

## **VHF Impedance Measurement - A Review**

**N M Ridler**

**November 1993**

# VHF Impedance Measurement - A Review

N M Ridler

Division of Electrical Science  
National Physical Laboratory  
% URA Malvern, St Andrews Road  
Malvern, Worcs, WR14 3PS  
United Kingdom

## ABSTRACT

This report reviews the current status of top echelon impedance measurement in the UK over the frequency range 30 to 300 MHz (VHF) of the electromagnetic spectrum. The term impedance is used to describe one of the complex electrical properties of one-port passive terminations. Some fundamental concepts of VHF impedance measurement are given along with discussions on the measuring instruments, calibration standards and existing UK national standard capabilities.

© Crown Copyright 1993

ISSN 0143-7305

National Physical Laboratory  
Teddington, Middlesex, UK, TW11 OLW

Extracts from this report may be reproduced  
provided that the source is acknowledged.

Approved on behalf of Chief Executive, NPL  
by Dr T G Blaney, Head, Division of Electrical Science

## CONTENTS

	Page
1 INTRODUCTION . . . . .	1
2 FUNDAMENTAL CONCEPTS . . . . .	1
2.1 DISCRETE CIRCUIT-ELEMENT CONCEPTS . . . . .	2
2.2 TRANSMISSION LINE CONCEPTS . . . . .	3
2.3 DISCRETE CIRCUIT-ELEMENT AND TRANSMISSION LINE EQUIVALENCE . . . . .	4
3 MEASURING INSTRUMENTS . . . . .	5
3.1 BRIDGE CIRCUITS . . . . .	6
3.1.1 <i>Historical perspective</i> . . . . .	6
3.1.2 <i>The Twin-T Bridge</i> . . . . .	7
3.2 SIX-PORT REFLECTOMETERS . . . . .	8
3.2.1 <i>General description</i> . . . . .	8
3.2.2 <i>VHF six-port reflectometer</i> . . . . .	9
3.2.3 <i>Calibration</i> . . . . .	11
3.3 VECTOR NETWORK ANALYSERS . . . . .	12
3.3.1 <i>General description</i> . . . . .	12
3.3.2 <i>Calibration techniques</i> . . . . .	12
4 CALIBRATION STANDARDS . . . . .	13
4.1 AIR LINES . . . . .	13
4.1.1 <i>Lossless line models</i> . . . . .	14
4.1.2 <i>Lossy line models</i> . . . . .	16
4.1.3 <i>Skin depth and resistivity</i> . . . . .	18
4.1.4 <i>Broadband considerations</i> . . . . .	20
4.2 TERMINATIONS . . . . .	22
4.2.1 <i>Short-circuits</i> . . . . .	22
4.2.2 <i>Open-circuits</i> . . . . .	22
4.2.3 <i>Matched loads</i> . . . . .	23
5 EXISTING CAPABILITIES . . . . .	23
6 CONCLUSIONS . . . . .	24
7 ACKNOWLEDGEMENTS . . . . .	25
8 REFERENCES . . . . .	25

## 1 INTRODUCTION

The use of VHF signals dates back to the late 19th century and the experiments of Heinrich Hertz validating the theory of guided electromagnetic waves [1]. Today's users of this frequency region include; land, maritime and aeronautical mobile radio; astronomy, satellite and space operations; terrestrial broadcasting, cordless telephones, public radiophones, international distress signalling and many more [2]. VHF impedance measurements provide an essential support service for these users, and many others, in existing and emerging technologies.

The VHF (Very High Frequency) region of the electromagnetic spectrum is understood generally to include frequencies between 30 and 300 MHz, or equivalently, wavelengths from 10 to 1 m. It is an historical classification dating back to the early days of broadcasting when these frequencies were very high compared with other utilised frequencies. This is not so for contemporary microwave engineering where 'very high' might be more suited to describe frequencies above 100 GHz. The term VHF has remained however, and makes a convenient designation for this decade of the radio frequency (RF) spectrum.

The term impedance is used in this report to describe one-port passive terminations having complex characteristics, *i.e.*, both real and imaginary properties. These items are used commonly as transfer standards disseminating traceability to calibration laboratories through national measurement systems. They are used also for international measurement comparisons, promoting understanding and global harmonisation of impedance standards at VHF and other RF bands.

This report begins by outlining some fundamental concepts of VHF impedance measurement. This is followed by a discussion of the instrument types used for these measurements, including; (i) bridge circuits, representing well-established methods, and (ii) vector reflectometers and network analysers, representing more recent techniques. The standards used to calibrate these vector instruments and existing UK national standard measurement capabilities are also discussed. Finally, a list of references is included to allow points of interest to be investigated in more detail.

## 2 FUNDAMENTAL CONCEPTS

There are many textbooks dealing with the theory underlying RF and microwave impedance. The author recommends particularly Somlo and Hunter [3] and Collin [4], for RF and microwave technologists. The derivation of fundamental expressions will not be given here as they are readily available in these, and other, textbooks.

VHF signals occupy an awkward band of the RF spectrum. At higher frequencies, such as the microwave region, it is convenient to use electromagnetic field theory to describe circuit behaviour; the engineer thinking instinctively in terms of electric and magnetic fields and the concept of wave propagation. At lower frequencies, such as audio frequencies, it is convenient to describe circuit behaviour in terms of discrete circuit-elements. Examples of passive circuit-elements include resistors, capacitors and inductors. At VHF however, the engineer must be aware, and consider the appropriateness, of both concepts for any given circumstance.

An important consequence of electromagnetic theory is the use of transmission lines for the efficient transference of energy. The choice of transmission medium, type and size, is governed usually by the application; in particular, the operating frequency and power level. Power handling capacity is not usually a consideration for precision measurements, leaving the choice of transmission medium dependent only on the range of operating frequencies. At VHF the most suitable transmission medium in common use is coaxial line; hollow waveguide is too large at these frequencies and media such as microstrip and CPW (co-planar waveguide) are more difficult to characterise.

## 2.1 DISCRETE CIRCUIT-ELEMENT CONCEPTS

It is often convenient to represent an impedance using a series electrical circuit consisting of a resistor and an inductor as in Figure 1.

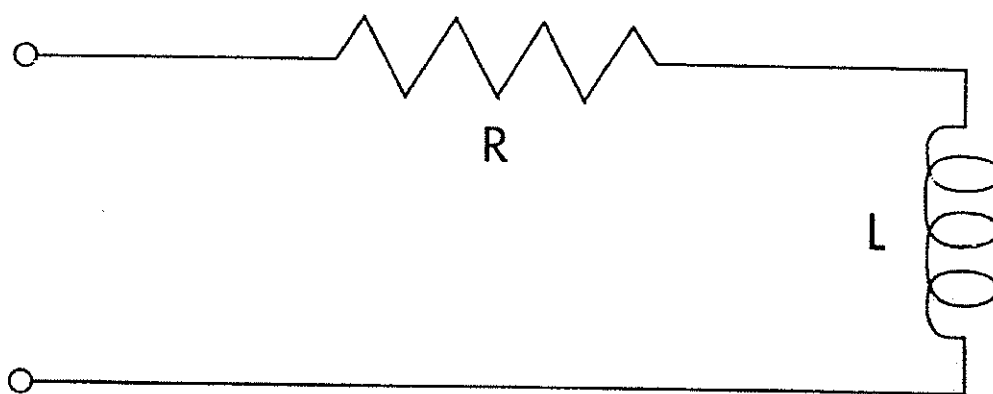


Figure 1: Series electrical circuit representing a terminating impedance.

The impedance  $Z$ , has both resistive and reactive properties and can be expressed as:

$$Z = R + jX \equiv R + j\omega L \text{ (Ohms)} \quad (1)$$

where  $R$  is the resistance,  $X$  the reactance,  $L$  the inductance and  $j = \sqrt{-1}$ . The angular frequency  $\omega$ , indicates that the impedance of the termination is a function of frequency.

Alternatively, it is sometimes convenient to treat a termination at VHF in terms of admittance parameters. Admittance is the reciprocal of impedance. It is conventional to represent an admittance using a parallel electrical circuit consisting of a conductor and a capacitor as in Figure 2.

The admittance  $Y$ , has both conductive and susceptive properties and can be expressed as:

$$Y = G + jB \equiv G + j\omega C \text{ (Siemens)} \quad (2)$$

where  $G$  is the conductance,  $B$  the susceptance and  $C$  the capacitance. A term used commonly to indicate the equivalence between impedance and admittance parameters is immittance, which implies the use of either form of parameter representation.

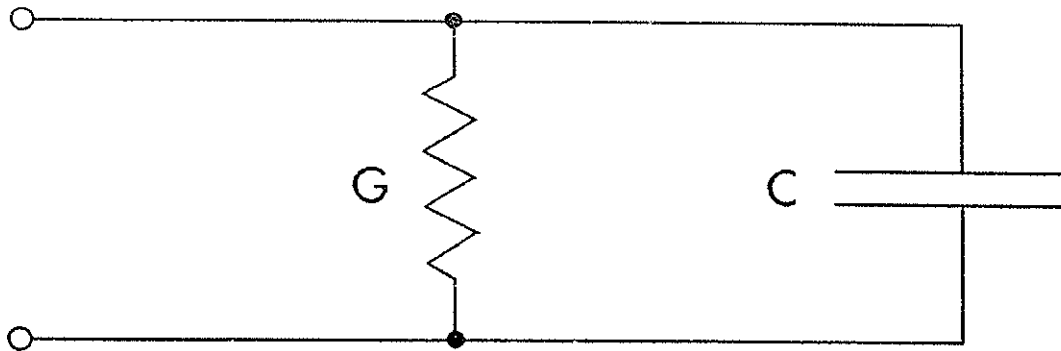


Figure 2: Parallel electrical circuit representing a terminating admittance.

An additional parameter used frequently in conjunction with immittance parameters is quality-factor, or Q-factor. It indicates the degree of purity for capacitors and inductors. The Q-factor for an impedance is defined as the ratio of reactive to resistive components:

$$Q = \frac{X}{R} \equiv \frac{\omega L}{R} \quad (3)$$

The Q-factor for an admittance is defined as the ratio of susceptive to conductive components. An alternative parameter used to indicate capacitor quality is the dissipation-factor, or loss tangent,  $\tan \delta$ . It is the reciprocal of Q-factor:

$$Q = \frac{1}{\tan \delta} = \frac{B}{G} \equiv \frac{\omega C}{G} \quad (4)$$

## 2.2 TRANSMISSION LINE CONCEPTS

The terminations modelled using discrete circuit-element concepts in the previous sub-section can be characterised in terms of their response to guided electromagnetic waves. Consider a coaxial transmission line of characteristic impedance  $Z_0$  terminated by an impedance  $Z_T$  as in Figure 3.

A signal generator connected to the input of the line provides an electromagnetic signal varying sinusoidally with time, *i.e.*,  $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ . The incident voltage and current at any point  $x$  along the line are described by  $V_i$  and  $I_i$  respectively. Similarly, the reflected voltage and current at any point along the line are described by  $V_r$  and  $I_r$  respectively.

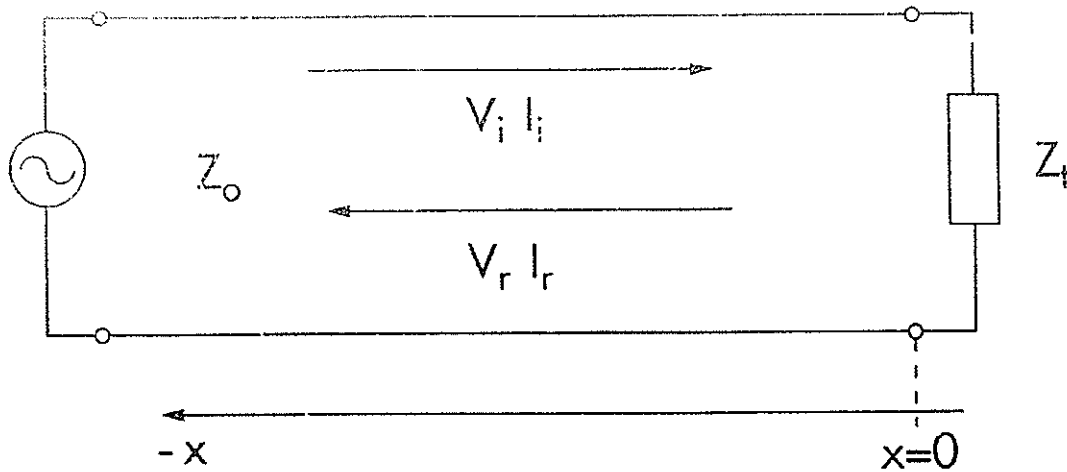


Figure 3: Transmission line of characteristic impedance  $Z_0$  terminated by an impedance  $Z_T$ .

The voltage reflection coefficient (VRC) of the terminating impedance is defined as the ratio of reflected to incident voltage. It is specified usually at the reference plane of the termination, *i.e.*,  $x = 0$ , although it can be given at any point along the line. Voltage reflection coefficient is denoted by  $\Gamma$ :

$$\Gamma = \frac{V_r}{V_i} \quad (5)$$

The current reflection coefficient is defined, for completeness, as the ratio of reflected to incident current. Current reflection coefficients are seldom used in RF and microwave engineering practice.

Two additional parameters relating to VRC are voltage standing wave ratio (VSWR) and return loss. They can be defined in terms of the magnitude of the VRC:

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (6)$$

$$\text{Return loss} = 20 \log_{10} \left[ \frac{1}{|\Gamma|} \right] \quad (\text{dB}) \quad (7)$$

### 2.3 DISCRETE CIRCUIT-ELEMENT AND TRANSMISSION LINE CONCEPT EQUIVALENCE

It can be shown that the VRC for an impedance  $Z_T$  terminating a line of characteristic impedance  $Z_0$ , as in Figure 3, can be expressed as:

$$\Gamma = \frac{Z_T - Z_0}{Z_T + Z_0} \quad (8)$$



Similarly, for an admittance  $Y_T$  terminating a line of characteristic admittance  $Y_0$ , the VRC is given by

$$\Gamma = \frac{Y_0 - Y_T}{Y_0 + Y_T} \quad (9)$$

The terminating impedance  $Z_T$  and admittance  $Y_T$  in equations (8) and (9) are equivalent to  $Z$  and  $Y$  in equations (1) and (2), respectively. These four equations allow VRCs to be transformed to immittance parameters and vice versa, illustrating alternative parameters expressing the same physical property.

Parameter equivalence is also apparent when charts are used to represent VRC and immittance properties. It is conventional to represent VRCs as points in the complex plane. The magnitude of VRC for a passive termination has a value between zero and one; zero when the termination absorbs all incident energy, and one when it reflects all incident energy. The phase of VRC varies between  $\pm 180^\circ$  depending on the termination's complex characteristics. VRC of a termination at any given frequency can be represented as a point within, or on the boundary of, a unit circle centred on the origin of the complex plane. Any impedance, or admittance, can be represented in the VRC plane by superimposing an appropriate grid on the unit circle. These grids, or charts, were used by Smith [5,6] in the 1930s to represent impedance characteristics, and Smith charts are still used as a convenient means to represent device immittance characteristics.

Parameter equivalence can be extended easily to transform from immittance and VRC to Q-factor, dissipation-factor, VSWR and return loss.

### 3 MEASURING INSTRUMENTS

The instruments used to measure impedance at VHF can be divided into two broad categories; bridges and reflectometers. Bridge techniques, in general, are understood best in terms of discrete circuit-element concepts, and reflectometer techniques in terms of transmission line concepts. However, the equivalence relationships discussed in the previous section should be considered when instrument performance is compared.

It is informative to examine trends of instrument usage given in reviews of top echelon VHF impedance measuring capabilities carried out over the past 30 years [7-15]. Bridge techniques, in general, represented the dominant technology throughout the 1960s and 1970s, and although still used in the 1980s and 1990s, they appear less common. Contemporary reflectometer techniques, represented by automatic network analysers based on either four-port or six-port principles, represent an emerging technology developing throughout the 1980s and 1990s.

This section reviews firstly RF bridge techniques; in particular, the Twin-T dual admittance bridge which is still used in some standards laboratories. This is followed by an examination of the six-port technique; in particular, the current UK national standard measuring instrument for VHF impedance measurement. Finally, commercially available vector network analysers based on four-port techniques are discussed. Although emphasis will be given to top echelon measurements performed by standards laboratories, some commercially available network analysers are rivalling these capabilities, as observed by Jones [16].

*3.1.1 Historical perspective.* The bridge circuit was first introduced by S H Christie in 1833 and later made famous by Wheatstone in 1843. Maxwell, in 1873, described several forms of bridge which represent the basis of the modern impedance bridge. Max Wein was the first to report applying AC signals to Maxwell's bridges in 1891, representing the development of the modern AC bridge. The general theory of the bridge circuit will not be given here as it is described in a large number of textbooks on elementary AC theory.

Several reviews of bridge networks can be found in the literature [8,9,17]. These reviews hint at the bewildering variety of networks which can be classified as bridges. In the most part, they are based on the four-arm impedance bridge in which, the components in two of the arms remain constant, the impedance to be determined is placed in the third arm, and the network restored to balance by adjustment of a variable component in the fourth arm (the numbering of the bridge arms is arbitrary). The bridge is energised by an RF source and uses a null detector to establish when the balance condition is reached. A generalised four-arm impedance bridge is illustrated in Figure 4.

Some of the bridge circuits used during the middle part of this century are discussed in references [18-25]. The Twin-T circuit proposed by Tuttle [18] and Sinclair [19] in 1940 formed the basis of a modern high-precision bridge - the Woods Twin-T dual admittance bridge [26]. Woods developed the Twin-T circuit and extended its frequency range to cover the complete VHF region. The instrument was used during the 1970s and 1980s for national standard immittance measurements at VHF in the UK. A description of this instrument type is given because of its importance at VHF for impedance measurement.

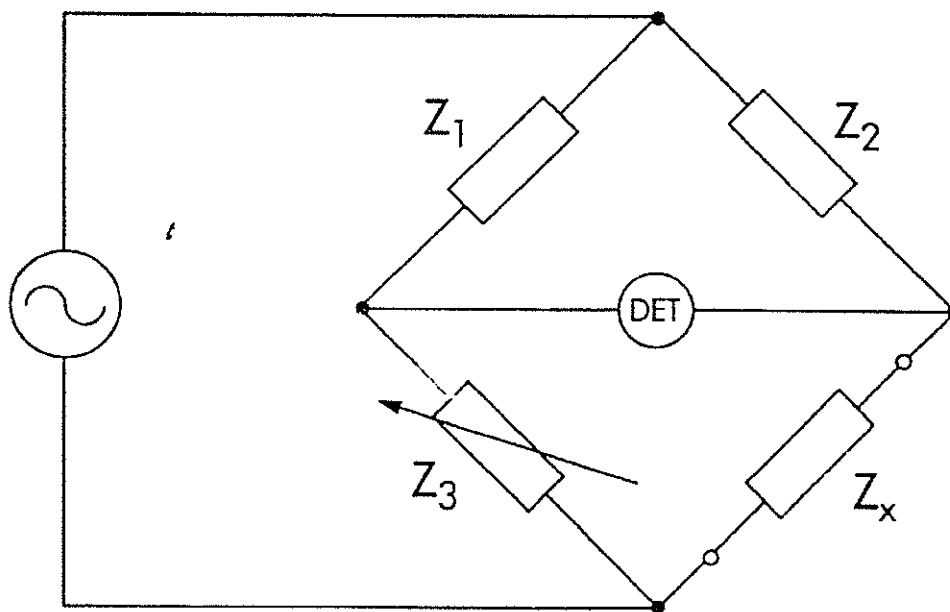


Figure 4: A four-arm impedance bridge containing two fixed components  $Z_1$  and  $Z_2$ , a variable component  $Z_3$ , the device under test  $Z_x$ , and a RF source and null detector.

1.1.2 The Twin-T bridge. A generic circuit diagram of a Twin-T bridge is shown in Figure 5.

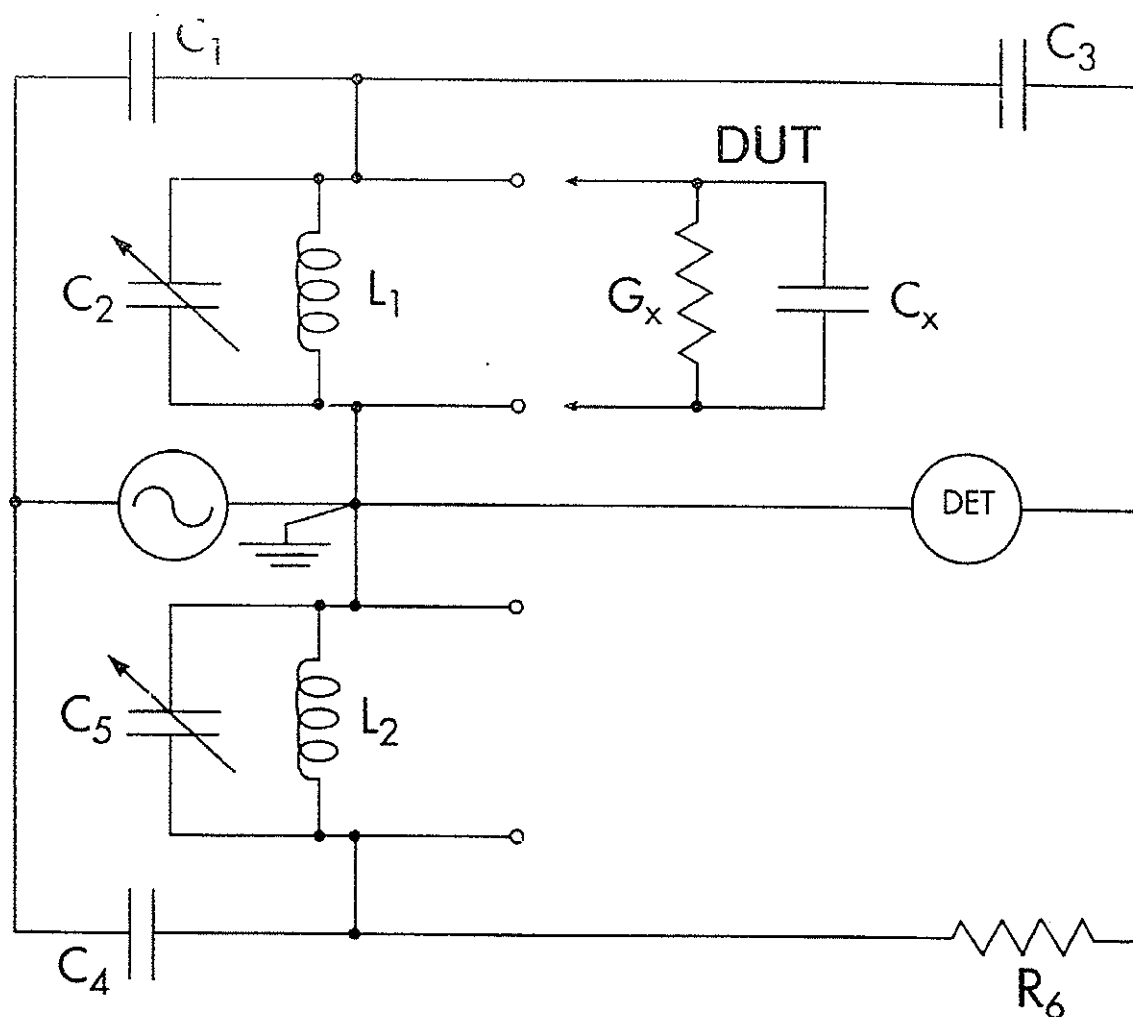


Figure 5: A generic circuit diagram of a Twin-T bridge

The circuit consists of two T-networks connected in parallel. The series elements in each T-network are  $C_1$ ,  $C_3$  and  $C_4$ ,  $R_6$  respectively. The parallel elements consist of variable susceptance standards (capacitors  $C_2$ ,  $C_5$ ), inductors  $L_1$ ,  $L_2$  and the measurement ports. One arm of both T-networks is connected to an ultra-stable RF source while the other arm of the networks is connected to a narrow bandwidth null detector. An advantage with this configuration is that both the parallel elements, including test ports, RF source and detector, share a common ground - aiding shielding and reducing potential earth-loop errors.

Initial balance is achieved against a shielded open-circuit on one of the test ports. The device under test (DUT) is connected to the other test port and balance restored by adjusting the variable capacitors  $C_2$ ,  $C_5$ . The DUT characteristics (admittance parameters) are determined from the balance equations, overleaf:

$$G_x = \omega^2 \left[ R_c \frac{C_1 C_3}{C_4} \right] \Delta C_x \quad (10)$$

$$C_x = \Delta C_2 \quad (11)$$

Conductance and capacitance for the DUT are both given directly in terms of capacitance changes. Hence, measurements can be traceable directly to national capacitance standards and therefore to the fundamental physical quantities of length and time.

The Woods version of this bridge [26] was used until recently as a national standard measuring instrument in the UK. Slack and Graham [27] evaluated the measurement uncertainty for the bridge establishing the useful operating frequency range from approximately 3 to 300 MHz. Measurement uncertainties were found to be unacceptably large above this frequency. There have been several notable contributions to the further development of the Woods Twin-T bridge; (i) Clarke [28] described a method whereby initial balance could be achieved with test port adaptors in place, facilitating measurements in different coaxial line sizes and characteristic impedances, (ii) Williams [29] extended the capabilities of the network and automated parts of the instrument's operation; this version is now available commercially, and (iii) the network's upper frequency limit has been extended to 1 GHz by Grno [30] using quarter-wave line techniques.

The Twin-T bridge is a high precision instrument and remains firmly in the domain of standards laboratories. It is laborious and tedious to use, and requires a high level of operator skill and judgement. These reasons, and the availability of fully automated technologies, led to the instrument's replacement as the UK national standard in 1991.

## 3.2 SIX-PORT REFLECTOMETERS

*3.2.1 General description.* The six-port technique has attracted a great deal of attention since its introduction by Hoer and Engen [31,32] in 1972. Since then, six-port instruments have been reported, or proposed, at operating frequencies from RF [33], microwave and millimetre wave [34,35], submillimetre wave [36] and even optical frequencies [37]. Circuit fabrication appears unaffected by the choice of transmission medium with designs appearing in most waveguide types in common use [38-42]. An overview of the six-port principle is given here which is sufficient to appreciate its use in a VHF impedance measuring instrument.

A block diagram of a six-port is given in Figure 6. A source is connected to one port, the DUT to another and scalar detectors to the remaining four ports. One detector monitors primarily the energy incident on the DUT whereas the remaining three detectors monitor a combination of incident and reflected energies, separated by different path lengths. The ratios of the incident wave detector to each of the three other detectors are used to determine the characteristics, *e.g.*, reflection coefficient, of the DUT. A computer is used to generate and apply correction factors to simulate perfect performance from the imperfect circuit components. A major advantage with the six-port technique is that vector information (magnitude and phase) can be obtained from scalar observations.

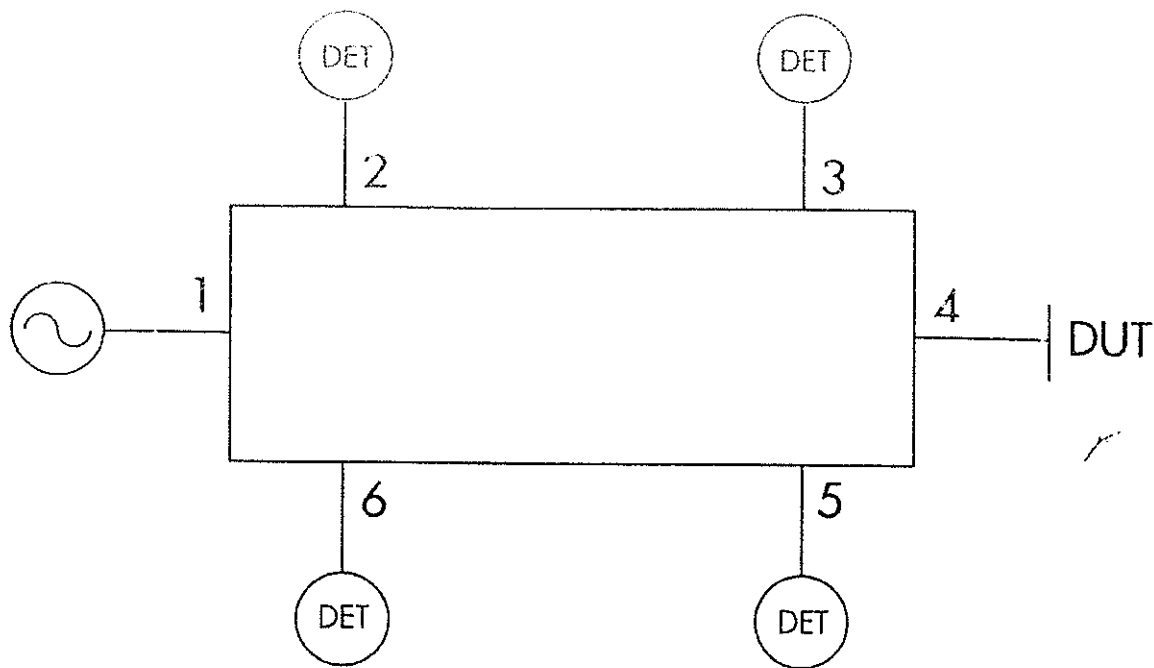


Figure 6: Block diagram of a six-port reflectometer.

Six-port junctions based on a wide range of circuits have been constructed. Designs depend upon required performance and available components. A large number of junctions have been constructed using 3 dB quadrature couplers, power dividers and directional couplers [32,43-46]. Several recent configurations use a single device, such as a five-port or six-port coupler, in conjunction with conventional directional couplers [47-49]. These devices have the advantage that circuit coupling occurs in the same physical space, minimising effects due to thermal expansion caused by temperature gradients existing throughout the circuit.

*3.2.2 VHF six-port reflectometer.* This sub-section describes briefly the six-port reflectometer which is the current UK national standard instrument for VHF impedance measurement. It superseded the Woods Twin-T bridge in 1991.

The RF circuit comprising the six-port junction is given in Figure 7. It is based on a circuit reported by Engen [43] and has been evaluated theoretically by Griffin and Hodgetts [50]. It uses five broadband 3dB quadrature hybrid couplers whose bandwidth is the limiting factor to the bandwidth of the six-port junction. The resistor symbols in Figure 7 represent nominally matched 50 ohm loads. The signal path lengths,  $\theta_i$  (for  $i = 1$  to 6), were adjusted during the instrument's development to ensure that only one of the detectors  $P_1, P_2, P_3$  can ever approach zero signal at any time at all the frequencies of operation, which in this case is any frequency in the range 50 to 250 MHz. These path lengths remain fixed when this criterion has been achieved.

The hermetically sealed couplers used for the six-port junction are temperature stabilised using a temperature-controlled plate in contact with all of the couplers [51]. The RF power sensors are also temperature controlled to minimise drift caused by external thermal fluctuations. Both temperature-controlled plates maintain a nominal temperature of 301K to within 50mK. The couplers and detectors are housed within an aluminium casing with connections for the RF input, the measurement test port and the four detectors. The entire

apparatus is kept in a temperature controlled laboratory maintained at 296K to within 1K. This degree of temperature stabilisation produces a very mechanically and electrically stable instrument, enabling calibrations up to a month old to be used without appreciable performance degradation.

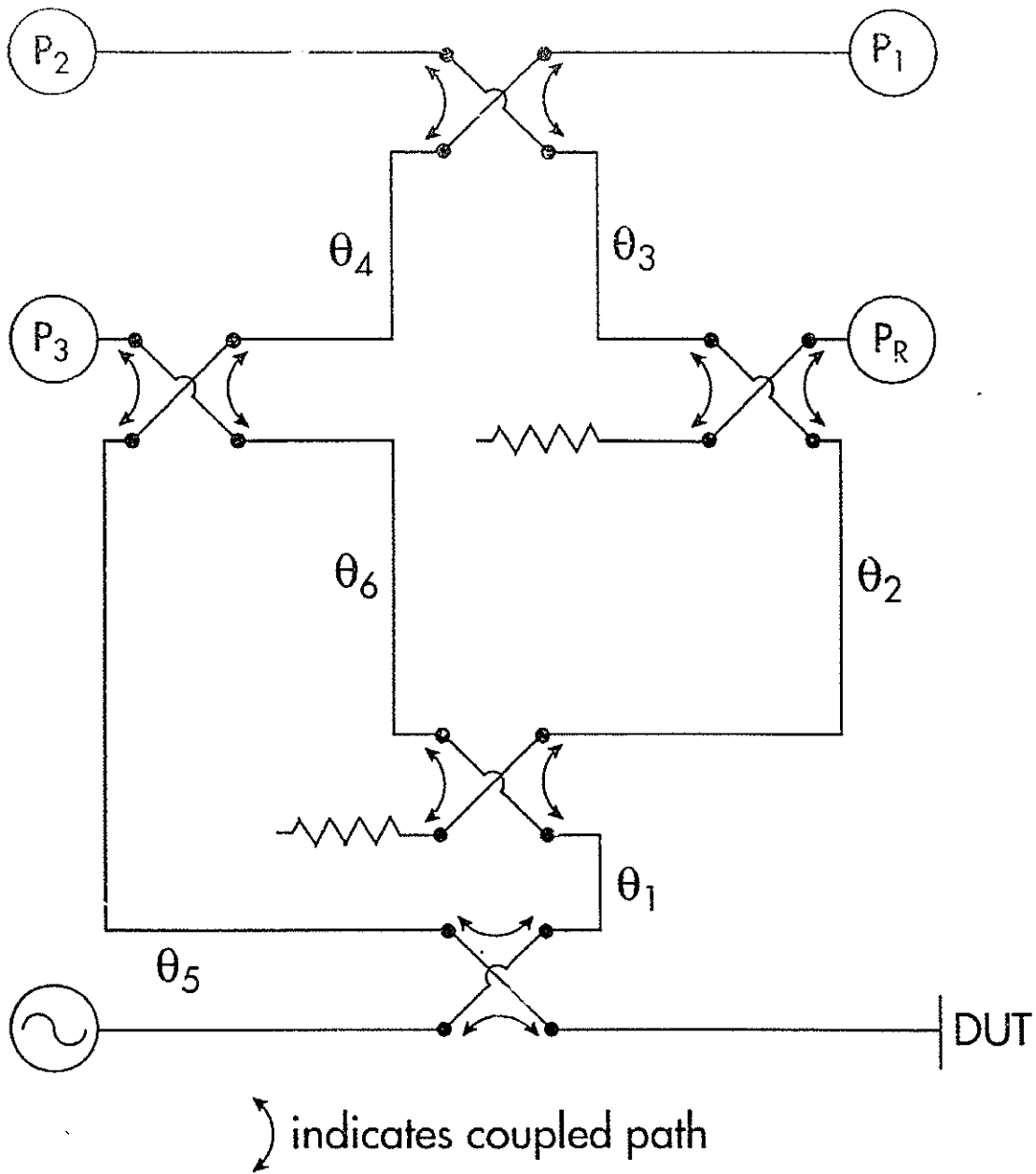


Figure 7: Diagram of the VHF six-port RF circuit.

A block diagram of the complete six-port system is given in Figure 8. The computer controls all the instrumentation via the GPIB. It also provides screen instructions for the operator when required to connect specific items (e.g., during calibration) or items for measurement.