Review of Methods and Analysis Software for the Determination of Modulus from Tensile Tests

Summary

This report has been carried out as part of the TENSTAND project *"Computer Controlled Tensile Testing Machine: Validation of European Standard EN 10002-1",* to review the current practice for making accurate modulus measurements from a tensile test.

The report considers the suitability of the current testing standards and provides recommendations and advice for improving the quality of modulus measurements. Testing issues and software analyses are addressed.

Further work is ongoing within the TENSTAND project to provide more detailed information and analysis of a range of data, but even at this stage it is clear that there is scope to develop a dedicated procedure for measuring modulus, which could be included in future revisions of EN 10002-1.

J D Lord

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1 INTRODUCTION

Although modulus is an intrinsic material property and a key parameter in engineering design and materials development, the current mechanical test methods for measuring it are not well developed. The existing tensile standards EN10002-1 [1] and ASTM E8 [2] focus predominantly on measuring the full stress-strain curve, of which the elastic part is often only a small proportion. A draft revision of EN10002-1, which is currently under formal vote and is being examined and validated within the current TENSTAND project, contains more detailed information on computer controlled testing, data sampling and uncertainty evaluation, but still does not cover modulus measurement in any detail. ASTM E111 [3] is the only standard currently addressing some of the issues relevant to making accurate modulus measurements from a tensile test, although a number of in-house proprietary procedures exist. These are often specialised tests, and it should be recognised that it might be neither feasible nor realistic to carry them out in a cost effective way in a high throughput computer controlled test machine or commercial test house. However it is becoming increasingly apparent that customers are expecting accurate values for modulus included in the material specification datasheet.

An accurate knowledge of the engineering value of Young's modulus is vital for design studies, for finite element and modelling calculations and for giving reliable fits to the constitutive equations for the stress-strain curve. Accurate values of modulus are also necessary for obtaining reliable values for proof stress, because inaccuracies in the slope or modulus fit can give significant errors in proof stress, particularly if the material has a high work hardening rate in the early stages of yield. There is considerable interest at present in the Formula 1 community, as regulations have been introduced to restrict the specific modulus of metallic components used in the cars. Accurate measurement of modulus is essential for getting the most out of the materials being developed.

A survey of TENSTAND partners [4] revealed that most did not measure modulus routinely from the tensile test. Of those who did, the modulus measurements were usually made using a different set up and test conditions, or by alternative dynamic methods.

This document therefore aims to highlight some of the issues necessary for making accurate measurements of Young’s modulus from the tensile test, with particular relevance to specific developments that could be included in future revisions of EN10002-1. **Throughout the document, key recommendations and issues are highlighted in bold.**

2 REVIEW OF EXISTING TENSILE TESTING STANDARDS

Young's modulus can be defined as the ratio of stress to strain during elastic loading. Traditionally, the modulus was determined 'by eye' from a straight line drawn on the linear part of the stress-strain curve, but more recently automatic testing machines using computer control and data acquisition use some form of curve fitting to get a best fit to the data. With the general tensile testing standards at present, there is little guidance on how modulus should be measured, and aspects of strain measurement are covered only briefly. Both EN10002-1 and ASTM E8 give no formal definition for modulus, and yet accurate measurement of the slope of the stress-strain or load-displacement curve is necessary for calculating reliable proof stress data. ASTM E111 covers the measurement of Young’s modulus, tangent modulus and chord modulus in more detail, the latter two being recommended for non-linear materials.

Table 1 below shows a comparison of the scope and test conditions of the current tensile testing standards, and their relevance to modulus measurement.

<table>
<tr><th></th><th>EN10002-1</th><th>ASTM E8</th><th>ASTM E111</th></tr>
<tr><td>Scope</td><td colspan="2">Tensile testing of metals at ambient temperatures

Modulus is not explicitly defined in either standard</td><td>Young’s modulus,
Tangent modulus
and Chord modulus.</td></tr>
<tr><td rowspan="3">Test conditions</td><td>Uniaxial tensile testing
Elevated temperature testing in EN10002- 5</td><td>Tensile testing of metallic materials
RT only</td><td>Tension or compression
RT, elevated temp (below creep), sub zero</td></tr>
<tr><td colspan="2">Continuous loading</td><td>Continuous or incremental loading (via dead weights).
Measurements during the loading or unloading cycle</td></tr>
<tr><td colspan="2">Hysteresis tests can be used to measure modulus/slope

Preloads permitted but the value must be noted</td><td>Preload is recommended,
Specimen should be free of residual stress.
Tests should be carried out below elastic limit, and below 0.25% strain</td></tr>
<tr><td>Speed of testing</td><td colspan="2">Recommendations given for various materials and conditions</td><td>Not specified</td></tr>
<tr><td>Extensometry</td><td>Class 1 for R_p

Class 2 elsewhere</td><td>Class B-2 for R_{eL}, R_{eH} and A_t
Averaging extensometry recommended for R_{eL}, R_{eH}</td><td>Class B-1
Averaging extensometry recommended</td></tr>
<tr><td>Strain uniformity and alignment</td><td colspan="2">No recommendations, other than general guidelines to reduce misalignment and bending</td><td>As ASTM E8, E9
Strain increments on opposite sides < 3%</td></tr>
<tr><td>Repeat measurements</td><td colspan="2">Not applicable</td><td>Minimum of 3 runs recommended, but single test is permissible</td></tr>
<tr><td>Uncertainty</td><td>Example calculations given for a number of parameters (not E) in Annex A of prEN10002-1.</td><td>Some advice regarding precision statistics, but no uncertainty calculations</td><td>Should be included with the report. No examples or guidelines.</td></tr>
<tr><td>Data fitting</td><td>No recommendations – some guidelines on data sampling and measuring the slope included in Annex A of prEN10002-1.</td><td>Only basic advise on data analysis.</td><td>Linear elastic: Least squares fit and/or strain deviation
Non-linear elastic materials Polynomial approximation and chord modulus</td></tr>
</table>

Table 1: Comparison of EN10002-1, ASTM E8 and ASTM E111

The main differences between ASTM E111 and EN10002-1 and ASTM E8 are the scope of testing and the level of detail related to testing at low strain values. ASTM E111 covers the measurement of modulus in both tension and compression testing and by the use of dead weight loading; EN10002-1 and ASTM E8 cover tensile testing only. In all three standards averaging extensometry is recommended, but only ASTM E111 gives specific guidelines for the uniformity of strain measurements over the range of the test, stating that the strain increments on opposite sides of the testpiece must not differ by more than 3%. ASTM E111 also advocates the use of a higher resolution extensometer compared to the conventional tensile test methods, and also gives detailed advice on data analysis.

Due to the difficulties in obtaining reliable modulus data from the existing tensile test standards, many users turn to dynamic methods. A variety of dynamic modulus methods are available including flexural resonance methods, the impulse excitation technique (IET), and various ultrasonic or acoustic wave propagation methods. Useful references are given in Refs. 5-8. The most commonly used methods for metals are the resonance techniques. The dynamic methods have the advantage that they are relatively quick and simple and involve small elastic strains and high strain rates. Some can be readily modified to enable high temperature measurements. They typically use a small and simple specimen geometry, but the methods can be sensitive to machining damage, surface finish and poor dimensional tolerances, all of which affect the accuracy of the result. A variety of commercial equipment is available and the theoretical errors in measurement of modulus by dynamic methods are small, typically of the order of ±1%.

Fig 1 shows modulus data for the BCR Nimonic 75 tensile certified reference material, CRM661 [9] measured by tensile tests and dynamic methods at NPL and BAM on the same testpieces. The BAM measurements were carried out using a high precision side-to-side averaging extensometer; the NPL tests were made using strain gauges bonded to both sides of a rectangular testpiece. Results generally show good agreement but the tensile data has greater scatter than the dynamic methods, due to the difficulties of measuring modulus at low strain values.

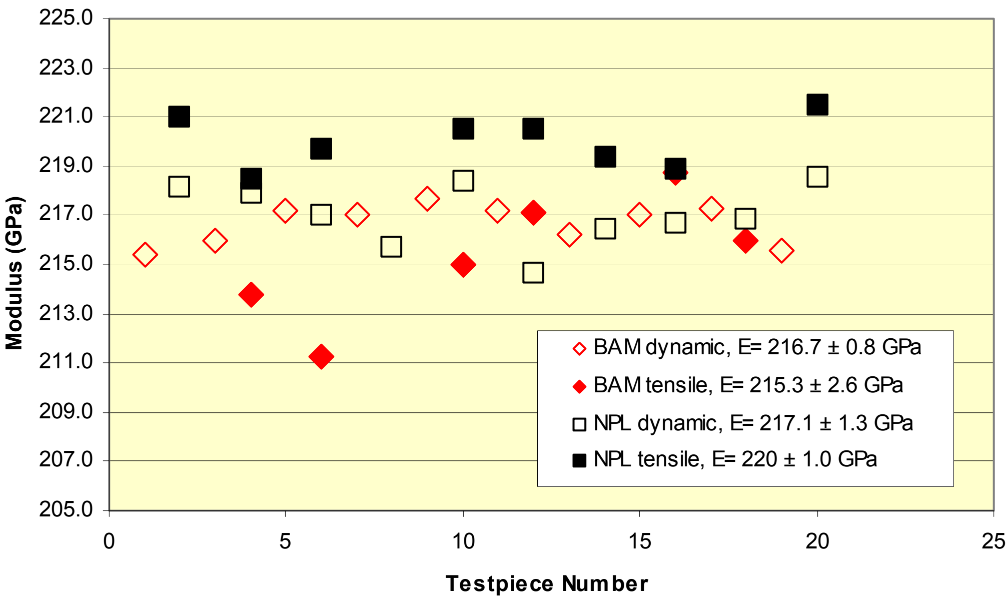


Fig 1: Modulus data for the BCR Nimonic 75 tensile certified reference material, measured by dynamic and tensile methods.

3 IMPROVING THE QUALITY OF MODULUS MEASUREMENTS FROM THE TENSILE TEST

Examination of the results from a series of intercomparison exercises and previous studies [9-16] show that a number of recommendations can be made for improving the quality of the modulus measurements obtained from the tensile test. These include:

- **More accurate strain measurement – averaging measurements are essential and higher Class extensometry is preferred**
- **More careful consideration of data sampling issues and data analysis methods**
- **Use of different specimen geometries – with longer gauge lengths and improved alignment, to reduce bending**
- **Checks and validation using certified reference materials**

Some aspects of strain measurement and data analysis will be covered in the following sections.

3.1 Strain Measurement Methods

Extensometry

According to EN ISO 9513 [17], the bias error associated with the various class of extensometer is summarised in Table 2 below.

Class of Extensometer	Bias Error	
	Relative, %	Absolute, µm
0.2	± 0.2	± 0.6
0.5	± 0.5	± 1.5
1	± 1.0	± 3.0
2	± 2.0	± 6.0

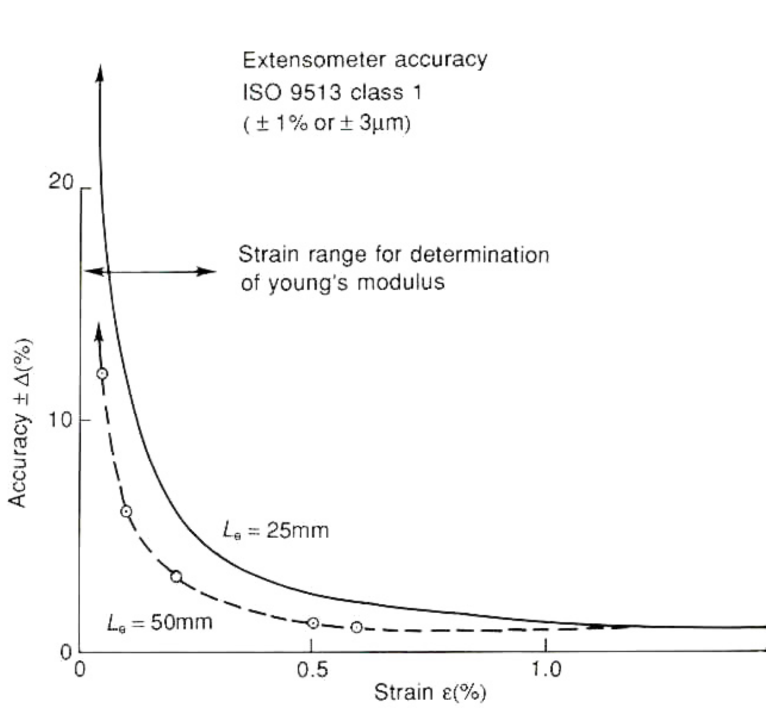
Table 2: Bias error associated with various class of extensometer (EN ISO 9513 [17])

For the highest possible accuracy, a Class 0.2 averaging high-resolution extensometer, calibrated according to EN ISO 9513 over the restricted strain range appropriate to the test, is recommended for modulus measurement. Unfortunately Class 0.2 and 0.5 extensometers are not widely available, nor do many users have the appropriate equipment to calibrate these devices over the low strain range encountered in modulus testing so in many cases Class 1 extensometers are used. For such devices the total bias error is ± 1% or 3 µm, whichever is the greater, and this can lead to significant errors at low strains. Consideration of Fig 2 illustrates the problem. Because of the 3µm lower limit, the absolute error in strain measurement increases as the strains become smaller. Typically, for a 25 mm gauge length and 0.1% strain, the error in modulus can be as high as ± 12% [14].

These are the typical uncertainties that can be attributed to the measurement of strain for a Class 1 extensometer. Other factors – such as the uncertainty in cross-sectional area, load measurements, and data fitting routines - also contribute to the overall uncertainty in the modulus measurement. These individual uncertainties are usually summed using the root mean square method, and then multiplied by a coverage factor to give an expanded uncertainty for the measurement to a known confidence level (typically 95% or 2 standard deviations). It is good practice for users to develop their own uncertainty budget for the modulus measurement, as it is a useful mechanism for identifying which parameters contribute most to the uncertainty in their own particular test set up. Some guidelines are included as an annex in the proposed revision to EN10002-1, but more general advice on uncertainty calculations can be found in Refs 18-21.

To summarise for extensometry:

- **Averaging extensometry is essential (example in Fig 3)**
- **Class 0.5 or better is recommended**
- **Devices must be calibrated specifically over the limited strain range relevant to the modulus measurement**



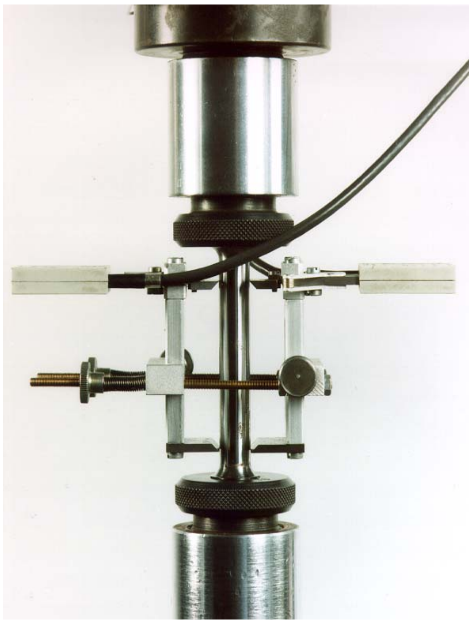


Fig 2: Typical errors likely in determining Young's Modulus with a Class 1 extensometer [14]

Fig 3: Typical high resolution averaging extensometer (courtesy of BAM)

3.2 Strain Gauges

Due to the difficulties associated with calibrating and setting up a high precision averaging extensometer, consideration should be given to using strain gauges bonded to each side of the testpiece to measure the strain during the test. At present none of the tensile testing standards directly advocate or support the use of strain gauges, however they are an attractive alternative to the high-resolution extensometer. A number of practical issues must be considered however to ensure accurate and reliable results:

- **The strain gauge is a precision instrument and installation should only be carried out by suitably qualified staff.**

- **The maximum amplifier gain should be chosen to give the greatest strain resolution and full-scale output over the limited strain range during the modulus test.**

- **An accurate gauge factor must be used and consideration should be made of the variation of gauge factor with temperature.**

- **Calibration of the strain gauge instrumentation should be carried out over a similar strain range to that used in the test.**

- **Strain gauges cannot be readily used for machine control.**

Strain gauges are only suitable for measuring the full tensile properties if the testpiece failure strains are less than about 3% and the resolution of the strain gauge reading depends on the gauge factor and instrumentation gain. Modern strain gauge instrumentation typically has a resolution of $\pm$ 1 $\mu\varepsilon$, but the maximum strain that can be measured may be limited to only 0.5% (5000 $\mu\varepsilon$). If gauges are used to measure a larger part of the stress-strain curve then a compromise must be reached between the maximum strain that can be measured and the measurement resolution required.

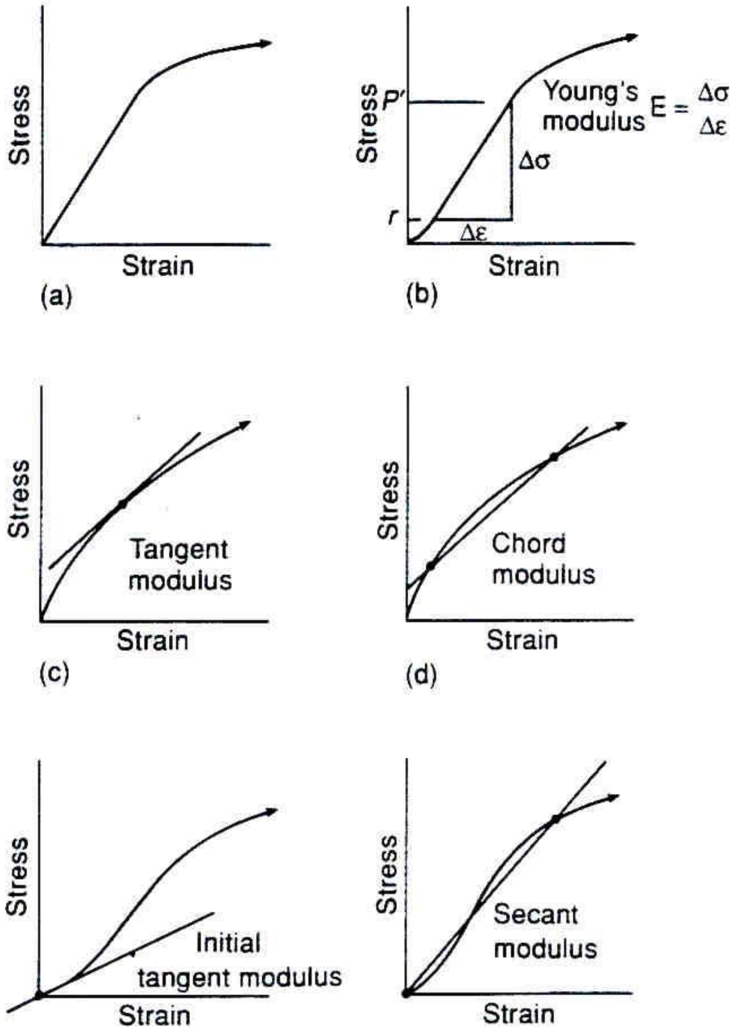
Strain gauge installations may also be susceptible to other uncertainties that are difficult to quantify. The instrumentation itself can be calibrated by using a shunt resistor, but the individual gauge installation on the testpiece itself cannot be calibrated easily. Errors can arise due to misalignment of the gauge, poor gauge installation and bonding, temperature effects, Wheatstone bridge non-linearities and transverse sensitivity. All are important factors but are difficult to quantify. However, for modulus measurements at low strain levels, uncertainties of better than ~ $\pm$ 1% should be readily achievable. **It is vital however, that the gauges should be applied to both sides of the testpiece and averaged to take account of out-of-plane bending.**

4 DATA ANALYSIS METHODS

The method used to analyse the data has an important effect on the calculated modulus value. A wide variety of methods can be used, including graphical techniques but most systems today use computer-based analyses. Commercial test machine software such as *Zwick testXpert* and *Instron Merlin* offer a wide range of options relating to the calculation of the slope or modulus, some of which are illustrated schematically overleaf. .

It is not the intention of this report to discuss in detail the aspects of all the individual options, but to highlight some of the issues that the user should consider in their choice of algorithm.

The preferred procedure is to examine the whole of the stress-strain data (below the elastic limit) and optimise the fit of the modulus line to the data by consideration of least squares regression analyses or other statistical fitting techniques.



Some of the typical analysis options available to the user include:

- Maximum slope
- Tangent modulus
- Chordal modulus
- Secant modulus
- Segment modulus
- Initial tangent modulus
- Hysteresis loop measurements
- Combined tangent/secant
- Variations and combinations of the above

Fig 3: Schematic definitions of modulus definitions and calculations [14]

The accuracy in modulus calculation is affected by the quality of the data. Ideally the data should be linear, free from excessive noise and contain sufficient data points in the elastic range for detailed analysis. The latter depends on the loading rate, sampling rate, and characteristics of the material behaviour; and may typically fall in the range of 20-200 datapoints [4]. Some of the data analysis procedures are carried out between discrete data points, others use some sort of fitting or interpolation between automatically or user selected limits, either as a simple straight line or by use of least squares regression.

- **Some knowledge of the suitability of the particular algorithm to the stress-strain behaviour is desirable.**

- **Ideally the software should analyse the data without operator intervention.**

Some algorithms only consider part of the curve, some split the curve into a fixed number of discrete regions (which may or may not overlap) and calculate values of the slopes according to particular criteria, such as the region with maximum slope, minimum deviation between the stress-strain curve and the particular fit. Others are designed to take account of anomalies at the start of the test such as non-linearity associated with bedding in, specimen straightening and initial slack in the load train etc.

Some require user defined input parameters; others are totally independent of user input. Best accuracy can be obtained if the analysis methods are focused on the linear portion of the curve below the elastic limit, but this might raise issues related to data sampling rates, if there are insufficient data points in the linear part of the curve for analysis.

A recent development is to look simultaneously at the tangent and secant moduli, calculated at each data point, to produce the best fit for modulus, and this has been adopted by a number of users and commercial software packages [15,16]. For a good fit to the linear part of the curve, the tangent and secant moduli should coincide. The analysis of the secant and tangent moduli data is a very sensitive method for checking whether the value chosen is a good fit to the stress-strain curve. The full procedure for this method is given in Ref. 15.

5 STATISTICAL ANALYSES

NPL has recently implemented a procedure that automatically selects the best fit to the modulus using a "bootstrapping" technique combined with the simultaneous examination of the tangent and secant moduli to calculate the uncertainty associated with the fit. Bootstrapping is a statistical technique that can be used to examine the variability of data and data fitting procedures without making any assumption about the shape of the error probability distribution [22,23]. It is ideally suited to computer-based analyses since it uses repeated calculations of parameters, rather than simple analytical solutions that are used in most other statistical calculations. Generally analytical solutions are calculated assuming a normal distribution of errors, but bootstrapping can deal with arbitrary distribution shapes. A further advantage is that is can be used for estimating error distributions of output results even from very complex algorithms.

Bootstrapping can be used in conjunction with a simple curve fitting algorithm such as a linear least squares fit to determine the likely distribution of errors of both intercept and gradient. This is achieved by generating information about the whole population of data from the sample of data. Since the sample of data contains all the information available about the population it is the best starting point for generating a much larger supply of data. The synthesised population data samples are then processed and the parameters from the curve fit algorithm stored. This process can be repeated many times and a distribution of the curve fit parameters can be constructed. The width of these distributions can be used as measures of the success of the curve fitting procedure and the quality of the data.

Bootstrapping has been implemented in the NPL analysis system that has been developed as part of the TENSTAND project to measure the robustness of fit using particular modulus algorithm fitting parameters. Various functions of tangent modulus, secant modulus, final modulus and a variance parameter in the form of a standard deviation value are used as minimisation variables. Software is currently available to access directly from the website address http://materials.npl.co.uk/modulus/

Table 3 shows data calculated from two representative ASCII datafiles that have been generated as part of the current project to evaluate the quality and accuracy of different computer software algorithms for measuring various parameters in the tensile test. Each file was analysed applying different algorithms and criteria for selecting the best modulus fit to the data. The optimised modulus value and associated uncertainty, calculated from 50 repeat applications of the bootstrapping routine, are presented for the Nimonic 75 reference material data file and a 316 Stainless steel.

Analysis Method	Modulus ± 2 Std.Dev (GPa)	
	Nimonic 75	316 Stainless
A	212.5 ± 6.4	190.6 ± 4.8
B	215.7 ± 8.1	190.8 ± 5.4
C	210.0 ± 7.0	192.8 ± 6.4
D	212.0 ± 3.6	192.8 ± 4.6

Table 3: Variation of Modulus values calculated using different algorithms

Method A minimises the error between the secant modulus and the calculated Young's modulus value; Method B minimises the difference between the tangent and secant moduli, Method C the error between the tangent modulus and the calculated Young's modulus and Method D uses analyses A,B and C but uses the criteria of minimising the standard deviation. At this stage there is some concern about the differences in values from the various algorithms, although all fit within the 2 standard distributions quoted with the results. Over 1200 tensile test data sets will be produced as part of the project and it is hoped to have more robust recommendations on the suitability of the different algorithms after detailed analysis of the results.

SUMMARY

A review of the current practice and modulus results from a series of exercises and the current project shows that there are still major issues associated with obtaining reliable modulus measurements from the tensile test. The two key recommendations for improving the accuracy of the data are:

- **To use averaging strain measurement methods**
- **Take careful consideration of the data analysis techniques used.**

Work is ongoing within the TENSTAND project focusing on more detailed analysis and testing of a large set of representative materials data but it is clear that there is scope to develop a dedicated procedure for modulus measurement, which could be progressed via a future revision of EN10002-1. Further details are available on the project website at .. *http://www.npl.co.uk/npl/cmmt/projects/tenstand*

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