Measurement Note

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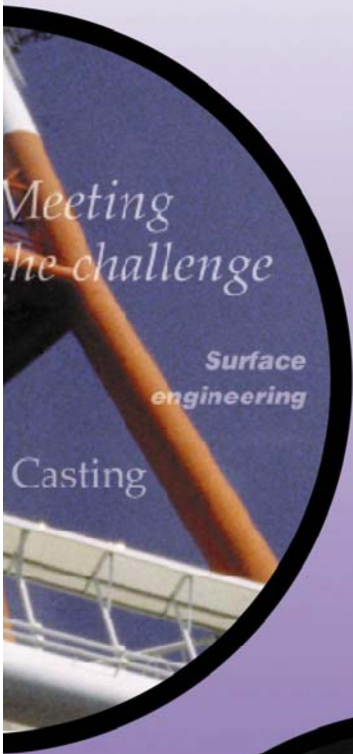
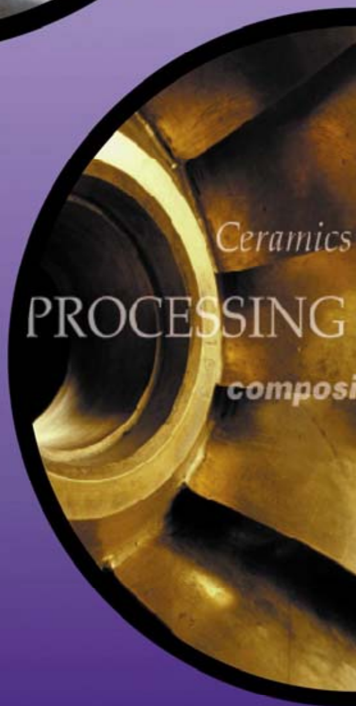


DETERMINING RESIDUAL STRESS IN HARD BRITTLE MATERIALS BY THE THIN STRIP CURVATURE METHOD

The flexural strength of hard brittle materials is often controlled by the level of surface residual stress caused by grinding. In order to evaluate these residual stresses in a simple way, the method of thin strip curvature has been used on a range of alumina ceramics and tungsten carbide hardmetals.

By approximating the depth profile of the residual stresses with a nominal average stress operating over a nominal depth, simple numerical computation of relative levels of stress between different materials, or between different surface finishing procedures, can be achieved.



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Introduction

Machining surfaces results in the generation of residual stress because of a combination of plastic work and thermal effects. When hard, brittle materials such as ceramics and hardmetals are subjected to a grinding operation, the depth of damage may be small, but the residual stresses that result may be very high, around 1 GPa. These residual stresses, which may be anisotropic, can influence the flexural strength of the material when this is determined by surface or near surface defects, such as pores, grinding cracks or large grains.

In order to assess the effect of a given grinding regime on residual stress, a simple method is that of thin-beam curvature measurement. A thin strip is cut from the side of an annealed block of material. The cut surface is ground to a given finish while the strip is held flat. The strip is then released, and the residual stresses developed cause the strip to curve. Normally, the stresses developed are compressive, so the machined surface becomes convex. Alternatively, a larger block of material can be ground, and then a thin strip machined off the ground surface, *e.g.* by electrodischarge machining (EDM) if the material is electroconductive. In this case, the developed curvature is due to the difference in residual stresses between the ground surface and the EDM surface. In either case this curvature is easily measured. The height profile can be measured using an optical microscope with a calibrated focusing stage, or a non-contact profilometer. Increasing curvature (or reducing radius of curvature) for a given thickness of strip implies increasing stress.[1]

Theory

In order to relate the radius of curvature ρ developed in a thin strip of thickness h to the stress in the surface, a mathematical model needs to be established. For any in-plane stress distribution, two conditions must be upheld:

$$\int_0^h \sigma(y)dy = 0, \quad \int_0^h y\sigma(y)dy = 0 \qquad (1)$$

In-plane surface residual stresses are normally an unknown function of depth into the material, being a maximum at the surface and declining with increasing depth. In order to relate this distribution to the net curvature of a thin strip by simple models, this distribution has to be approximated by an average or nominal stress, $\overline{\sigma}_r$, operating over an average depth, h_1. In the absence of knowledge of the depth profile of residual stress, an estimate of the average residual stress can be determined only if a given effective thickness is selected.

A formula originating from Stoney [1] has been used for coatings which suffer thermal mismatch stresses on change of temperature, and thus bend the substrate to a biaxial radius of curvature ρ:

$$\rho = \frac{\overline{E}_s h_2^2}{6\sigma_c h_1} \qquad (2)$$

where $\overline{E}_s$ is the substrate biaxial modulus = $E/(1 - \nu)$, E is Young's modulus, ν is Poisson's ratio, h_2 is the substrate thickness, h_1 is the coating thickness, and σ_c is the stress developed in the coating. This formula assumes that the coating is 'thin' compared with the substrate and subject to uniform biaxial stress. In the case of a thin narrow strip cut from a block of material, it is unclear whether this formula would overestimate for the biaxial effects. An alternative, more-rigorous approach can be derived from Timoshenko's analysis [2] of the uniaxial bi-metallic strip:

$$\rho = \frac{(h_1 + h_2)[3(1+m)^2 + (1+mn)(m^2 + 1/mn)]}{6\Delta\alpha\Delta T(1+m)^2} \qquad (3)$$

where h_1 and h_2 are the respective layer thicknesses for materials 1 and 2, $m = h_1/h_2$, $n = E_1/E_2$, and $\Delta\alpha\Delta T$ is the thermal expansion strain mismatch on changing the temperature by ΔT. By replacing the thermal strain element by an imposed average elastic strain σ/E in the thickness h_1 due to a mechanically induced

[1] The principle is very similar to the long-standing 'Almen' technique for recording the generation of residual stress due to shot peening processes in metal alloys (see *e.g.* 'Shot peening applications', 8th edition, Metal Improvement Co. Inc., Paramus, NJ07652, USA, 2001.)

residual stress, and using $n = 1$, it can be shown that:

$$\sigma \approx \frac{E}{\rho}\left(\frac{h_1 + h_2}{2} + \frac{h_1^3 + h_2^3}{6h_1h_2}\right) = \frac{E}{\rho} f(h_1, h_2) \quad (4)$$

In this case there is no allowance for biaxiality, with E representing the uniaxial Young's modulus along the length of the strip. Equation 4 becomes Equation 2 if biaxiality is re-introduced and one of the layers becomes very thin compared with the other, *e.g.* $h_1 < h_2/100$.

Baratta [3] has discussed the relationship between curvatures of monolithic beams and plates used in flexural strength testing. He concludes that, for a typical brittle material with $\nu = 0.25$, there is less than 1% deviation from the simple solution for a narrow beam if the beam cross-section shape (width/depth ratio) is less than 12 for an axial outer-fibre strain of 0.8%, or greater for smaller strains. Consequently, for the aspect ratios that are practical for experiments on ceramics and hardmetals (width 2 mm to 5 mm, thickness 0.2 mm to 0.5 mm), the thin strip method can be assumed, *i.e.* Equation 4.

Finally, of course, the stress in the original surface, constrained flat, is higher than that in the curved strip by an amount determined by the flexibility of the strip. To correct for this, a mechanical bending moment is required in the opposite sense to apply an equal reverse radius of curvature. This generates mechanical bending stresses in the strip which reach σ_m in the surface layer, where:

$$\sigma_m = \frac{Eh}{2\rho} = \frac{E}{\rho}\left(\frac{h_1 + h_2}{2}\right) \quad (5)$$

Using the superposition principle, this stress should be added to that shown in Equation 4 to give the effective residual stress in the surface of the original tile. To visualise the stress/curvature behaviour, Figure 1 illustrates the dependence of the average residual stress on strip thickness for a given curvature in a material of elastic modulus 400 GPa, and the raised stress if the strip were constrained flat in a thick body. The effect of being constrained flat is small, especially in the thicker strips.

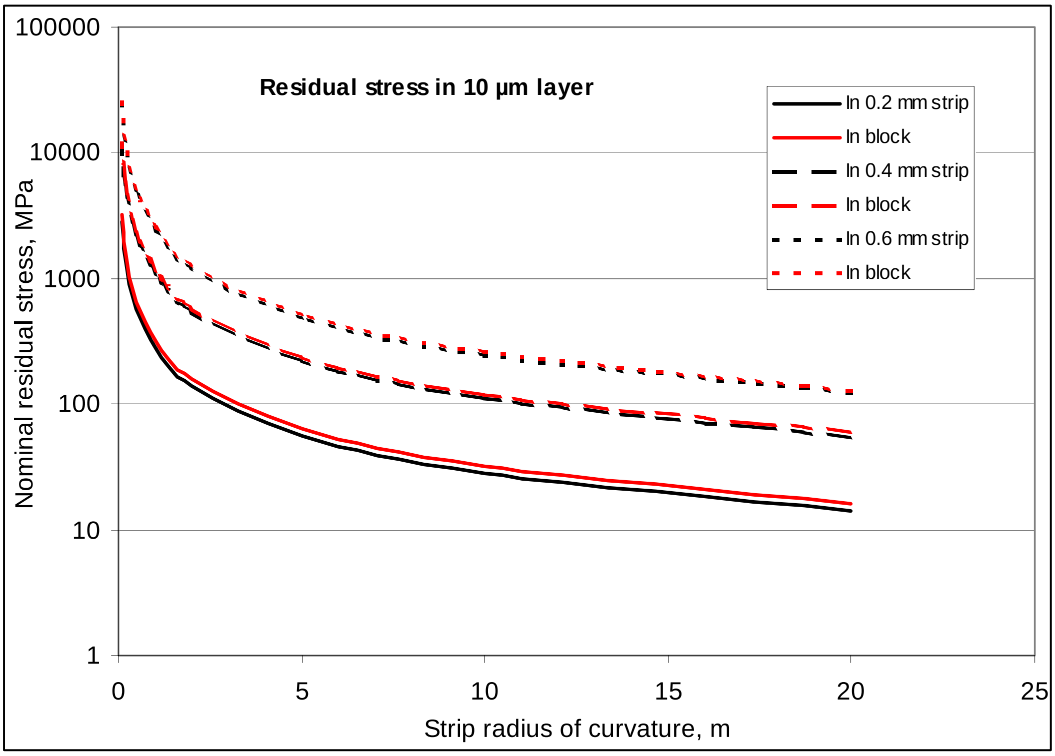

Figure 1: *Average residual stress in a 10 µm layer required to induce a given radius of curvature in a material of Young's modulus 400 GPa.*

Experiment

The simplest procedure is to machine flat the narrow face of a ceramic or hardmetal tile or block with a length of typically 40 mm to 50 mm and thickness of 2 mm to 5 mm. The tile is then annealed to remove residual stresses from this operation. A thin strip is then cut from the tile edge using diamond sawing or, for electrically conducting materials, electro-discharge machining. The removed strip will then curve as a consequence of the cutting operation. If it is straightened by gluing to a flat plate, the cut surface can be ground, and the new curvature on release is that due solely to the last grinding operation.

The strip curvature can be measured most easily by obtaining a height profile of the strip laid on the bed of a microscope incorporating height measurement as focus. Alternatively, a scanning profilometer can be used. A plot of height against lateral position, which is strictly circular, can be approximated by a parabolic fit $y = ax^2 + bx + c$, the second derivative of which gives the curvature, C, which is equal to the reciprocal of the radius of curvature, ρ, *i.e.*:

$$\rho = \left(\frac{d^2 y}{dx^2} \right)^{-1} = \frac{1}{2a} \qquad (6)$$

As noted above, the residual stress developed by machining is likely to be a maximum near the surface, declining into the bulk of the material. In employing Equation 4, an assumption has to be made concerning the effective ‘thickness’ of the stressed layer. In most brittle materials this is unlikely to be more than about 10 µm to 15 µm from evidence of damage in cross-sections of machined or abrasively worn surfaces. The nominal stress that is then computed is an average over this depth. If the surface is subsequently polished away in stages, and the curvature re-measured at each stage, the effective depth profile of the residual stress can in principle be determined. This will require uniform material removal at each stage, and accurate knowledge of the thickness of material removed. The former of these requirements is hard to achieve, but the latter can be done by following the size of small hardness indentations as material is removed.

Results

Tests have been done on both ceramics and hardmetals. In the case of ceramics, two commercial 95% aluminas were used in the form of tiles 50 mm x 50 mm x 5 mm thick. Two edges of each tile were machined, and then the tiles were annealed at about 1200 °C. Thin strips were cut from the edges, and these were glued to a steel plate and surface ground with a 320 grit bronze-bonded diamond wheel, either along or across their lengths. They were carefully released from the steel plate by dissolving the glue. The convex side curvature was measured using a Nikon measuring microscope with a vertical digital height read-out. Measurements were made every 2 mm to 3 mm along their lengths, and the curvature derived as described above. An example of ‘Sintox FA’ 95% alumina is shown in Figure 2. The results of curvature and nominal residual stress calculations are summarised in Table 1 where a stressed layer thickness of 10 µm has been assumed. No correction has been made using Equation 5 for constraining the strip flat.

It can be seen that the two ceramics behave rather differently, with the residual stress in the finer-grained ‘Fibralox’ being considerably higher than those in the ‘Sintox FA’ under the same grinding condition. Residual stress levels are typically 500 MPa, more than adequate to affect flexural strength measurements.

Five different hardmetals were machined on a 40 x 2 mm surface of a 40 x 5 x 2 mm test bar, using both benign and harsh grinding conditions as detailed in Table 2. These surfaces were then sliced off using electro-discharge machining (EDM) at three different thicknesses, nominally 0.2, 0.4 and 0.6 mm.

The results for the curvature measurements are given in Table 3, and for the stress calculations in Table 4 and Figure 3. The same procedure was used for estimating the residual stress difference between the two faces of the strips. A 10 µm stress layer thickness was assumed with no correction for being constrained flat.

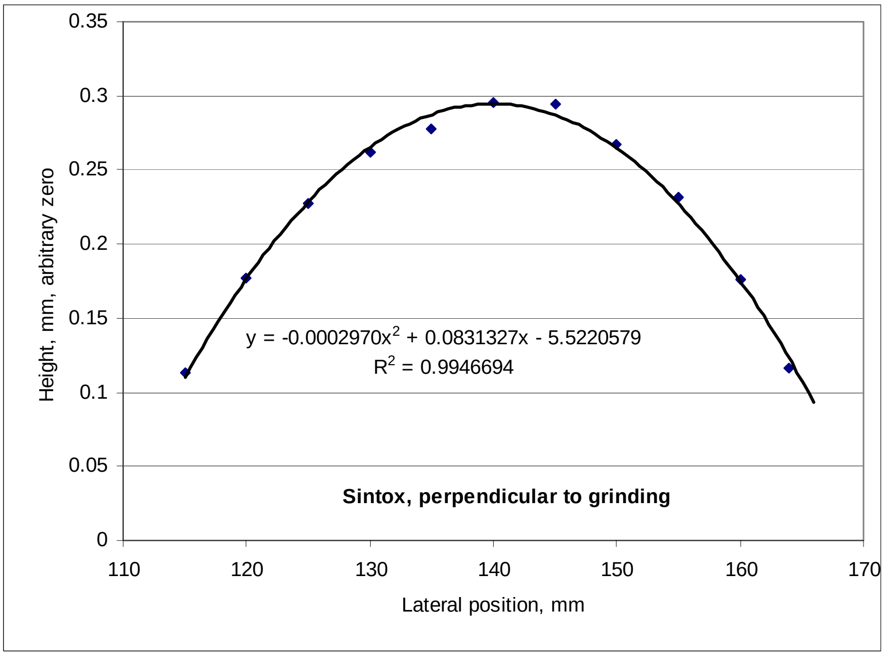

Figure 2: *Strip curvature measurements on 'Sintox FA' 95% alumina, grinding direction across the strip length.*

Table 1: Stress calculations for alumina ceramics

Material	Grinding direction	Curvature, mm^{-1}	Total thickness, mm	Modulus, GPa	Assumed layer thickness, µm	Nominal residual stress, MPa
Sintox FA (95% alumina)	parallel	0.000708	0.290	360	10	370
	perpendicular	0.00055	0.308	360	10	323
Fibralox (96% alumina)	parallel	0.001009	0.305	360	10	582
	perpendicular	0.001375	0.295	360	10	743

Table 2: Grinding conditions for hardmetal test-pieces

Condition	Wheel	Rotation speed	Traverse speed	Depth of cut
'Benign'	D107 75 R, 200 mm diameter, 13 mm face	2840 rpm	~100 mm/s	0.01 mm, five passes, five passes at final cut, machine feed
'Harsh'			~50 mm/s	0.1 mm, single pass, hand feed

Table 3: Curvature results for hardmetals in two finishes and three different thicknesses

Material code	Nominal Co content, %	Curvature, mm^{-1}, three strip nominal thicknesses					
		'Benign' ground			'Harsh' ground		
		0.2 mm	0.4 mm	0.6 mm	0.2 mm	0.4 mm	0.6 mm
mars6	6	0.00376	0.00150	0.00064	0.00612	0.00102	0.00044
dyf10	10	0.00622	0.00152	0.00068	0.00322	*	*
mars11	11	0.00634	0.00121	0.00045	0.00512	0.00086	0.0003
tuf12	12	0.00706	0.00156	0.00068	0.00104	0.00003	*
dyc16	16	0.01188	0.00202	0.00080	0.00548	0.00072	0.00045

* Curvature too small to measure

Table 4: Stress calculations for hardmetals, assuming layer thickness of 10 µm

Material	Modulus GPa	Longitudinal nominal residual stress, GPa, compressive					
		Fine ground			Coarse ground		
		0.2 mm	0.4 mm	0.6 mm	0.2 mm	0.4 mm	0.6 mm
mars6	626	2.76	2.51	2.29	2.43	1.75	1.68
dyf10	575	2.27	2.27	2.31	1.34	-	-
mars11	565	2.23	1.84	1.87	2.30	1.31	1.05
tuf12	540	2.68	2.30	2.17	0.46	0.05	-
dyc16	510	4.05	2.75	2.45	2.06	1.05	1.45

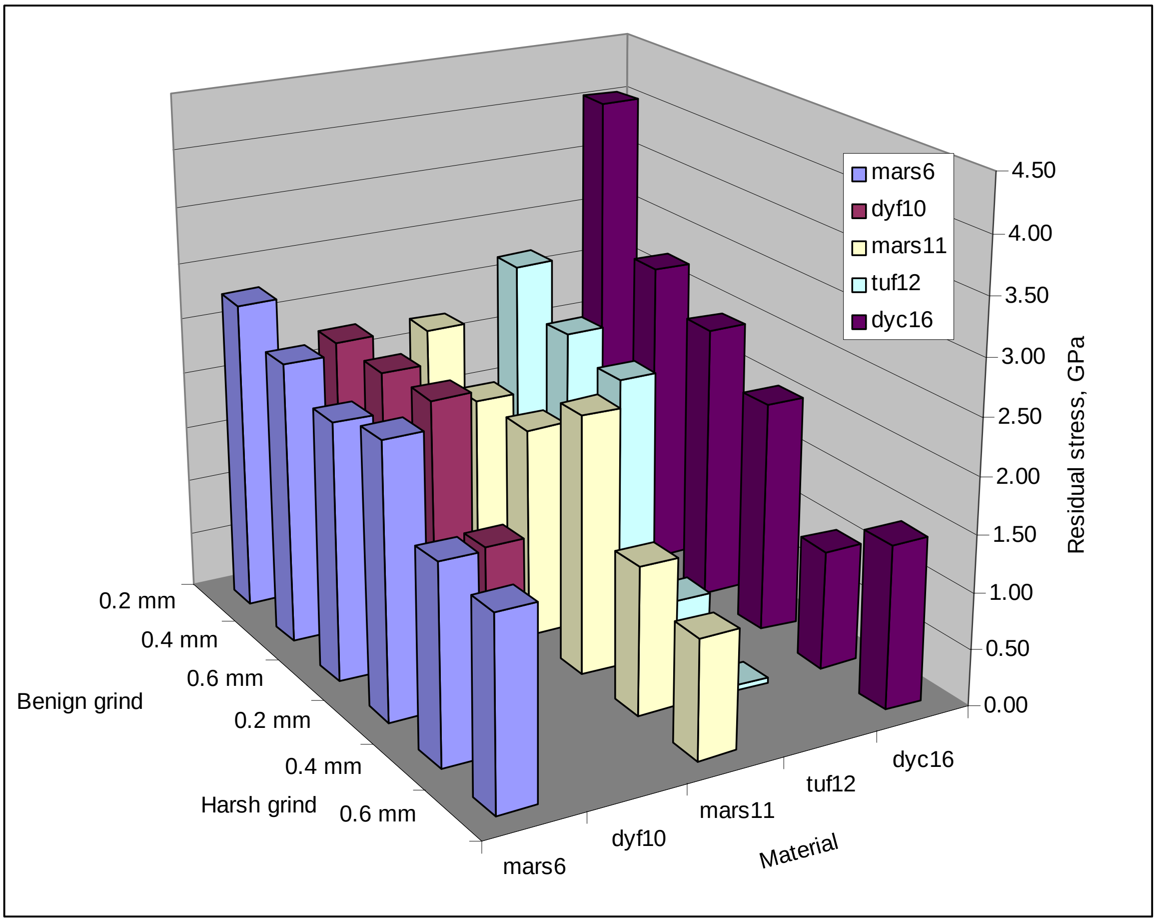

Figure 3: *Computed stresses in hardmetal test samples*

The nominal compressive residual stresses computed for a 10 µm layer thickness are up to 4 GPa in magnitude, but are typically 2 GPa. Such levels will clearly influence flexural strength measurements from surface origins which are subject to this stress. In principle, the nominal residual stress should be independent of the test-piece thickness, but the results vary somewhat. An assumption has been that the EDM process places no residual stress on the cut surface, and this may not in fact be the case. The strip thickness measurement is also an important parameter, a 1% error in thickness resulting in a 2% error in computed stress.

It also seems remarkable that for the stresses for the single-pass finish, harsh grinding condition, the nominal residual stress seems generally lower than for the multipass finish with small cuts.

Conclusions

The strip curvature method has been employed to investigate the development of residual stress in machining of hard and brittle materials. Using strip thicknesses of typically 0.2 to 0.4 mm, it has been found that the method is appropriate for demonstrating differences in response between materials, and the effects of machining direction. Average nominal residual stress levels have been estimated using the Timoshenko bi-metallic strip analysis and assuming a typical depth of residual stress generation of 10 µm. The depth profile of the residual stress can in principle be determined by polishing away the surface if means can be found to do this in a very uniform manner.

Acknowledgements

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