)\Delta T + \phi_o(\Delta T) \right], \tag{41}$$

where the function $\phi_o(\Delta T)$ is the Gibbs free energy per unit volume of the laminate when $\sigma = \sigma^* = p = 0$. It takes account of the thermal residual stresses and is defined by (see [4, eqs.(B31, B35)])

$$\phi(\Delta T) = -\frac{\sigma_o^2}{2E_A^c} - \frac{(\sigma_o^*)^2}{2E_T^c} + \frac{\nu_A^c}{E_A^c}\sigma_o\sigma_o^* + \frac{h}{2E_A}\left(\frac{\sigma_o^2}{h_2} + \frac{(\sigma_o^*)^2}{h_1}\right) + g_o(\Delta T) , \tag{42}$$

where $g_o(\Delta T)$ is the Gibbs free energy per unit volume for the stress-free state, of the unidirectional composite used for both the 0° and 90° plies. The stresses σ_o and σ_o^* are the values of the axial and transverse applied stresses for which transverse stresses in the 0° plies, the axial stress in the 90° ply and the through-thickness pressure p are all zero. For this state any cracks in the 0° plies and the 90° plies would have just closed. The stresses σ_o and σ_o^* are given by (see [4, Appendix C])

$$\sigma_o = \frac{h_2 E_A}{h}\frac{\alpha_T - \alpha_A - \nu_A(\alpha_T - \alpha_A)}{1 - \nu_A^2}\Delta T , \tag{43}$$

$$\sigma_o^* = \frac{h_1}{h_2}\sigma_o . \tag{44}$$

While (43) was derived in [4] for the case when $E_t = E_T$, $\nu_a = \nu_A$, $\nu_t = \nu_T$, $\alpha_t = \alpha_T$, the result remains valid for the more general case when these quantities are not equal.

10. Gibbs free energy for Region II

In Region II the analysis is more simple than for Region I as the plies are regarded as being detached so that they can slip relative to one another. The interface is, however, assumed to be freely slipping (i.e. there is no frictional stress transfer even when a through-thickness compressive stress p is acting). It is assumed that in region II both the 0° and 90° plies share the same transverse strain ε_{II}^* that will differ from the transverse strain ε_I^* encountered in Region I.

Since the axial stress in the 90° ply is zero, it follows from (8) and (9) that the state of stress and strain in the 90° ply is described by

$$\varepsilon_1 = \frac{\nu_t}{E_T} p - \frac{\nu_A}{E_A} \sigma_1^* + \alpha_T \Delta T , \tag{45}$$

$$\varepsilon_{II}^* = \frac{\nu_a}{E_A} p + \frac{\sigma_1^*}{E_A} + \alpha_A \Delta T , \tag{46}$$

where σ_1 (= 0) and σ_1^* are respectively the axial and transverse stresses in the 90° ply, and where ε_1 and ε_{II}^* are the corresponding axial and transverse strains respectively.

On using (8) and (9) the state of stress and strain in the 0° plies is described by the equations

$$\varepsilon_2 = \frac{\nu_a}{E_A} p + \frac{\sigma_2}{E_A} - \frac{\nu_A}{E_A} \sigma_2^* + \alpha_A \Delta T , \tag{47}$$

$$\varepsilon_{II}^* = \frac{\nu_t}{E_T} p - \frac{\nu_A}{E_A} \sigma_2 + \frac{\sigma_2^*}{E_T} + \alpha_T \Delta T , \tag{48}$$

where σ_2 and σ_2^* are respectively the axial and transverse stresses in the 0° plies, and where ε_2 and ε_{II}^* are the corresponding axial and transverse strains respectively.

From (15) mechanical equilibrium in the axial direction demands that

$$\sigma_2 = \frac{h}{h_2} \sigma , \tag{49}$$

and in the transverse direction

$$h_1 \sigma_1^* + h_2 \sigma_2^* = h \sigma_t . \tag{50}$$

From (46) it follows that

$$\sigma_1^* = E_A\left[\varepsilon_{II}^* - \frac{\nu_a}{E_A}p - \alpha_A\Delta T\right], \tag{51}$$

and from (48) and (49)

$$\sigma_2^* = E_T\left[\varepsilon_{II}^* - \frac{\nu_t}{E_T}p + \frac{\nu_A}{E_A}\frac{h\sigma}{h_2} - \alpha_T\Delta T\right]. \tag{52}$$

On substituting (51) and (52) in (50) it follows that

$$\left[h_1E_A + h_2E_T\right]\varepsilon_{II}^* = h\left[\sigma_t - \nu_A\frac{E_T}{E_A}\sigma\right] + \left[h_1\nu_a + h_2\nu_t\right]p + \left[h_1E_A\alpha_A + h_2E_T\alpha_T\right]\Delta T, \tag{53}$$

thus enabling the transverse strain ε_{II}^* to be calculated in terms of the prescribed quantities σ, σ_t, p and ΔT. The non-zero stresses in the plies are then given in terms of σ, σ_t, p and ΔT on substituting (53) in (49), (51) and (52).

The following expression for the Gibbs free energy for unit length of the 90° ply (having volume $2h_1$) in Region II is obtained on using (38)

$$G_1^{II} = 2h_1\left[-\frac{(\sigma_1^*)^2}{2E_A} - \frac{p^2}{2E_t} - \frac{\nu_a}{E_A}\sigma_1^*p - (\sigma_1^*\alpha_A - p\,\alpha_t)\Delta T\right] + 2h_1\,g_o(\Delta T). \tag{54}$$

The corresponding Gibbs free energy for unit length of the 0° ply (having volume $2h_1$) in Region II

$$G_2^{II} = 2h_2\left[-\frac{\sigma_2^2}{2E_A} - \frac{(\sigma_2^*)^2}{2E_T} - \frac{p^2}{2E_t} + \frac{\nu_A}{E_A}\sigma_2\sigma_2^* - \frac{\nu_a}{E_A}\sigma_2 p - \frac{\nu_t}{E_T}\sigma_2^*p - (\sigma_2\alpha_A + \sigma_2^*\alpha_T - p\,\alpha_t)\Delta T\right] + 2h_2\,g_o(\Delta T). \tag{55}$$

The Gibbs free energy per unit length of laminate for region II is then given by

$$G^{II} = G_1^{II} + G_2^{II}. \tag{56}$$

11. Prediction of steady state delamination stress

On substituting (41) and (56) into the delamination condition (40) using (54) and (55), the resulting complicated equation can be written in the form

$$f(\sigma, \sigma_t, p, \Delta T) = 4\gamma. \tag{57}$$

It is assumed that the temperature difference ΔT is prescribed at a fixed value that does not vary during loading. It should be noted that the function f is independent of the function g_o characterising the Gibbs free energy of an unloaded sample of ply material.

It is convenient to allow the axial, transverse and through-thickness applied stresses to be proportional during loading by letting

$$\sigma = s, \quad \sigma^* = K_1 s, \quad p = K_2 s, \tag{58}$$

where K_1 and K_2 are constants characterising the nature of the biaxial and through-thickness loading and where s is the axial loading parameter. It follows from (49), (51) and (52) that the stresses σ_1^*, σ_2 and σ_2^* are then linear functions of s, in which case the equation (57) may be expressed in the form $F(s) = 4\gamma$ where F is a quadratic function of s, i.e.

$$F(s) = Ls^2 + Ms + N = 4\gamma, \tag{59}$$

where the coefficients A, B and C can be obtained from the function F as follows

$$N = F(0), \quad L = \frac{1}{2}Y - X, \quad M = 2X - \frac{1}{2}Y, \tag{60}$$

where

$$X = F(1) - F(0), \quad Y = F(2) - F(0). \tag{61}$$

The values of the coefficients are calculated numerically from the function F that is defined using (40) and (57). The required solution of the quadratic equation (59) is the least positive root, which is identified with the stress for steady state delamination for the triaxial stress state defined by (58) using the appropriate values of the parameters K_1 and K_2.

Numerical calculations have shown that the quadratic form F(s) defined by (59) is in fact a perfect square which is expressed in the form

$$F(S) = \left[\alpha s + \beta \Delta T\right]^2, \tag{62}$$

where α is a linear function of K_1 and K_2, and where β is independent of K_1 and K_2 but dependent on the ply properties and the geometry. The parameter α is written in the form

$$\alpha = q_0 + q_1 K_1 + q_2 K_2, \tag{63}$$

where q_0, q_1 and q_2 also depend on the ply properties and geometry. It then follows from (59) that the criterion governing steady state delamination growth is

$$\left[(q_0 + q_1 K_1 + q_2 K_2)s + \beta \Delta T\right]^2 = 4\gamma. \tag{64}$$

Thus on selecting the positive branch when taking the square root, the steady state delamination stress is given by

$$s = \frac{\sqrt{4\gamma} - \beta \Delta T}{q_0 + q_1 K_1 + q_2 K_2}. \tag{65}$$

The influence of the stress ratios K_1 and K_2, and the temperature difference ΔT, on the steady state delamination stress is now clearly identified. It is emphasised that the relationship (64) has been derived by careful examination of numerical results. To establish the result by analytical methods is exceedingly difficult and has not yet been attempted.

12. Preliminary results

Preliminary predictions are made using sets of values for the thermoelastic properties of two different types of composite. The first set of materials properties selected for predictions are for a laminate whose ply properties correspond to a filament wound composite where Silenka E-glass 1200tex fibre reinforces MY750/HY917/DY063 epoxy [6], namely

$E_A = 45.6$ GPa $\quad E_T = 16.2$ GPa $\quad \nu_A = 0.278$
$\nu_T = 0.4$ $\quad \mu_A = 5.83$ GPa
$\alpha_A = 8.6 \times 10^{-6}/°C$ $\quad \alpha_T = 26.4 \times 10^{-6}/°C$.

The second set of properties is for a typical carbon fibre/epoxy laminate whose ply properties are given by

$E_A = 136.6$ GPa $\quad E_T = 9.79$ GPa $\quad \nu_A = 0.286$
$\nu_T = 0.455$ $\quad \mu_A = 6.474$ GPa
$\alpha_A = -0.458 \times 10^{-6}/°C$ $\quad \alpha_T = 38.2 \times 10^{-6}/°C$.

These values (used in [7]) are for plies having a 65% volume fraction of HTA 5131 carbon fibres in Ciba Fiberdux F922 resin system. The above material properties for both GRP and CFRP are assumed to be transverse isotropic so that $E_t = E_T$, $\mu_a = \mu_A$, $\mu_t = \mu_T$, $\nu_a = \nu_A$, $\nu_t = \nu_T$, $\alpha_a = \alpha_A$, $\alpha_t = \alpha_T$, such that $E_T = 2\mu_T(1 + \nu_T)$. In order to determine the effect of composite type on properties all predictions have taken the temperature difference $\Delta T = 0$, and the fracture energy (2γ) for delamination has been taken as 150 J/m^2 (also used in [7] for the case of a brittle matrix carbon fibre composite).

Fig.2 shows the results of predictions of the steady state delamination stress, as a function of the through-thickness pressure ratio K_2 defined by (58), for a GRP composite for three different values of the transverse stress ratio K_1. It is seen that for all positive values of the pressure ratio K_2 the delamination stress is enhanced if the transverse applied stress is compressive with $K_1 = -1$, but lowered if it is tensile with $K_1 = 1$. Fig.3 shows the corresponding predictions for the CFRP laminate showing that higher steady delamination stresses are predicted for CFRP when compared to GRP, assuming the same value for the fracture energy for delamination. It should also be noted that much lower through-thickness pressure ratios are needed to achieve the same delamination stresses that are predicted by the corresponding GRP laminate. If the through-thickness pressure ratio is increased further it is found that the delamination stress tends to infinity for a finite value of the pressure ratio. This behaviour is to expected from (65) for values of the stress ratios K_1 and K_2 for which

$$q_0 + q_1 K_1 + q_2 K_2 = 0 \,. \tag{66}$$

Beyond this critical value the delamination stress predicted by (65) is negative, monotonically increasing from the value $-\infty$ at the critical point. Beyond the critical point positive values for the delamination stress can be predicted if the negative branch is taken when solving (64). This second possible solution is discounted as the laminate would have completely delaminated before such transverse pressure ratios could be reached, a situation for which the model is invalid.

The effect of laminate lay-up is investigated by considering GRP laminates of the type

$[0_m/90_n]$ where a single ply has thickness 0.125 mm. Fig.4 shows the dependence of the delamination stress on through-thickness pressure ratio when $K_1 = 0$ and $\Delta T = 0$. Increasing the thickness of the 90° plies while keeping the total ply thickness fixed at 1 mm leads to a significant reduction in the delamination stress for all values of the through-thickness pressure ratio. A significant increase in the delamination stress occurs when the thickness of the 0° plies is increased.

Having investigated the effects of varying the stress ratios for proportional loading defined by (58) and ply lay-up, it is appropriate to consider now the effects of varying the fracture energy 2γ and the temperature difference ΔT. These calculations are carried out for the GRP laminate with the above-specified properties for the case when $h_1 = h_2 = 0.25$ mm. On selecting the temperature difference $\Delta T = 0$, Fig.5 shows the steady state delamination stresses predicted by the model for three values of the fracture energy $2\gamma = 150$, 300 and 450 J/m^2. For the case $\Delta T = 0$ it follows from (65) that the delamination stress is proportional to the square root of the fracture energy.

Finally, the effect of varying the temperature difference ΔT is considered for a single value of the fracture energy given by $2\gamma = 300\ J/m^2$. The predicted delamination stresses are shown in Fig.6 where it is seen for values ΔT = 0°C, -50°C, -100°C that increasing the magnitude of the temperature difference decreases the steady state delamination stress. It should be noted from (65) that, for any given value of the through-thickness pressure ratio, the delamination stress is to within numerical error linearly related to the temperature difference.

13. Conclusions

1) Predictions, based on energy balance methods, can be made for the steady state delamination stress associated with transverse cracks in simple cross-ply laminates for triaxial loading conditions, taking account of thermal residual stresses.

2) Laminates made from stiff plies (as for carbon fibre laminates) are more resistant to steady state delamination when compared to lower stiffness plies (as for glass fibre laminates).

3) The steady state delamination stress is linearly related to the temperature difference ΔT.

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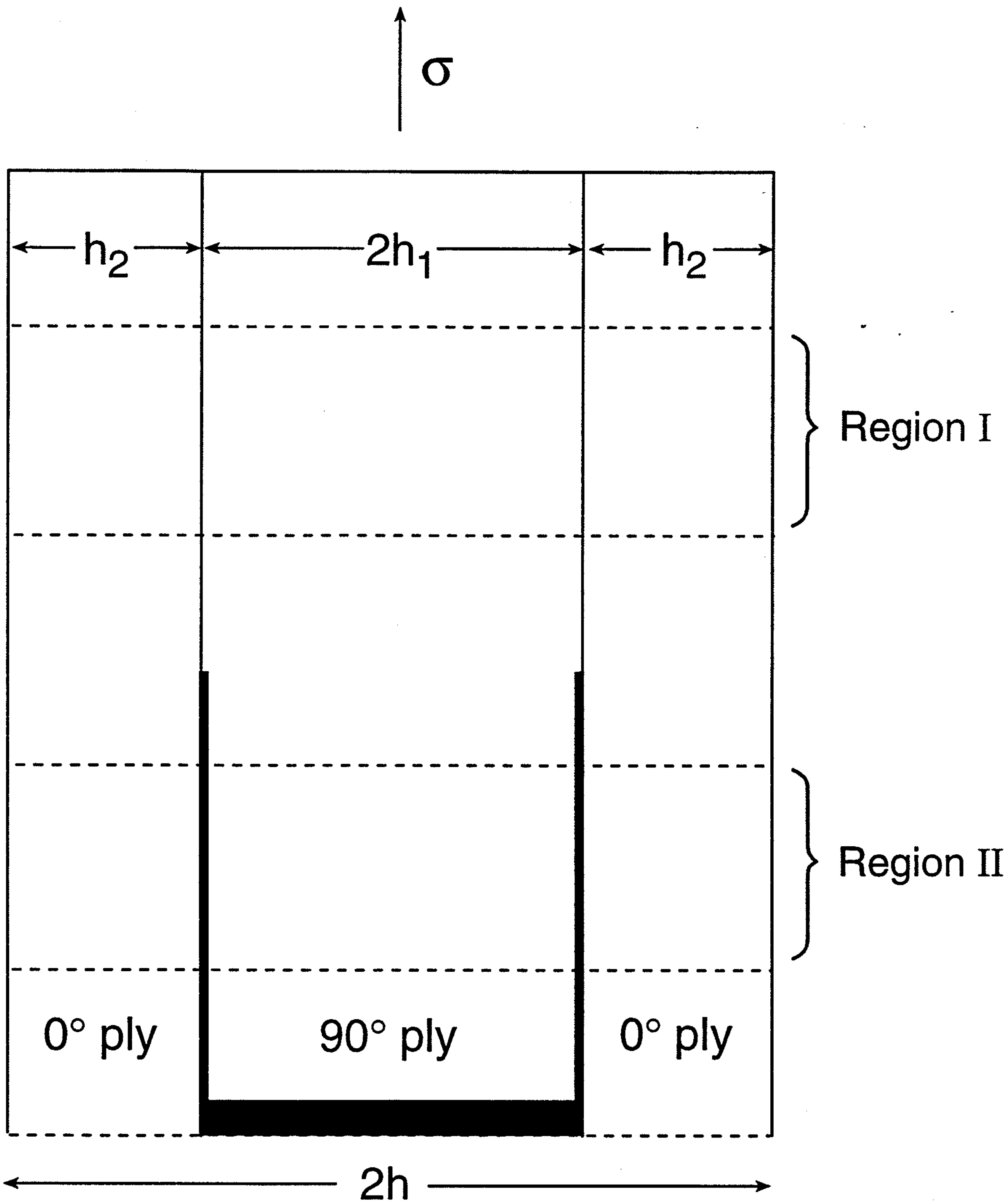

FIG.1 : Schematic diagram of the edge view of a cross-ply laminate showing delaminations initiated from a transverse crack.

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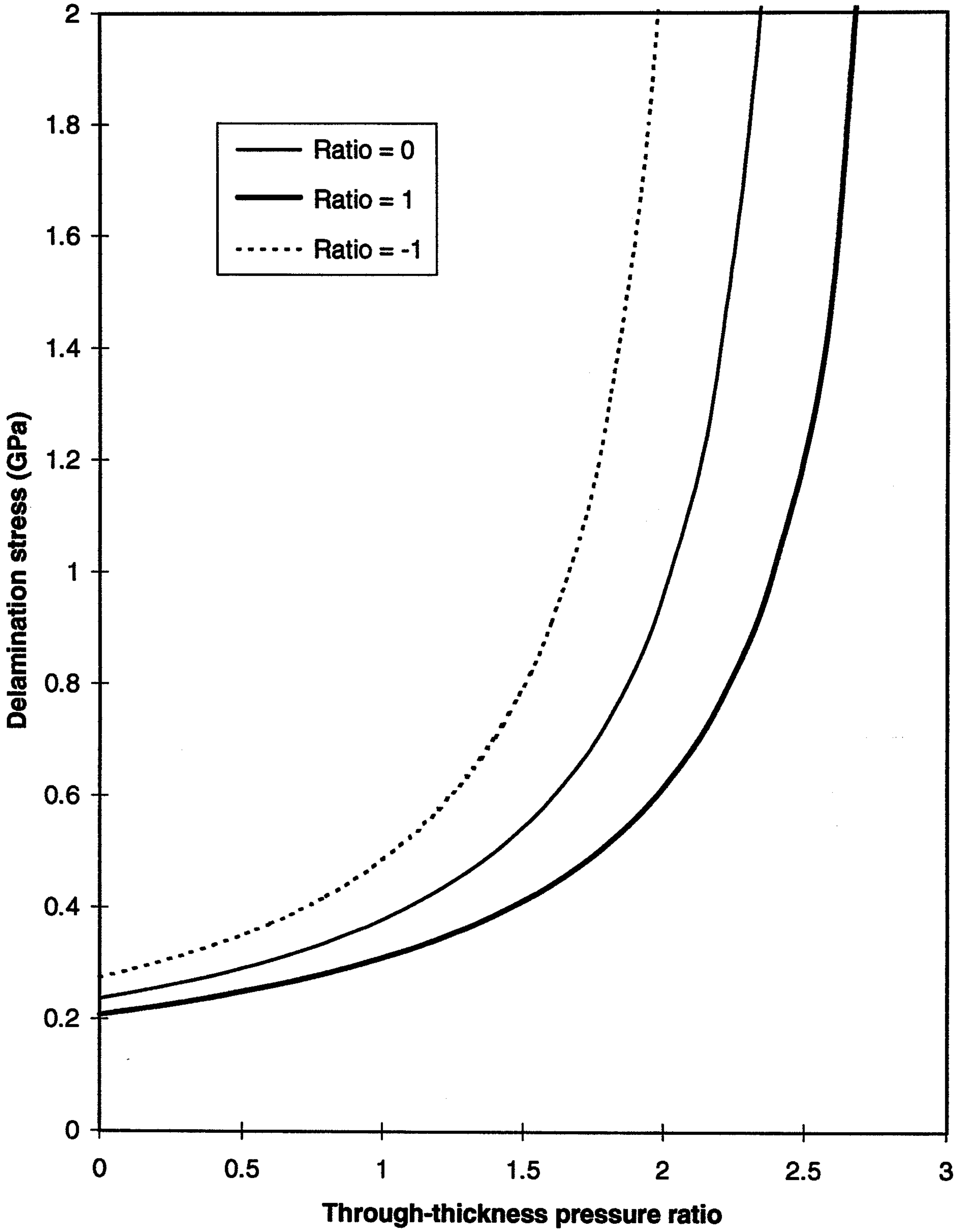

FIG.2 : Dependence of steady state delamination stress for a GRP cross-ply laminate on the through-thickness pressure ratio K_2 when $\Delta T = 0$ for three different ratios for transverse loading given by $K_1 = 0, 1, -1$.

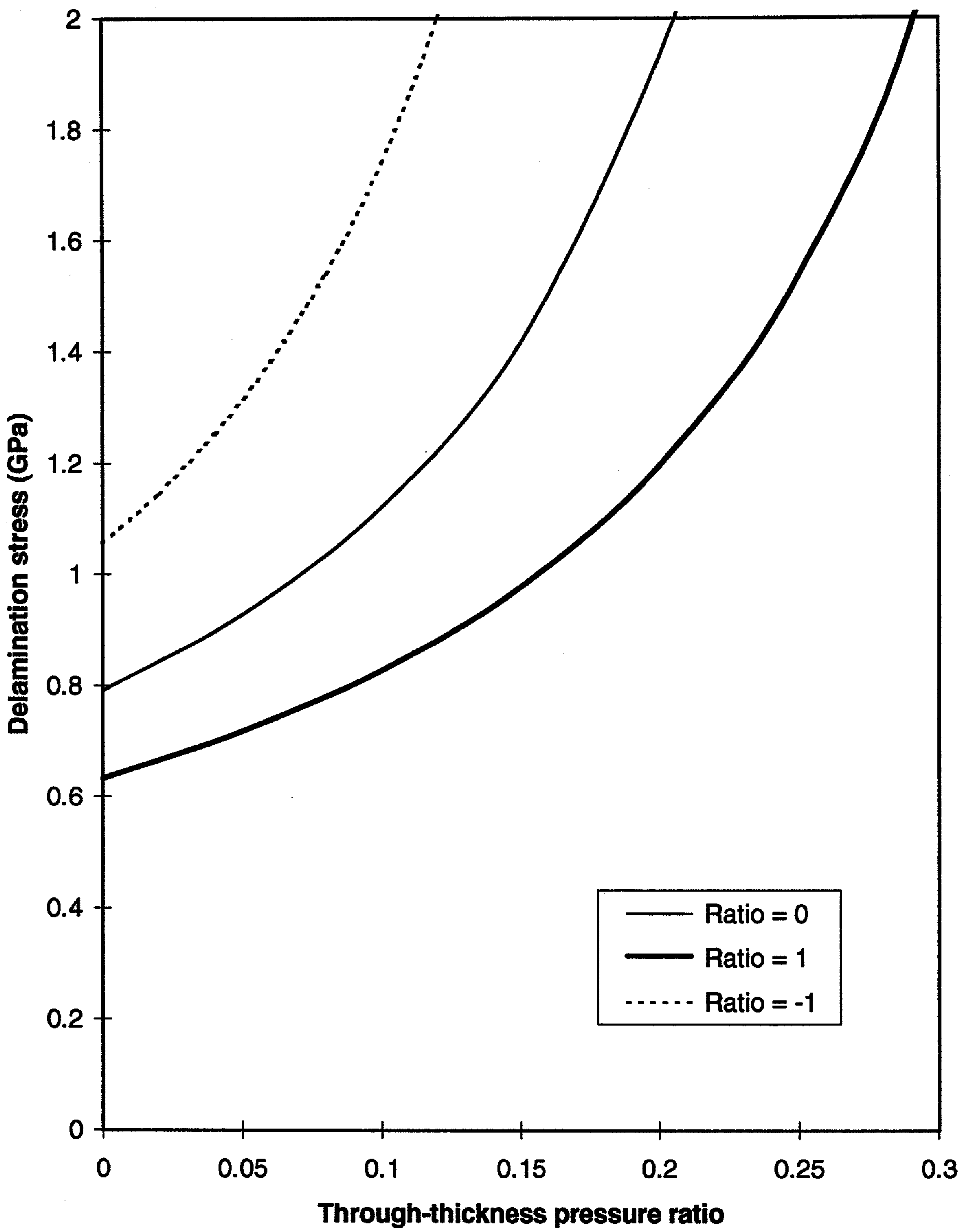

FIG.3 : Dependence of steady state delamination stress for a CFRP cross-ply laminate on the through-thickness pressure ratio K_2 when $\Delta T = 0$ for three different ratios for transverse loading given by $K_1 = 0, 1, -1$.

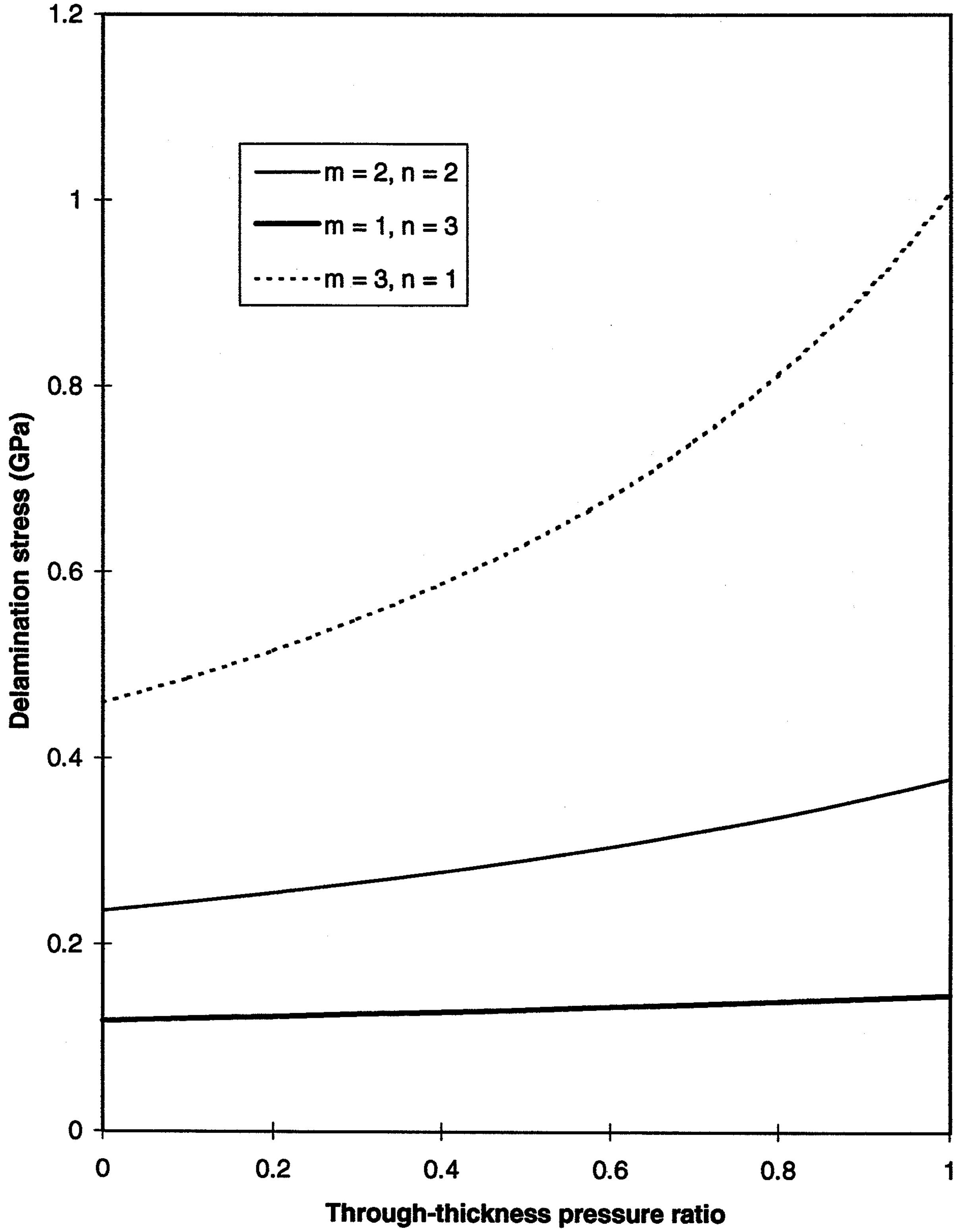

FIG.4: Dependence of steady state delamination stress for a CFRP cross-ply laminate on the through-thickness pressure ratio K_2 when $K_1 = 0$ and $\Delta T = 0$ for three different laminates of the type $[0_m/ 90_n]_s$ when ply thickness = 0.125mm.

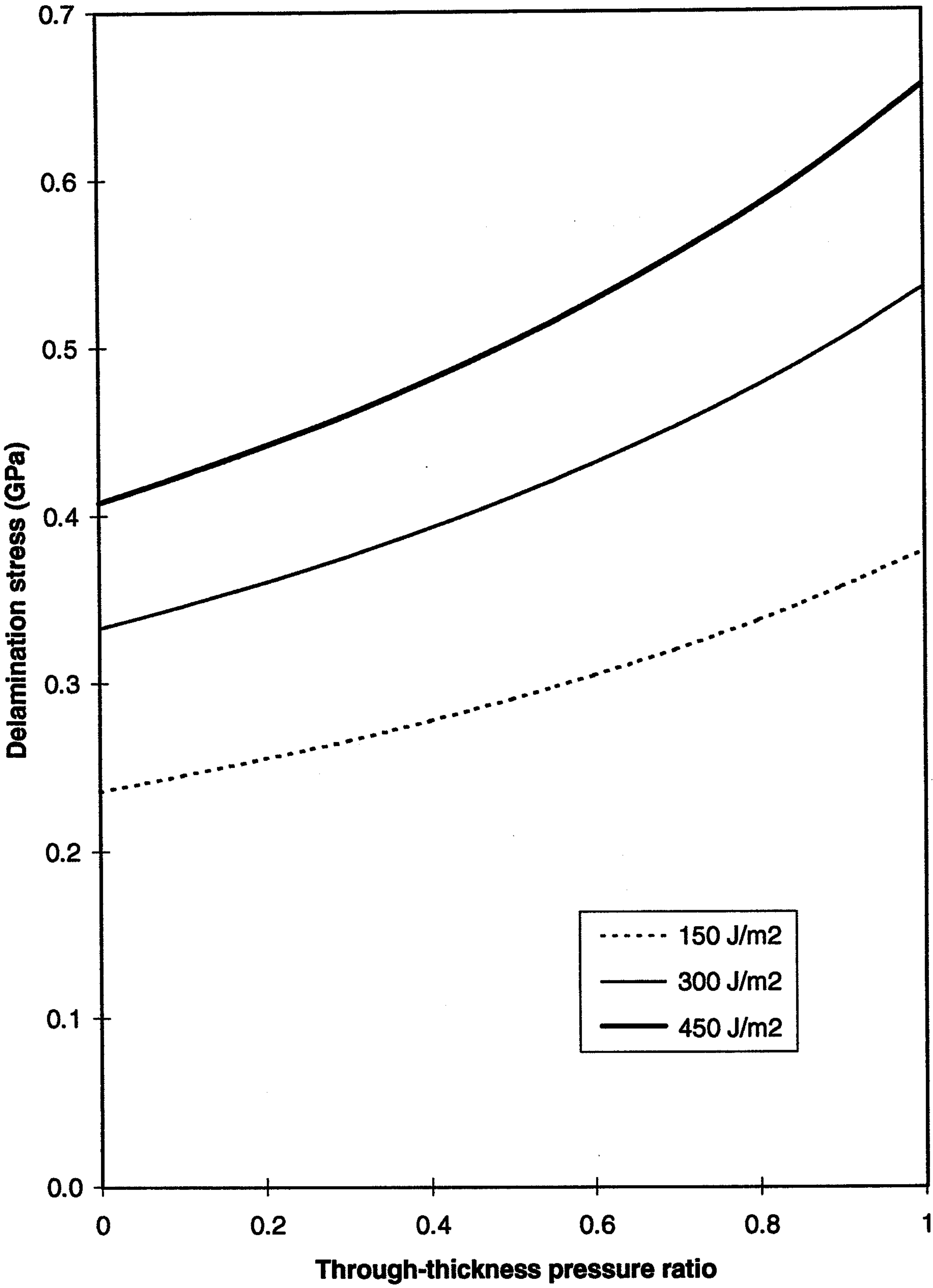

FIG.5 : Steady state delamination stresses as a function of the through-thickness pressure ratio K_2 for a GRP cross-ply laminate when $K_1 = 0$, $\Delta T = 0$ for values of $2\gamma = 150, 300$ and $450\ J/m^2$.

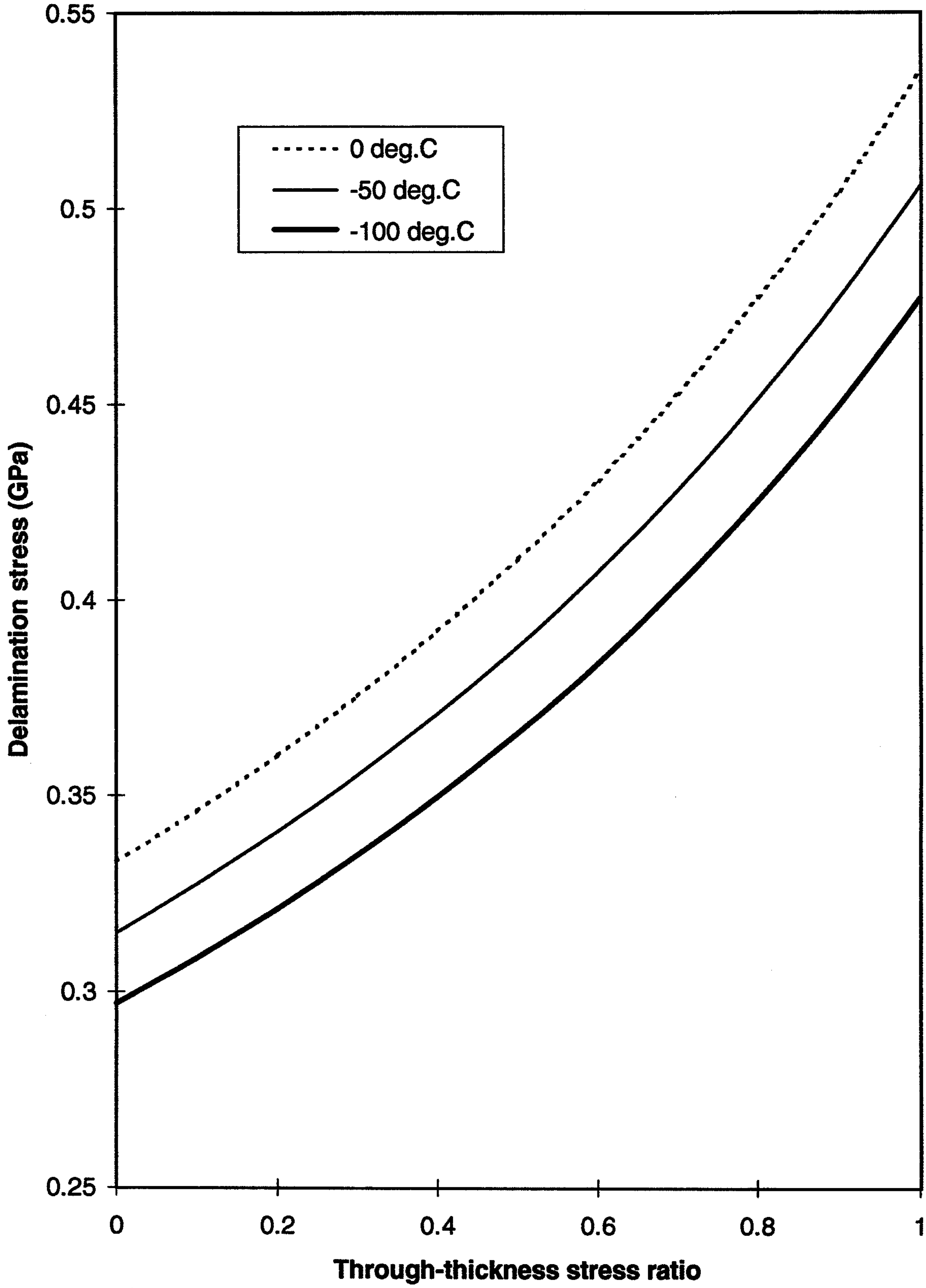

FIG.6 : Steady state delamination stresses as a function of the through-thickness pressure ratio K_2 for a GRP cross-ply laminate when $K_1 = 0$, $2\gamma = 300\ J/m^2$ for values $\Delta T = 0°C, -50°C, -100°C$.