National Physical Laboratory

NPL REPORT ENG 45

Areal Lateral Calibration Software: User Manual

C L Giusca and I M Smith

DECEMBER 2013

National
Measurement
System

NPL Report ENG 45

Areal Lateral Calibration Software: User Manual

C L Giusca and I M Smith
National Physical Laboratory

DECEMBER 2013

ABSTRACT

This report constitutes a *user manual* for software developed at the National Physical Laboratory to calculate the locations of the centres of gravity of the protrusions of a calibrated cross-grating artefact from measurements of the artefact. The calculated locations may then be compared with corresponding reference locations. The software enables users to process an ASCII file containing values of a surface measured on a two-dimensional grid of points. The software is provided in the form of a standalone executable and the locations of the centres of gravity of the protrusions are written to an ASCII results file. The user may also provide a second ASCII file containing reference locations of the centres of gravity of the protrusions. The software applies a transformation to the reference locations, determines the differences between the calculated locations and the transformed reference locations, and writes those differences to the ASCII results file.

NPL Report ENG 45

ISSN 1754–2987

National Physical Laboratory,
Hampton Road, Teddington, Middlesex, United Kingdom TW11 0LW

We gratefully acknowledge the financial support of the UK Department for Business, Innovation and Skills (National Measurement Office)

Approved on behalf of NPLML by Andrew Lewis,
Science Area Leader (Dimensional) for the Engineering Measurement Division

Contents

1 Introduction

1.1 Background

Areal surface topography measuring instruments, widely used in metrology and industry, provide three dimensional maps of surfaces from which surface texture parameters may subsequently be calculated. Current international standardisation in the field of areal surface texture measurement includes the development of a series of documents describing the nominal characteristics of, and calibration methods for, the scales of areal surface topography measuring instruments. Working group ISO/TC 213/WG 16, responsible for the development of such documents, has identified a set of metrological characteristics that are applicable to all these instruments regardless of their design or operation. The set of characteristics includes measurement noise, residual flatness, amplification of the scales, linearity of the scales, squareness of the axes, and resolution. For each of three of the above characteristics, viz., amplification of the scales, linearity of the scales, and squareness of the axes, the paper [1] provides methodologies for the calculation of a parameter that quantifies each characteristic – the *amplification coefficient*, the *linearity*, and the *squareness*. In practice, values of these parameters can be determined by measuring the locations of the centres of gravity of the protrusions of a calibrated cross-grating artefact.

This report is a user manual for software developed at the National Physical Laboratory (NPL) to calculate the locations of the centres of gravity of the protrusions of a cross-grating artefact from measurements of the artefact. The software may also be used to compare the calculated locations of the centres of gravity with a set of reference locations. The report is organised as follows. The remainder of this section describes the main functions of the software. Section 2 gives information on installing and uninstalling the software. Section 3 describes how to use the software. An appendix to the report describes the mathematical approach implemented in the software.

1.2 Software User Licence Agreement

The software is provided with a Software User Licence Agreement (Ref: MSC/L/13/001) and the use of the software is subject to the terms laid out in that agreement. By installing and running the software, the user accepts the terms of the agreement.

To run the software, the user must first install MATLAB's Component Runtime (MCR) libraries (section 2). Note that the MCR installer file is not included in the Areal Lateral Calibration Software distribution – it may be obtained separately by contacting NPL. The user must accept the terms of the MCR Library License as part of the installation of the MCR libraries.

1.3 Main functions of the software

The main functions of the software are as follows:

- Read data from a measurement data file;
- Identify the regions containing protrusions and calculate the locations of the centres of gravity (CGs) of the protrusions (referred to subsequently in this document as the "calculated CGs");
- Read the reference locations of the CGs of the protrusions (referred to subsequently in this document as the "reference CGs") from a reference data file;
- Determine the transformation that aligns (in a least squares sense) the reference CGs to the calculated CGs, and calculate the distances (in the x- and y-directions) from the transformed reference CGs to the calculated CGs;
- Write results to an output data file.

The following conditions are assumed to apply:

1. The measurement data file is provided in ASCII format, and consists of $n_x n_y$ lines of $\{x, y, z\}$ values ordered as follows

$$\begin{array}{ccc}
x_{1,1} & y_{1,1} & z_{1,1} \\
x_{2,1} & y_{2,1} & z_{2,1} \\
\vdots & \vdots & \vdots \\
x_{n_x,1} & y_{n_x,1} & z_{n_x,1} \\
x_{1,2} & y_{1,2} & z_{1,2} \\
x_{2,2} & y_{2,2} & z_{2,2} \\
\vdots & \vdots & \vdots \\
x_{n_x,2} & y_{n_x,2} & z_{n_x,2} \\
\vdots & \vdots & \vdots \\
x_{1,n_y} & y_{1,n_y} & z_{1,n_y} \\
x_{2,n_y} & y_{2,n_y} & z_{2,n_y} \\
\vdots & \vdots & \vdots \\
x_{n_x,n_y} & y_{n_x,n_y} & z_{n_x,n_y}
\end{array}$$

   The z-values represent the height of the cross-grating artefact measured on an approximately rectangular n_x by n_y grid of x- and y-coordinate values.

The jth row of (x, y) points

$$(x_{1,j}, y_{1,j}) \quad (x_{2,j}, y_{2,j}) \quad \ldots \quad (x_{n_x-1,j}, y_{n_x-1,j}) \quad (x_{n_x,j}, y_{n_x,j}),$$

with

$$x_{1,j} < x_{2,j} < \ldots < x_{n_x-1,j} < x_{n_x,j},$$

is nominally parallel to the x-axis.

The ith column of (x, y) points

$$\begin{array}{c} (x_{i,n_y}, y_{i,n_y}) \\ (x_{i,n_y-1}, y_{i,n_y-1}) \\ \vdots \\ (x_{i,2}, y_{i,2}) \\ (x_{i,1}, y_{i,1}), \end{array}$$

with

$$y_{i,1} < y_{i,2} < \ldots < y_{i,n_y-1} < y_{i,n_y},$$

is nominally parallel to the y-axis.

The rows and columns of protrusions of the cross-grating artefact are therefore aligned nominally parallel to the x- and y-axes, respectively.

2. The reference data file is provided in ASCII format, and consists of the following information

$$\begin{array}{ccc} q_x & & \\ q_y & & \\ & & \\ \widehat{x}^{\text{ref}}_{1,q_y} & \ldots & \widehat{x}^{\text{ref}}_{q_x,q_y} \\ \vdots & \vdots & \vdots \\ \widehat{x}^{\text{ref}}_{1,1} & \ldots & \widehat{x}^{\text{ref}}_{q_x,1} \\ & & \\ \widehat{y}^{\text{ref}}_{1,q_y} & \ldots & \widehat{y}^{\text{ref}}_{q_x,q_y} \\ \vdots & \vdots & \vdots \\ \widehat{y}^{\text{ref}}_{1,1} & \ldots & \widehat{y}^{\text{ref}}_{q_x,1} \end{array}$$

Note that there is a single blank line immediately below both the line containing q_y and the final line of $\widehat{x}^{\text{ref}}_{k,1}$ values.

The values q_x and q_y indicate the number of protrusions in the x and y-directions, respectively. The arrays of x- and y-coordinate values contain, respectively, the x- and y-coordinates of the reference CGs. The coordinates satisfy

$$\widehat{x}^{\text{ref}}_{1,l} < \widehat{x}^{\text{ref}}_{2,l} < \ldots < \widehat{x}^{\text{ref}}_{q_x-1,l} < \widehat{x}^{\text{ref}}_{q_x,l}, \quad l = 1, \ldots, q_y,$$

and

$$\widehat{y}^{\mathrm{ref}}_{k,1} < \widehat{y}^{\mathrm{ref}}_{k,2} < \ldots < \widehat{y}^{\mathrm{ref}}_{k,q_y-1} < \widehat{y}^{\mathrm{ref}}_{k,q_y}, \quad k = 1, \ldots, q_x.$$

3. In the measurement data file, all x- and y-coordinate values are provided in the same unit of measurement, which then determines the unit of measurement of the calculated CGs. In the reference data file, all x- and y-coordinate values are provided in this unit of measurement, but not necessarily in the same frame of reference as the measured data.
4. The measurement data file must not contain any “non-measured points”, i.e., points represented by NaNs (not a number).
5. The reference data file must contain only numerical values.

Additional conditions assumed to be satisfied by the measured data are described in section A.1.

2 Installing and uninstalling the software

The software takes the form of an application program called

- `ArealLateralCalibrationv1.exe`

generated by compiling (using the MATLAB compiler) software implemented in the MATLAB programming language [2], together with the associated files

- `ArealLateralCalibrationSoftware-UMv10.pdf`
- `ArealLateralCalibration-MSC_L_13_001.pdf`
- `CCI_100um_10_1.txt`
- `CCI_100um_10_1_ref.txt`
- `CCI_100um_10_1.dat`
- `readme.txt`

The program has been created and tested on a personal computer running the 32 bit Microsoft Windows 7 Enterprise operating system.

To install the software, undertake the following steps:

1. Copy the application program and the above associated files to a working folder;

2. Install MATLAB's Component Runtime (MCR) libraries. This is done by running the MCR installation program

   `MCRInstaller.exe`

   once on the target machine, i.e., the machine on which it is intended to run the application program. It is necessary to have administrative privileges for the target machine because both the system registry and system path are modified as part of the installation process. The MCR installation program installs the MCR libraries, registers the components as needed, and updates the system path to point to the MCR binary directory. The installation process takes some time due to the number of files that are installed. The MCR installation program is about 388 MB in size, and the installed libraries require about 926 MB of disk space.

The software is uninstalled by

- running the MCR installation program `MCRInstaller.exe` and selecting 'Remove' to uninstall the MCR libraries, and
- deleting the application program and its associated files.

3 Using the software

3.1 General

The application program may be run in either of two ways:

(a) Double-clicking on the executable file (with the extension `.exe`) in Windows Explorer. An MS-DOS window opens and the software begins running;

(b) Opening an MS-DOS window, navigating to the folder containing the program, typing the name of the program (without the extension `.exe`), and pressing `Return`. The software begins running (and the MS-DOS command line may not be accessed again until the user has exited the software).

It may take several seconds for the software to begin running.

The software allows the user to perform the following operations:

1. Select a measurement data file;
2. Select a reference data file;

3. Load measured data from measurement data file;
4. Assign grid size for measured data;
5. Pre-process the measured data;
6. Process measured data;
7. Load reference CGs from reference data file;
8. Process reference CGs;
9. Write calculated CGs to output data file;
10. Write differences between calculated and reference CGs to output data file;
11. Exit.

Steps 7, 8 and 10 are only implemented if a reference data file is selected in step 2.

Each of the steps is described in the sections below.

3.2 Select a measurement data file

The user is first presented with a graphical user interface (GUI) and prompted to select a measurement data file. To select a measurement data file, left-click on the name of the file then left-click on Open.

If the user presses Cancel, the software displays an error message and exits.

If no measurement data file has been selected and the user presses Open, nothing happens and the software waits for further action from the user.

If the user selects a measurement data file having a `.res` file extension, the software displays an error message and exits (see section 3.10).

3.3 Select a reference data file

The user is then presented with a second GUI and prompted to select a reference data file. To select a reference data file, left-click on the name of the file then left-click on Open.

If the user presses Cancel, the software continues running without a reference data file.

If no reference data file has been selected and the user presses Open, nothing happens and the software waits for further action from the user.

3.4 Load measured data from measurement data file

The measured data is read from the measurement data file.

3.5 Assign grid size for measured data

The user is presented with a GUI and prompted to enter values n_x and n_y for the number of data points in the x- and y-directions, respectively. To enter values, edit (if necessary) the default values in the two text boxes and left-click on OK.

The default grid size for the measurement data file is 1024×1024.

The software displays an error message and exits if:

- The user presses Cancel;
- The total number of lines of measured data read in from the measurement data file is not equal to the product $n_x n_y$;
- No value is present in at least one text box and the user presses OK;
- A non-numerical value is present in at least one text box and the user presses OK;
- A non-integer numerical value is present in at least one text box and the user presses OK;
- A non-positive integer value is present in at least one text box and the user presses OK.

3.6 Pre-process measured data

The measured z-values are shifted such that the shifted z-values have zero mean.

3.7 Process measured data

The software identifies protrusions and determines the p_x by p_y array of calculated CGs.

Appendix A provides a detailed description of the underlying mathematics.

3.8 Load reference CGs from reference data file

The q_x by q_y array of reference CGs is read from the reference data file.

The software displays an error message and exits if:

- The size of the array of reference CGs differs from that defined by q_x and q_y;
- The size of the array of reference CGs differs from that of calculated CGs, i.e., $q_x \neq p_x$ or $q_y \neq p_y$;
- Blank lines are not found in the expected locations in the reference data file (see section 1.3).

3.9 Process reference CGs

The software determines the transformation (comprising a translation followed by a rotation) to apply to the reference CGs so that the sum of squares of distances from the transformed reference CGs to the calculated CGs is minimised. The differences in the transformed reference and measured CGs are then calculated.

Appendix A provides a detailed description of the underlying mathematics.

3.10 Write calculated CGs to output data file

Results, viz. the x- and y-coordinates of the calculated CGs, are written to a file in the same folder as the measurement data file with the same filename as that file apart from having the `.res` extension.

If the file does not exist, it will be created. If it does exist, results will be appended to the end of the file.

The following information is written to the output data file:

- Details (name, release number and date of release) of the software;
- The date and time at which the software was run;
- The path to the measurement data file;
- The path to the reference data file (if a reference data file has been selected);
- The p_x by p_y array of x-coordinates of the calculated CGs

$$\begin{array}{ccc} \widehat{x}_{1,p_y} & \dots & \widehat{x}_{p_x,p_y} \\ \vdots & \vdots & \vdots \\ \widehat{x}_{1,1} & \dots & \widehat{x}_{p_x,1} \end{array}$$

- The p_x by p_y array of y-coordinates of the calculated CGs

$$\begin{array}{ccc} \widehat{y}_{1,p_y} & \cdots & \widehat{y}_{p_x,p_y} \\ \vdots & \vdots & \vdots \\ \widehat{y}_{1,1} & \cdots & \widehat{y}_{p_x,1} \end{array}$$

3.11 Write differences between calculated and reference CGs to output data file

The following information is written to the output data file:

- The p_x by p_y array of differences between the calculated and reference CGs in the x-direction

$$\begin{array}{ccc} \widehat{e}^{x}_{1,p_y} & \cdots & \widehat{e}^{x}_{p_x,p_y} \\ \vdots & \vdots & \vdots \\ \widehat{e}^{x}_{1,1} & \cdots & \widehat{e}^{x}_{p_x,1} \end{array}$$

- The p_x by p_y array of differences between the calculated and reference CGs in the y-direction

$$\begin{array}{ccc} \widehat{e}^{y}_{1,p_y} & \cdots & \widehat{e}^{y}_{p_x,p_y} \\ \vdots & \vdots & \vdots \\ \widehat{e}^{y}_{1,1} & \cdots & \widehat{e}^{y}_{p_x,1} \end{array}$$

3.12 Example files

The example measurement data file `CCI_100um_10_1.txt` and reference data file `CCI_100um_10_1_ref.txt` are provided as part of the software distribution. Also included is the example output file `CCI_100um_10_1.dat` which was generated by running the software on the provided example files.[1]

Having installed the software, users may run it on the example input data and reference data files to check that installation has been successful (when prompted, the user should assign the default grid size of 1024×1024). The values of the results should agree with those in the provided example output data file.

---

[1] The file extension for the example output data file has been changed from its default `.res` file extension. Users may run the software on the provided example measurement and reference data files to generate an output data file with the `.res` file extension and then easily compare that file with the example output data file.

Acknowledgements

The National Measurement Office of the UKs Department for Business, Innovation and Skills supported this work as part of its Engineering and Flow programme.

References

[1] Giusca C L, Leach R K and Helery F Calibration of the scales of areal surface topography measuring instruments: part 2. Amplification, linearity and squareness *Meas. Sci. Technol.* **23** (2012) 065005

[2] The MathWorks, Inc. *MATLAB* `www.mathworks.co.uk`

A Underlying mathematics

Upon selection of a measurement data file by the user, the software implements the following steps:

- Identification of regions enclosing protrusions;
- Determination of calculated CGs;
- Removal of "false" protrusions.

If a reference protrusions file has also been selected by the user, the software implements the following additional steps:

- Alignment of calculated and reference CGs;
- Assignment of "central" protrusion;
- Determination of differences between calculated and reference CGs.

In this appendix, for each of the six steps listed above, a description of the underlying mathematics is provided.

A.1 Identification of regions enclosing protrusions

The (pre-processed) z–values are first processed to simplify the identification of regions enclosing protrusions. Figure 1 summarises the processing steps. The left-hand diagram shows the side view of the protrusion that has the lowest measured z–value, z_{min}. The mean plane ($z = 0$) is also shown. In the middle diagram, the z–values are shifted by subtracting one fifth of z_{min}. This shifting has no effect on the CGs of the protrusions. In the right-hand diagram, all positive (shifted) z–values are set to zero while all other (shifted) z–values are left unchanged.[2]

Figure 2 shows the results of processing the z–values for the example measurement data file provided as part of the software distribution. The top figure shows the original measured data. The middle figure shows the data after the z–values have been shifted. The bottom figure shows the data after all positive (shifted) z–values have been set to zero. The same colormap is used on all three plots.

Assuming that the protusions are aligned (approximately) parallel to the x– and y–axes, subsets $I_k,\ k = 1, \ldots, p_x$, of the indices of the grid points in the x–direction and subsets

---

[2] Strictly speaking, it is not essential to shift the data prior to zeroing. The inclusion of the shifting step is included only to make coding of protrusion identification simpler in MATLAB.

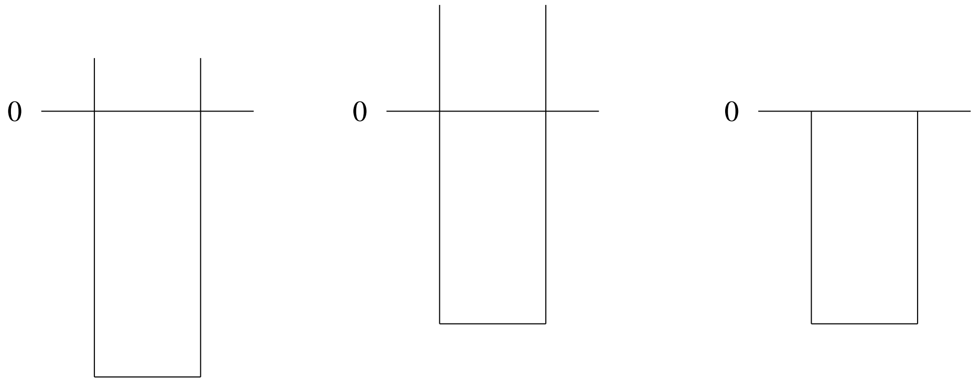

Figure 1: Schematic showing processing of z–values: original measured data (left), shifted z–values (middle), positive (shifted) z–values set to zero (right).

$J_l,\ l = 1, \dots, p_y$, of the indices of the grid points in the y–direction are determined such that

- each subset I_k encloses p_y protrusions in the y–direction,
- each subset J_l encloses p_x protrusions in the x–direction,
- subsets I_k and J_l together determine a rectangular region that encloses a single protrusion, identified as the (k, l) protrusion, and
- a total of $p_x p_y$ protrusions are identified.

To facilitate the identification of regions enclosing protrusions within the software, the following conditions are assumed to apply:

1. $\min(I_1) \geq 3$.
2. $\min(J_1) \geq 3$.
3. $\max(I_{p_x}) \leq n_x - 2$.
4. $\max(J_{p_y}) \leq n_y - 2$.
5. For $k = 1, \dots, p_x - 1$, $\min(I_{k+1}) \geq \max(I_k) + 2$.
6. For $l = 1, \dots, p_y - 1$, $\min(J_{l+1}) \geq \max(J_l) + 2$.

These conditions ensure that, in both the x- and y-directions, there is a separation of at least two data points between the "start" of the data and the first subset of indices (conditions 1 and 2); between the final subset of indices and the "end" of the data (conditions 3 and 4); and between adjacent subsets of indices (conditions 5 and 6).

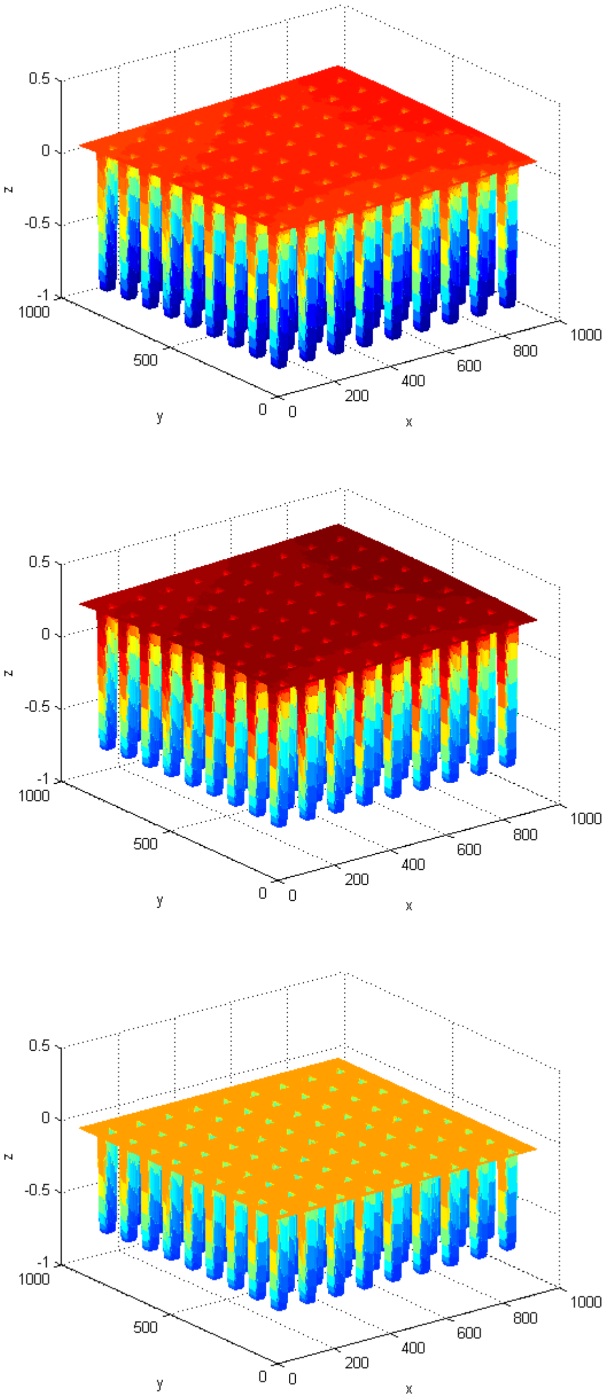

Figure 2: Processing of z–values of example measurement data file: original measured data (top), shifted z–values (middle), positive (shifted) z–values set to zero (bottom).

Figure 3 shows the shifted and zeroed data of figure 2 (bottom), viewed from above, with vertical and horizontal black lines added to indicate the endpoints of the subsets $I_k,\ k = 1, \ldots, p_x = 9$, and $J_l,\ l = 1, \ldots, p_y = 9$.

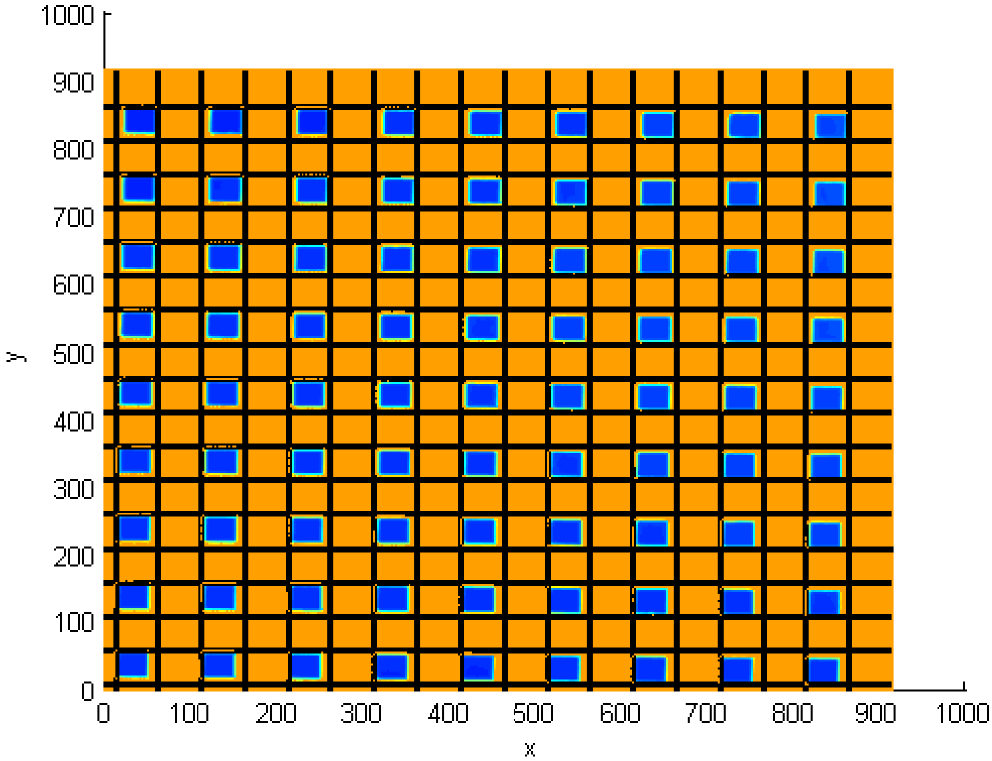

Figure 3: Shifted and zeroed data of figure 2 (bottom) with black lines added to indicate the endpoints of subsets identifying protrusions.

A.2 Determination of calculated CGs

The x-coordinate $\widehat{x}_{k,l}$ and y-coordinate $\widehat{y}_{k,l}$ of the CG of the (k, l) protrusion are then given by

$$\widehat{x}_{k,l} = \frac{\displaystyle\sum_{i \in I_k} \sum_{j \in J_l} x_i z_{i,j}}{\displaystyle\sum_{i \in I_k} \sum_{j \in J_l} z_{i,j}}, \quad \widehat{y}_{k,l} = \frac{\displaystyle\sum_{i \in I_k} \sum_{j \in J_l} y_j z_{i,j}}{\displaystyle\sum_{i \in I_k} \sum_{j \in J_l} z_{i,j}}.$$

A.3 Removal of "false" protrusions

For some data sets, implementing the identification process described in section A.1 may result in the identification of "false" protrusions. Such protrusions are caused by the presence of a (typically very small) number of shifted and zeroed points having small negative values.

Consider a subset I_{k_r} of the indices of the grid points in the x-direction that is identified as a result of the presence of shifted and zeroed points having small negative z-values. I_{k_r} and the subsets $J_l,\ l = 1, \ldots, p_y$, of the indices of the grid points in the y-direction then identify (what are considered to be) the p_y (k_r, l) protrusions in the y-direction. However, typically all but one of these "protrusions" will have zero z–values throughout their rectangular regions. Calculation of the x- and y-coordinates of the CGs therefore involves division by zero, and each coordinate is recorded by MATLAB as NaN (representing "not a number"). The widespread occurrence of NaNs in the coordinates of the CGs can therefore be used to infer the presence of "false" protrusions.[3] The subset I_{k_r} should then be removed from the list of subsets $I_k,\ k = 1, \ldots, p_x$, and the number of subsets can be reduced by one, i.e., $p_x := p_x - 1$.

Similar considerations apply to the y-direction.

A.4 Alignment of calculated and reference CGs

The calculated and reference CGs are not necessarily provided in the same frame of reference. Therefore, in order to determine the differences between the calculated and reference CGs, a transformation (comprising a translation followed by a rotation) is applied to the reference CGs to align them to the calculated CGs.

The transformation is defined by three parameters: rotation angle Θ and translation parameters X_0 and Y_0.

A transformed reference CG is then given by

$$\begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} \widehat{x}_{i,j}^{\mathrm{ref}} - X_0 \\ \widehat{y}_{i,j}^{\mathrm{ref}} - Y_0 \end{bmatrix}.$$

Alignment of the calculated and reference CGs is implemented by determining estimates (θ, x_0, y_0) of (Θ, X_0, Y_0) that minimise

$$\sum_{i=1}^{p_x}\sum_{j=1}^{p_y} \left\| \begin{bmatrix} \widehat{x}_{i,j} \\ \widehat{y}_{i,j} \end{bmatrix} - \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} \widehat{x}_{i,j}^{\mathrm{ref}} - X_0 \\ \widehat{y}_{i,j}^{\mathrm{ref}} - Y_0 \end{bmatrix} \right\|^2.$$

---

[3] In the software, the presence of a "false" protrusion is assumed if more than $p_y/2$ of the x-coordinates of the calculated CGs are NaNs. For some extreme cases, this assumption may not be sufficiently strong, and a number of calculated CGs may be recorded as NaNs in the output data file.

A.5 Assignment of "central" protrusion

The "central" protrusion depends on the values of p_x and p_y and is assigned to be:

- The $((p_x+1)/2,(p_y+1)/2)$ protrusion if both p_x and p_y are odd;
- The $(p_x/2,p_y/2)$ protrusion if both p_x and p_y are even;
- The $((p_x+1)/2,p_y/2)$ protrusion if p_x is odd and p_y is even;
- The $(p_x/2,(p_y+1)/2)$ protrusion if p_x is even and p_y is odd.

For the example input data file provided as part of the software distribution, $p_x = p_y = 9$, and the "central" protrusion is the $(5,5)$ protrusion.

A.6 Determination of differences between calculated and reference CGs

Let $\widehat{x}_{k,l}^{\mathrm{ref,T}}$ and $\widehat{y}_{k,l}^{\mathrm{ref,T}}$ be, respectively, the x- and y-coordinates of the transformed (k,l) reference CG, i.e.,

$$\left[\begin{array}{c} \widehat{x}_{k,l}^{\mathrm{ref,T}} \\ \widehat{y}_{k,l}^{\mathrm{ref,T}} \end{array}\right] = \left[\begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array}\right] \left[\begin{array}{c} \widehat{x}_{k,l}^{\mathrm{ref}} - x_0 \\ \widehat{y}_{k,l}^{\mathrm{ref}} - y_0 \end{array}\right],$$

where (θ, x_0, y_0) are the estimates determined in section A.4.

The differences between the calculated and reference CGs in the x-direction are then given by

$$\widehat{e}_{k,l}^{x} = \left(\widehat{x}_{k,l}^{\mathrm{ref,T}} - \widehat{x}_{c_x,c_y}^{\mathrm{ref,T}}\right) - \left(\widehat{x}_{k,l} - \widehat{x}_{c_x,c_y}\right), \quad k = 1,\ldots,p_x, \quad l = 1,\ldots,p_y,$$

where the indices c_x and c_y indicate the "central" protrusion identified in section A.5.

Similarly, the differences between the calculated and reference CGs in the y-direction are given by

$$\widehat{e}_{k,l}^{y} = \left(\widehat{y}_{k,l}^{\mathrm{ref,T}} - \widehat{y}_{c_x,c_y}^{\mathrm{ref,T}}\right) - \left(\widehat{y}_{k,l} - \widehat{y}_{c_x,c_y}\right), \quad k = 1,\ldots,p_x, \quad l = 1,\ldots,p_y.$$