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**THE EFFECT OF TEST-PIECE CHAMFERS ON THE
DETERMINATION OF ELASTIC MODULI USING THE IMPACT
EXCITATION METHOD**

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ABSTRACT

An experimental analysis of the effect of chamfers on the long edges of bar test-pieces employed for the determination of elastic properties by the impact excitation or resonance methods given in various standards (ASTM C1259 Annex, ASTM C1198 Annex) has been undertaken. The published flexural mode chamfer correction based on moment of inertia works appropriately, but the suggestion that the nominal density value has also to be corrected applies only if the density version of the calculation equation is used, and not if the mass version, as written in the standards, is used.

These standards have no equivalent chamfer correction for determining shear modulus by torsional vibration. Generic correction equations have been developed from experimental results that can be applied to cross-sectional aspect ratios $4 > b/t > 1.3$ and chamfer sizes $c/t < 0.3$. These enable Poisson's ratio to be determined correctly from chamfered bars.

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Approved on behalf of NPLML by
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1 INTRODUCTION

The impact excitation of vibration method for determining elastic moduli using a prismatic bar or disc test-piece is now well-established. Commercial equipment is available. The method has been incorporated into international standards, such as CEN EN843-2 [1] and ASTM C1259 [2] for advanced technical ceramics, and ASTM E1876 [3] for metals and alloys. In the case of bar test-pieces, the test-piece is typically supported at the fundamental flexural nodal positions (0.224 of the length from each end), and is struck centrally to induce flexural vibrations, the fundamental flexural frequency is then determined and used to calculate dynamic (adiabatic) Young's modulus. If the test-piece is supported centrally and struck on one corner, torsional mode vibrations can be induced, and the fundamental mode torsional frequency is used to calculate shear modulus. Young's modulus and shear modulus can then be used to calculate Poisson's ratio provided that the material is elastically isotropic:

$$\mu = \frac{E}{2G} - 1$$

There are no prescribed dimensions for test-pieces, but ideally they need to be large enough to ensure that the fundamental mode frequencies can be struck reliably and detected by an appropriate means, typically a high-frequency microphone. In order to induce torsional mode vibrations, the test-piece generally has to be of sufficient width that the sound intensity is detectable among the other frequencies.

The standards assume the use of perfect prismatic bar test-pieces. In many cases, the fabrication of suitable test-pieces is non-trivial. In the case of ceramics and glass test-pieces, it is usually necessary to apply a long-edge chamfer to remove edge chipping, especially for the case of flexural strength testing. ASTM C1259 [2] contains a correction for the effects of a chamfer on the calculation of flexural Young's modulus, based on [4]. This correction comprises two parts, a correction for the change in cross-sectional moment of inertia, and a correction for a change to nominal density. However, there is no correction function for the effects of chamfers on the torsional behaviour. Clearly, if the method is to produce the correct Poisson's ratio for chamfered bars, the torsional mode also requires correction. However, this is a non-trivial problem for which there is no prior information or solution in the literature.

We initiated a study of chamfers using mild steel bars of a variety of cross-sectional aspect ratios, commencing with the unchamfered condition, and progressively applying an increasing size of chamfer to all four long edges. This report gives the results of this study.

2 THEORY

The theory for the natural vibration frequencies of beams is long-standing and well established, and is based on the solution of equations of motion of thin beams. A review was prepared by Spinner and Tefft [5] which states that the best equation for rectangular section prisms is that of Pickett [6], where:

$$E = 0.94642 \left\{ \frac{\rho L^4 f_{f1}^2}{t^2} \right\} T$$

where:

E is Young's modulus

ρ is the density

L is the length

f_{f1} is the fundamental flexural mode frequency

t is the test-piece thickness

and T is a complex numerically fitted function incorporating the ratio t / L and Poisson's ratio μ , and has a value ~ 1 for thin beams:

$$T = 1 + 6.585(1 + 0.0752\mu + 0.8109\mu^2)\left(\frac{t}{L}\right)^2 - 0.868\left(\frac{t}{L}\right)^4 - \left\{ \frac{8.34(1 + 0.2023\mu + 2.173\mu^2)(t/L)^4}{1 + 6.338(1 + 0.1408\mu + 1.536\mu^2)(t/L)^2} \right\}$$

Torsional vibration modes for prismatic bars are more difficult to analyse. Whereas a cross-sectional plane in a cylindrical rod stays planar, that in any other cross-sectional shape does not, and this requires a complex function of shape. The basic equation is:

$$G = 4\rho L^2 f_{t1}^2 R$$

where:

f_{t1} is the fundamental torsional mode frequency

and $R = 1$ for rod, but is a complex function of cross-sectional shape not readily determined other than by experimental means, as explained in [5]. The formulation now being used in ASTM C1259 and ASTM E1876 derives from Spinner and Valore [7]:

$$R = \left[\frac{1 + \left(\frac{b}{t}\right)^2}{4 - 2.521\left(\frac{t}{b}\right)\left(1 - \frac{1.991}{\exp\left(\frac{\pi b}{t}\right) + 1}\right)} \right] \left(\left(1 + \left[\frac{0.00851b^2}{L^2}\right]\right) - 0.060\left(\frac{b}{L}\right)^{3/2}\left(\frac{b}{t} - 1\right)^2 \right)$$

where b is the test-piece width. This is considered to be accurate to 0.2% for $b/L < 3$ and $b/t < 10$. Earlier versions of these standards, and also the existing EN standards for impact excitation, all ISO and some ASTM standards for the resonance method, use a different, earlier formulation given by Pickett [6]:

$$R = \left[\frac{\frac{b}{t} + \frac{t}{b}}{4\left(\frac{t}{b}\right) - 2.52\left(\frac{t}{b}\right)^2 + 0.21\left(\frac{t}{b}\right)^6} \right] / (1 + A)$$

where A :

$$A = \frac{0.5062 - 0.8776\left(\frac{b}{t}\right) + 0.3504\left(\frac{b}{t}\right)^2 - 0.0078\left(\frac{b}{t}\right)^3}{12.03\left(\frac{b}{t}\right) + 9.892\left(\frac{b}{t}\right)^2}$$

which is the range 0 to 0.01 for the typical b/t range. It is a correction factor for torsion/flexural coupling effects.

The basic formulae presented in standards are based on the more conveniently measured mass, m , rather than density ρ . For a rectangular section bar, the latter parameter is substituted by m / Lbt .

The stiffness of the beam in flexure is effectively controlled by the cross-sectional moment of inertia, I , which for a rectangular section is given by:

$$I = \frac{bt^3}{12}$$

Quinn and Swab [4] have computed the effect of placing 45° chamfers or roundings of various radii on the cross-sectional moment of inertia as a correction factor to the calculation of Young's modulus E . In the case of equal 45° chamfers to all four long edges:

$$I' = \frac{bt^3}{12} - \frac{c^2}{9} \left(c^2 - \frac{1}{2}(3t - 2c)^2 \right)$$

where c is the size of the chamfer as seen on the face or edges of the bar. This effective reduction in moment of inertia means that an upward correction to Young's modulus is required by multiplying the result computed assuming no chamfer by I/I' . This correction appears in Annex C1259 for advanced ceramics, and C1198 (resonance method) for advanced ceramics [8].

These authors have also made a correction for the change in effective density; for equal 45° chamfers:

$$\rho' = \frac{m}{L(bt - 2c^2)}$$

but this would only apply if the density version of the modulus formula were to be used. As the present work will demonstrate, it is not appropriate to use this second correction factor if the mass version of the equation, as appears in the standards, is used, so this can be misleading.

There are no equivalent corrections for the presence of chamfers in shear modulus determination, which means that it is currently not possible to compute a correct value for Poisson's ratio from the apparent Young's modulus and shear modulus.

3 TEST-PIECES

These were prepared from standard flat mild steel bar stock by EDM machining and minimal fine-grit surface grinding in NPL Workshops to produce a set of bars approximately 100 mm in length by 4.85 mm thickness by various widths from 20 mm down to 7.8 mm, initially without chamfers.

Cross-sectional dimensions of the test-pieces were measured using a calibrated micrometer reading to the nearest 0.001 mm. The test-piece length was measured using calibrated digital calipers reading to the nearest 0.01 mm. The test-piece mass was measured to the nearest 0.0001 g using a calibrated five-figure balance operating on the four-figure scale.

Chamfering was undertaken by hand on flat SiC paper laps, attempting to achieve equal 45° angles of similar size on all four long edges. The chamfer widths were measured on each face at the test-piece centre, giving a total of eight measurements, using a Nikon measuring microscope. For the purposes of analysis the average chamfer size was employed. Each test-piece was subject to four different chamfer levels.

4 IMPACT EXCITATION TECHNIQUE

The measurements were made using equipment supplied by IMCE, Belgium, comprising a high-frequency microphone and software triggered to detect the impact sound on striking the test-piece. The frequency spectrum was determined using a fast Fourier transform method, and the frequency peaks were identified.

Each test-piece in turn was supported on taut nylon strings at the fundamental flexural nodal positions, and struck centrally to produce predominantly the flexural modes, or was struck on one corner to produce predominantly torsional modes. The frequency peaks were automatically recorded. At least five strikes were used for each chamfer size, and the average frequencies determined.

All tests were undertaken at controlled ambient temperature of 22 ± 0.5 °C in order to restrict the effects of variable thermal expansion.

5. RESULTS

5.1 EFFECT OF CHAMFERING ON YOUNG'S MODULUS DETERMINATION

The test-piece and chamfer dimension data, the mass, and the average frequencies (Table 1) from several strikes were incorporated into a spreadsheet which was used to calculate the 'nominal' Young's modulus and shear modulus.

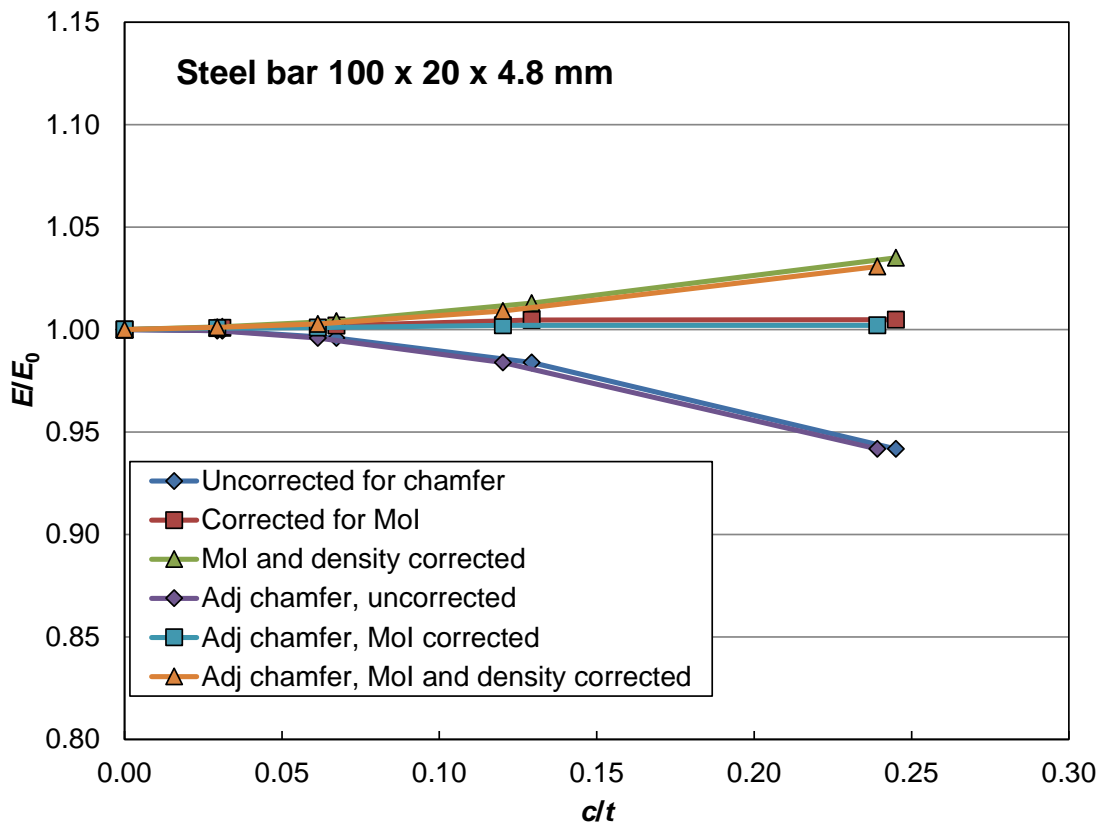
Table 1: Fundamental mode natural frequencies (F1, T1) determined for chamfered bars

Nominal bar size, mm	Chamfer size* and frequencies	Chamfer no.				
		0	1	2	3	4
100 x 20 x 4.8	<i>c/t</i>	0, 0	0.0311, 0.0294	0.0673, 0.0614	0.1293, 0.1202	0.2450, 0.2391
	F1, Hz	2593.28	2593.14	2590.24	2581.50	2552.44
	T1, Hz	7161.16	7167.25	7182.29	7230.68	7378.19
100 x 17 x 4.8	<i>c/t</i>	0, 0	0.0377, 0.0320	0.0702, 0.0664	0.1580, 0.1458	0.2399, 0.2380
	F1, Hz	2592.12	2590.48	2587.01	2569.94	2544.88
	T1, Hz	8137.98	8143.25	8166.59	8259.42	8419.85
100 x 14 x 4.8	<i>c/t</i>	0, 0	0.0431, 0.0367	0.0748, 0.0741	0.1394, 0.1371	0.2483, 0.2332
	F1, Hz	2591.63	2589.98	2584.08	2566.23	2538.26
	T1, Hz	9460.66	9474.41	9511.13	9607.16	9841.55
100 x 11 x 4.8	<i>c/t</i>	0,0	0.0330, 0.298	0.0656, 0.0632	0.1506, 0.1481	0.2392, 0.2345
	F1, Hz	2615.28	2613.82	2608.07	2579.45	2546.48
	T1, Hz	11247.38	11262.84	11302.06	11497.53	11790.82
100 x 7 x 4.8	<i>c/t</i>	0, 0	0.0454, 0.0361	0.0870, 0.0784	0.1615, 0.1594	0.2179, 0.2124
	F1, Hz	2616.23	2613.27	2602.07	2560.76	2535.63
	T1, Hz	13638.00	13691.33	13782.24	14078.35	14357.15
100 x 6 x 6	<i>c/t</i>	0, 0	0.0153, 0.0217	0.0633, 0.0523	0.1259, 0.1126	0.1636, 0.1603
	F1, Hz	3109.23	3103.73	3095.82	3043.46	2987.33
	T1, Hz	~15000**	-	-	-	-

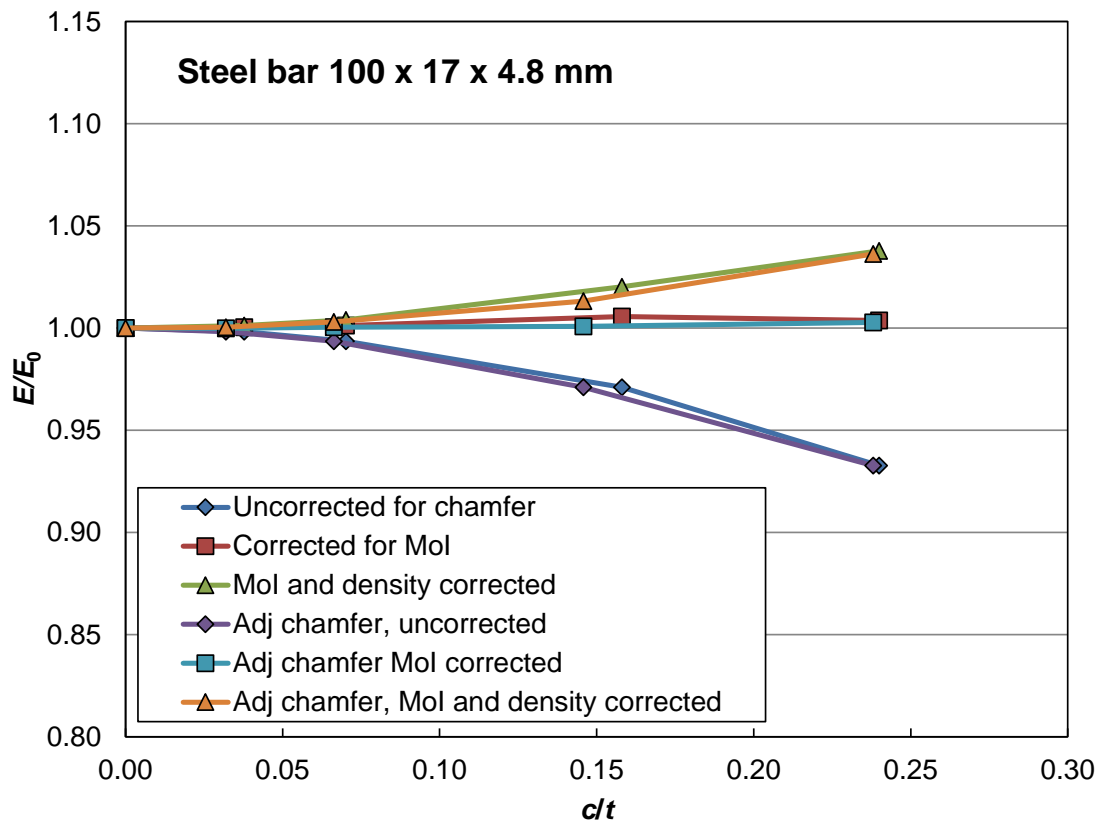
* First figure average measured chamfer, second figure chamfer calculated from mass and unchamfered density.

** The bar aspect ratio was such that reliable torsional modes could not be struck.

The flexural mode moment of inertia and density correction factors as given by Quinn and Swab were computed and used to correct the 'nominal' values of Young's modulus. Figure 1 shows plots of Young's modulus, normalised by the unchamfered value, versus chamfer size normalised by the test-piece thickness. Two sets of data are shown, one set using the averaged chamfer cross-sectional area, and the other using the chamfer size computed from the change in mass from the unchamfered state. The latter route was taken to examine the effect of unequal hand-made chamfers, since it proved experimentally difficult to ensure that a 45° angle was maintained on each edge and that they were of exactly equal sizes, when made by hand. The values of Young's modulus uncorrected for the chamfers clearly fall with increasing chamfer size as a consequence of a reduced mass and a reduced fundamental mode frequency. Correcting for the cross-sectional moment of inertia change results in closely consistent moduli values, independent of chamfer size. Correcting additionally for the change in 'nominal' density in accordance with the ASTM C1259 procedure produces an increase in moduli with increasing chamfer size. These results show that the moment of inertia correction alone is adequate to correct for the chamfers. The second correction for density is clearly incorrect, requiring a change to the instruction in the standard, unless of course one is using the density formulation as given in the basic equation above.

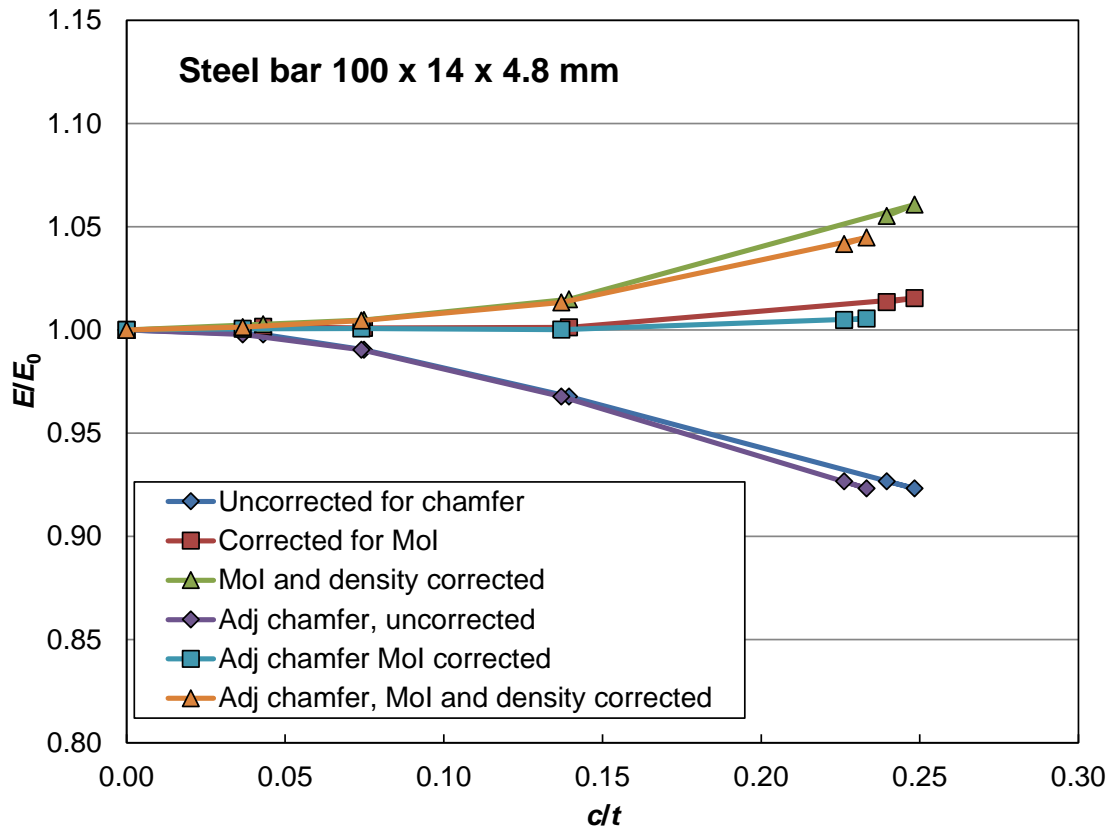


(a)

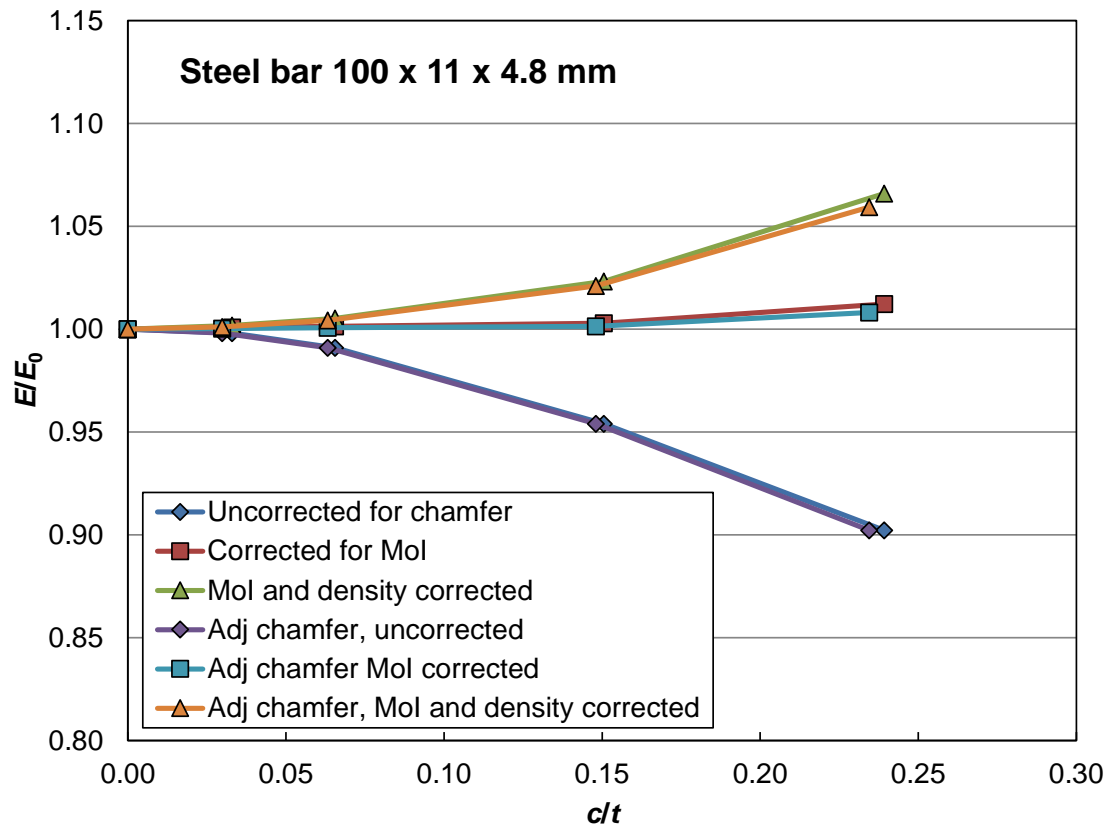


(b)

Figure 1: Normalised Young's modulus as a function of normalised chamfer width for various sizes of steel bars, (a) 20 x 4.8 mm, (b) 17 x 4.8 mm, using the averaged measured chamfer size, and the adjusted size computed from the change in mass, showing the effect of the moment of inertia and density corrections.

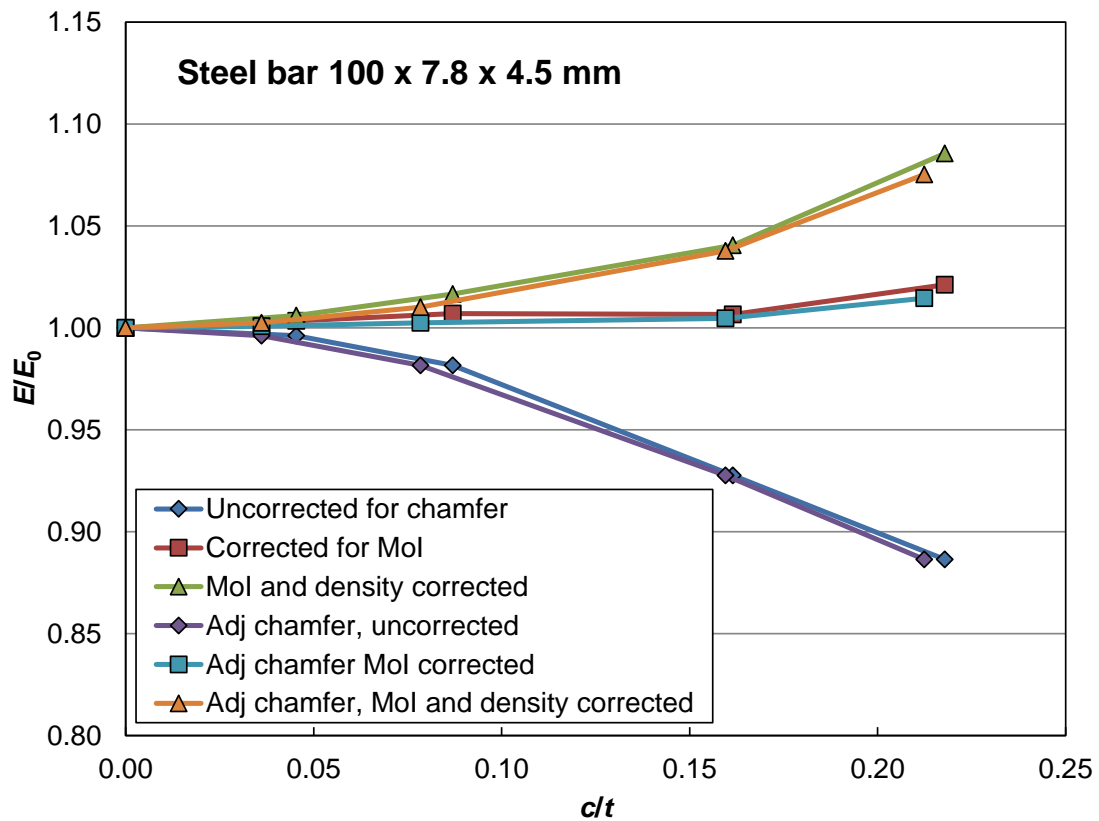


(c)

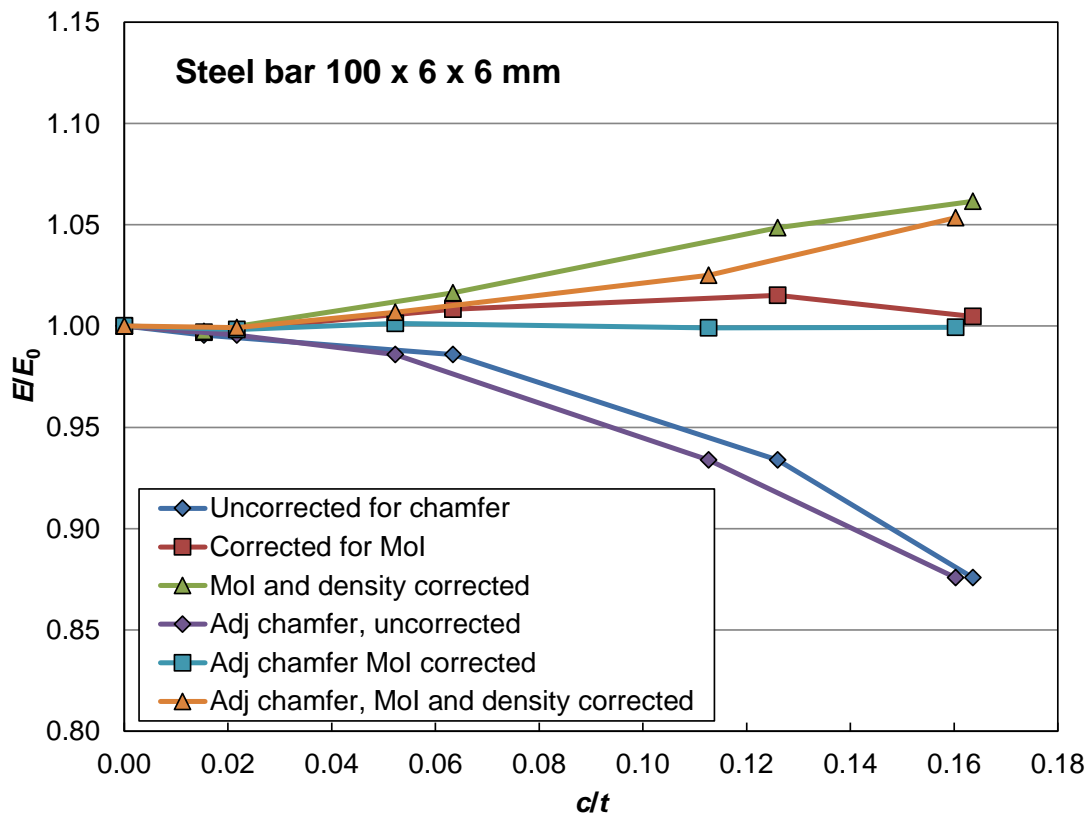


(d)

Figure 1 (cont.): Normalised Young's modulus as a function of normalised chamfer width for various sizes of steel bars, (c) 14 x 4.8 mm, (d) 11x 4.8 mm, using the averaged measured chamfer size, and the adjusted size computed from the change in mass, showing the effect of the moment of inertia and density corrections.



(e)



(f)

Figure 1 (cont.): Normalised Young's modulus as a function of normalised chamfer width for various sizes of steel bars, (e) 7.8 x 4.8 mm, (f) 6 x 6 mm, using the averaged measured chamfer size, and the adjusted size computed from the change in mass, showing the effect of the moment of inertia and density corrections.

5.2 EFFECT OF CHAMFERING ON SHEAR MODULUS DETERMINATION

Table 1 shows that the effect of chamfering on the torsional mode frequency is quite marked, because the most effective stiffening elements of the cross-section, the corners, have been removed. There is a much more marked change compared with the flexural case. When the apparent shear modulus is calculated based on the chamfered mass it increases approximately parabolically with increasing chamfer size. When it is calculated using the chamfer-corrected density rather than the mass, it increases almost twice as rapidly with increasing chamfer size. The factor difference is effectively equivalent to the density correction factor given by Quinn and Swab [4], as might be expected. Figure 2 shows an example of the trend on the largest of the set of test-pieces, and also shows the effect of using the calculated chamfer size based on the unchamfered density.

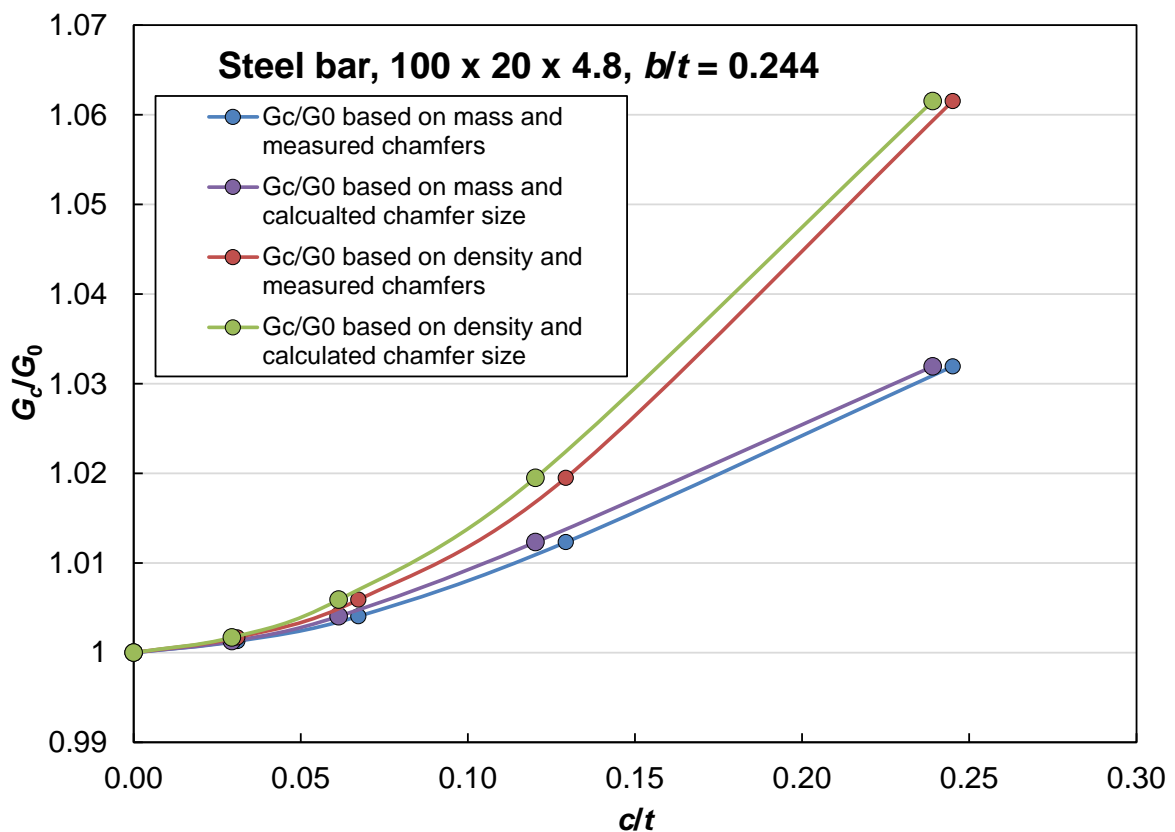


Figure 2: Variation in normalised shear modulus, G_c/G_0 , with increasing normalised chamfer size c/t .

Assuming that the calculated chamfer size based on mass and unchamfered density is a more reliable estimate of the true average chamfer size than physical measurements, Figure 3 shows the two cases of using the mass and using the unchamfered density for the calculation of shear modulus for the five test-pieces in which a clear torsional response could be detected (this was not possible on the square test-piece). It is immediately clear that the effect becomes more marked with decreasing b/t ratio, which can be explained by the relatively greater fraction of the cross-section being removed for a given size of chamfer.

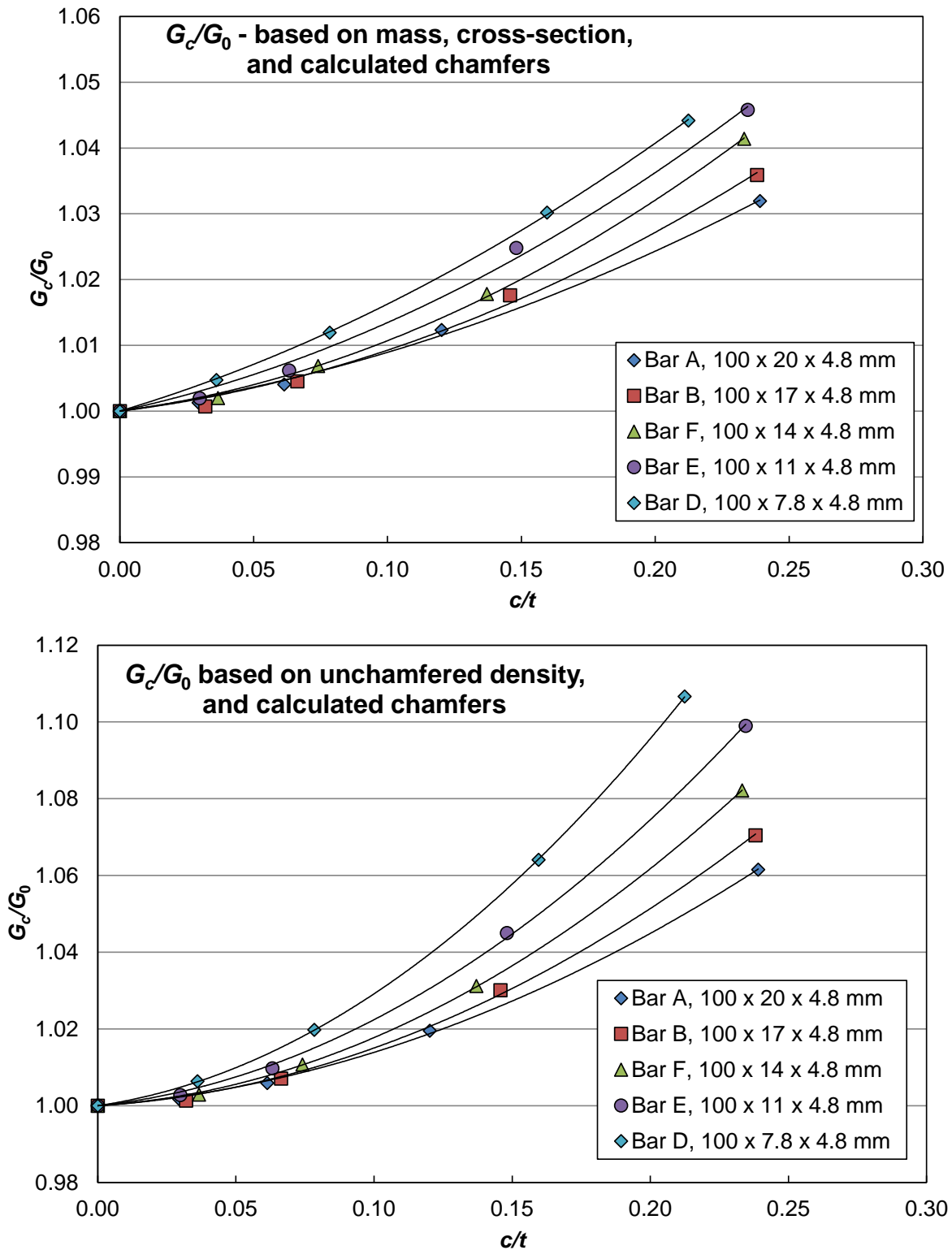
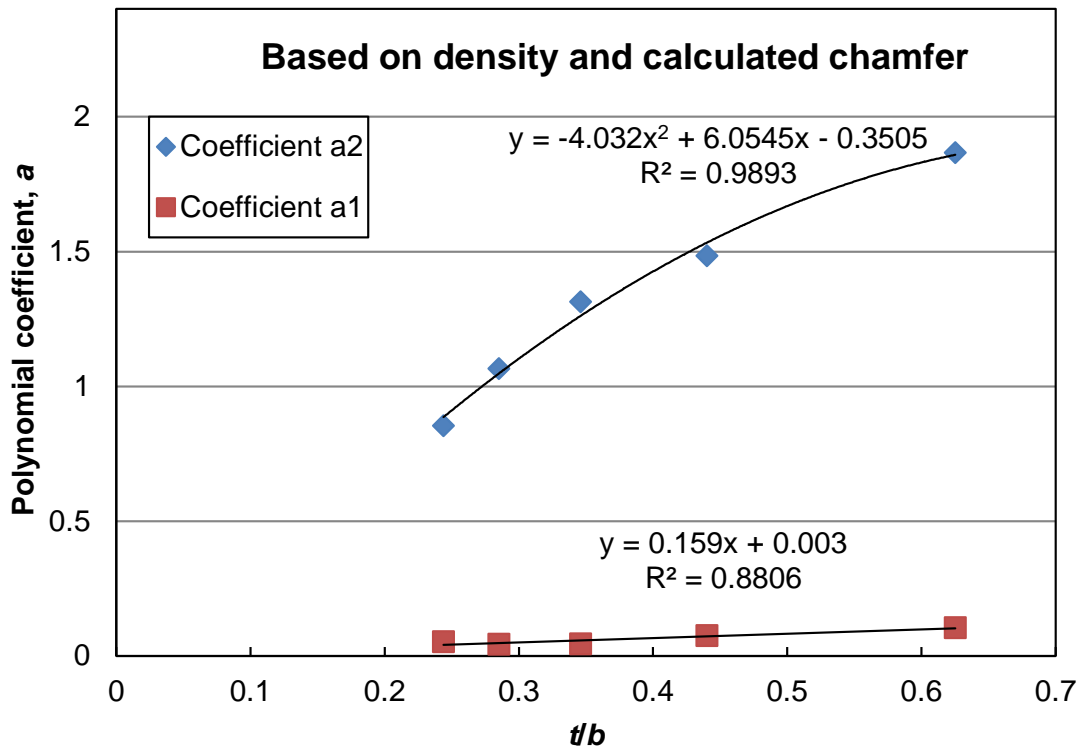
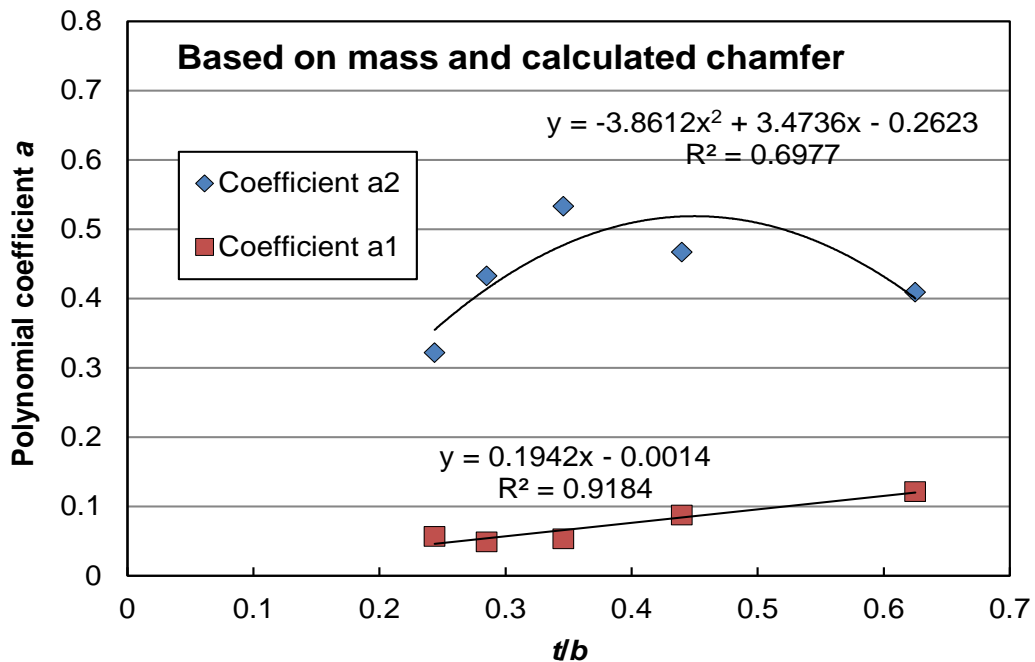


Figure 3: The increase in normalised apparent shear modulus with increasing normalised chamfer size calculated using (a) actual test-piece mass, and (b) unchamfered density, both assuming calculated chamfers based on mass change.

These trends were curve-fitted with simple second order polynomials: $y = 1 + a_1x + a_2x^2$, *i.e.* constrained to the value of $G_c/G_0 = 1$ for no chamfer. The polynomial terms a_1 and a_2 were then plotted against the cross-section aspect ratio t/b to develop generic correction factors for all cross-sectional aspect ratios (Figure 4).



(a)



(b)

Figure 4: Plots of the polynomial coefficients a_2 and a_1 from curvefits to the trends in Figure 3 against cross-sectional aspect ratio t/b , (a) for calculations based on mass and (b) for calculations based on density.

This analysis leads to the generic shear modulus corrections:

$$\text{For mass: } \frac{G_c}{G_0} = \left(-3.861 \left(\frac{t}{b} \right)^2 + 3.474 \left(\frac{t}{b} \right) - 0.2623 \right) \left(\frac{c}{t} \right)^2 + \left(0.194 \left(\frac{t}{b} \right) - 0.001 \right) \left(\frac{c}{t} \right) + 1$$

$$\text{For density: } \frac{G_c}{G_0} = \left(-4.032 \left(\frac{t}{b} \right)^2 + 6.055 \left(\frac{t}{b} \right) - 0.3505 \right) \left(\frac{c}{t} \right)^2 + \left(0.159 \left(\frac{t}{b} \right) + 0.003 \right) \left(\frac{c}{t} \right) + 1$$

As a test of the appropriateness of these curve-fits, these expressions have been evaluated for the smallest cross-section test-piece, *i.e.* the one which would be most affected by chamfering, and are compared with the experimental results in Figure 5. It can be seen that there is a match well within about 0.2% between the experimental results and the global curve-fits.¹

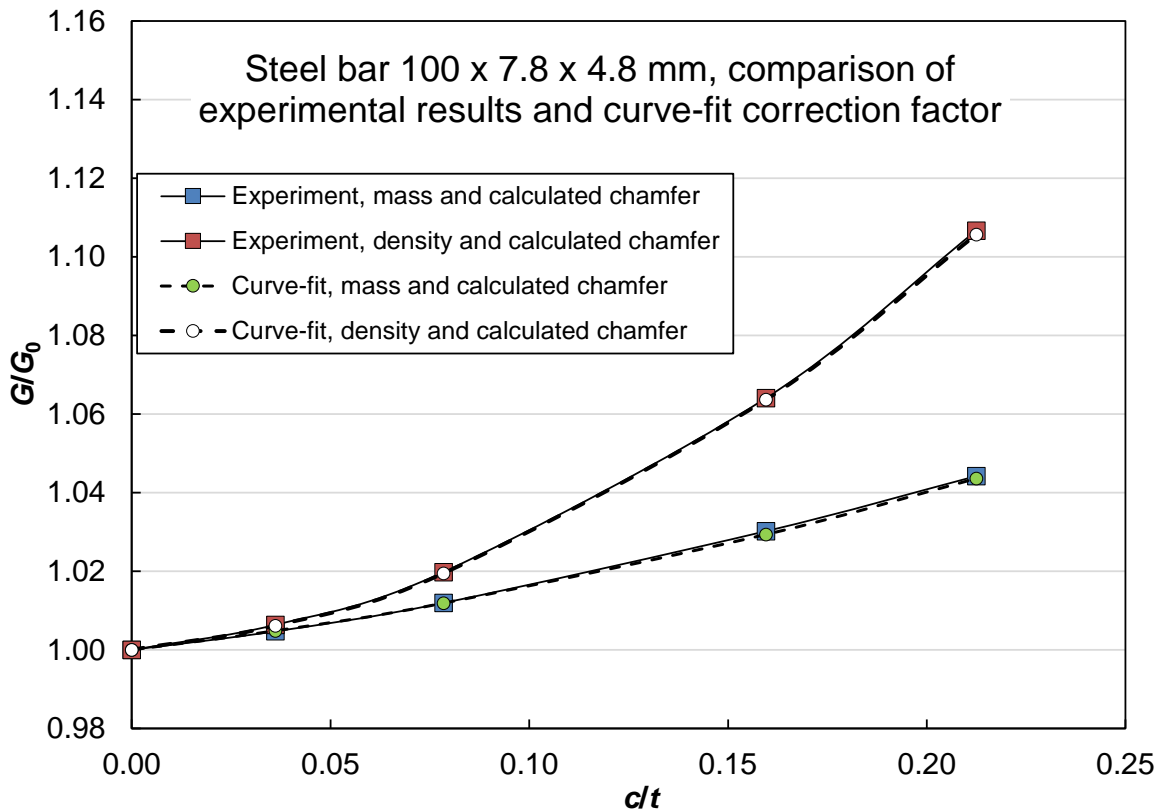


Figure 5: Comparison of the apparent change in experimental shear modulus with increasing chamfer size, and the evaluation of the curve-fit relationship from all experiments.

Thus, depending on whether the mass formulation (as given in the standards) or the density formulation (as typically used in the scientific literature) is selected, the apparent shear modulus of a chamfered test-piece, G_c can be corrected to the true shear modulus G_0 by the use of a relatively simple correction function.

These curves also clearly demonstrate that the uncorrected chamfer-induced error in shear modulus is smaller when using the mass version of the equation, rather than one involving density, but nevertheless can exceed 1% even for a chamfer size c/t of 0.05 in this worst-case cross-sectional shape. For this chamfer size, the apparent uncorrected drop in Young's modulus of about 2%, which combined with the

¹ It has been pointed out that the tangent of the fitted curves should probably be forced to 0 at zero chamfer size, *i.e.* the coefficient $a_1 = 0$, rather than the small value obtained by unconstrained fitting, but this actually makes a very small difference only (P. Supancic, ISFK, MU Leoben).

apparent uncorrected increase in shear modulus of 1%, leads to a very significant drop in the calculated Poisson's ratio of ~0.04. The effects are smaller for wider bars.

Thus if test-pieces are chamfered to a significant size, there is no point correcting only for effect on the flexural-determined Young's modulus if Poisson's ratio is required; it is necessary to correct also for the effects on apparent shear modulus.

6 CONCLUSIONS

An experimental study of the role of chamfers in the determination of elastic properties by the impact excitation or resonance method has been conducted. By using a series of test-pieces characterised with increasing chamfer sizes it has been shown that:

- For Young's modulus determined using rectangular bar flexure in accordance with extant standards, only a correction for cross-sectional moment of inertia changes is required when employing the calculation equation based on test-piece mass. The density correction factor as given in ASTM C1259 (impact excitation), and ASTM C1198 (resonance) is required only if a density-based version of the calculation is used. The impression given in the standard to employ both corrections with the required mass-based calculation is incorrect.
- For shear modulus determination using rectangular bar torsion, increasing chamfer size leads to a marked increase in natural frequencies. For a range of cross-section aspect ratios b/t from 1.6 to 4.1 that might typically be used for the measurement, and for chamfer sizes up to c/t ratios of 0.2, generic correction factors have been computed for when mass is used in the calculation equation, as in ASTM C1259 and ASTM C1198, and for when density is used.

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