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The cover design depicts the microwave planar near-field scanner being developed at NPL.

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Development of a cryogenic current comparator for the measurement of small currents

by

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Introduction

Recent exciting developments in the field of single electron tunnelling, in very small junctions of Al-AlO$_x$-Al at ultra-low temperatures (≈50 mK) [1], have shown that it is feasible to "clock" single electrons in synchronism with an applied radio-frequency (RF) voltage and that the resulting very small current I_Q, typically of the order 1 pA, is accurately determined in terms of the fundamental charge e and the frequency f_c of the RF source, by $I=ef_c$. In order to confirm this relationship it is necessary to measure I_Q with very high precision in terms of the existing electrical units of resistance and voltage which are universally represented by the von Klitzing constant R_K, and the Josephson constant K_J, respectively. However, for this measurement to have a significant impact either as a determination of R_K in terms of h/e^2, or as a quantum standard of current, it will be necessary to measure I with an uncertainty of about 0.1 aA, much smaller than the capabilities of any present measurement system.

An effective measurement of small currents in low impedance circuits has been achieved recently [2] at the NPL. In an experiment for the direct comparison of the quantised Hall resistances of gallium arsenide and silicon an equivalent one-standard-deviation (1σ) random uncertainty of 10^{-14} A for a measurement time of twenty minutes was obtained using a cryogenic current comparator (CCC) of 10,000 turns and an rf SQUID (Superconducting QUantum Interference Device) as a detector. The purpose of this development is to construct a CCC measurement system to improve this resolution by about three orders of magnitude enabling a current of 1 pA to be measured with a relative uncertainty of 1 part in 10^5. There are alternative methods for the measurement of small currents but these involve the use of high value resistors whose resistances have to be stable since their values are determined in a separate measurement. By using a cryogenic current comparator bridge the necessity for these additional measurements is effectively bypassed.

Design principles

A schematic of the layout of a CCC coupled to a SQUID is shown in figure 1. The windings of the CCC are surrounded by a superconducting shield constructed, usually, of overlapping thin lead sheets with a layer of insulation between the sheets. The self-inductance of the resulting torus is:

$$L_{shield} = \mu_0 a[\ln(8a/r)-2] \tag{1}$$

where $2a$ is the diameter and r is the radius of the cross-section. The torus is closely coupled to a pick-up coil with N_{pu} turns of superconducting wire, radius ρ, and self-inductance:

$$L_{pu} = \mu_0 N_{pu}^2 b[\ln(8b/\rho)-2] \tag{2}$$

where $2b$ is the diameter. The pick-up coil is connected to the input coil of the SQUID. For a current change ΔI in a CCC winding of N turns the resulting change in circulating current, $\Delta i_{cc}=N\mathrm{x}\Delta I$, in the superconducting shield gives rise to a change, $\Delta\phi_s$, in the magnetic flux sensed by the SQUID given by:

$$\frac{\Delta\phi_s}{\Delta i_{cc}} = \frac{N_{pu}L_{shield}}{L_{pu}+L_s}\cdot\frac{\Delta\phi_s}{\Delta i_s} \tag{3}$$

where $\Delta\phi_s/\Delta i_s$ is the magnetic flux sensitivity of the SQUID when a current Δi_s flows in its input coil which has self-inductance L_s.

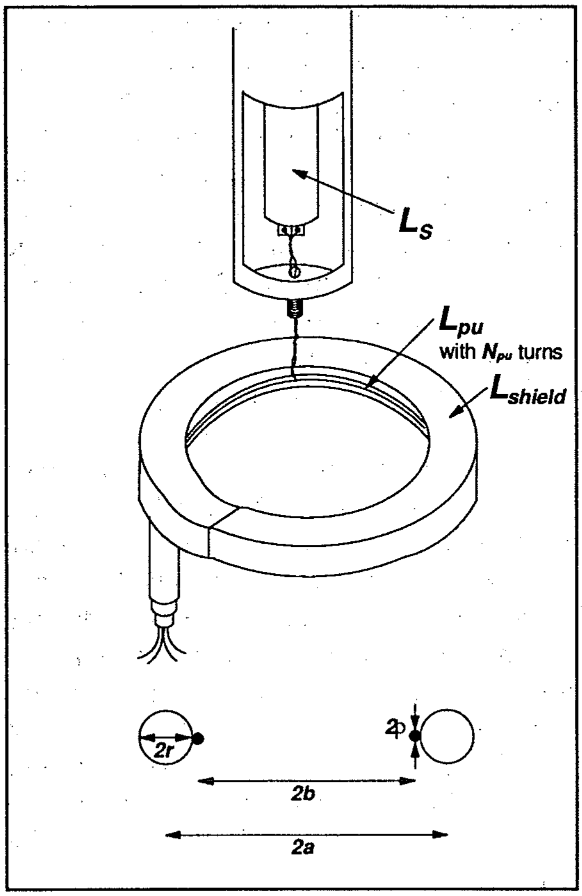

Figure 1. Schematic layout of CCC and SQUID

From equation (3) flux coupling to the SQUID is optimised when $L_{pu}=L_s$, and the flux-to-current transfer, $\Delta\phi_s/\Delta I$, is proportional to N, $2a$ and $\Delta\phi_s/\Delta i_s$. It has been shown [3] that the shape, usually rectangular, of the cross-section of the torus has an insignificant influence on $\Delta\phi_s/\Delta I$.

Construction

Since it is necessary to shield the pick-up coil from external magnetic fields the diameter, $2a$, of the comparator must be sufficiently less than that of the outer shield for the mutual-inductance between L_{shield} and L_{pu} not to be reduced. For this and other practical reasons the value chosen for $2a$ is 0.16 m. The comparator has 27 separate windings of Nb-Ti single core, copper clad and insulated superconducting wire with an overall diameter of 65 µm. The windings comprise 1,1,2,2,4,4, 10,20,.....4000,10x10000 turns for ease of self-checking. By joining all windings, except one of 1 turn, in series, a comparator ratio of 109,999:1 can be produced. The windings are shielded with 2½ overlapped turns of 0.13 mm thick lead sheet separated by 60 µm thick polyimide (Kapton) insulation resulting in a torus which has a rectangular cross-section, with an equivalent diameter estimated to be 30 mm. The 3 turn pick-up coil, wound with insulated, 125 µm diameter Nb-Ti wire and having a diameter of 0.135 m, is connected to the input coil, with self-inductance 2.2 µH, of a commercial dc SQUID (Quantum Design). The SQUID has "white" flux noise, $\langle\phi_n^2\rangle^{1/2}/(\Delta\nu)^{1/2} = 3.2\ \mu\phi_0/\sqrt{\mathrm{Hz}}$ and sensitivity, $1/(\Delta\phi_s/\Delta i_s) = 0.2\ \mu\mathrm{A}/\phi_0$, where ϕ_0 is the flux quantum. Substituting the quantities above into equations (1)-(3) gives a predicted sensitivity of 2.88 µA.turns/ϕ_0 and an equivalent current noise of $8.4\mathrm{x}10^{-17}$ A/$\sqrt{\mathrm{Hz}}$ for a 109,999 turn winding.

The CCC, screened by closed lead and semi-closed low-temperature µ-metal coaxial shields, is inserted into a liquid helium cryostat, which itself is surrounded by a semi-closed µ-metal cylinder.

Measurements

Preliminary measurements of the sensitivity and accuracy of the CCC have been made using a simple battery driven current source. The sensitivity, 2.33 μA.turns/ϕ_0 with N_{pu}=3, is higher than that predicted above indicating that L_{shield} is somewhat larger than the value calculated from equation (1). Simulated measurements of current resolution have been made by applying a square-wave signal directly to the SQUID with the CCC input floating. For a measurement time of about 50 minutes the equivalent input current in 109,999 turns has a 1σ standard error of the mean of $\pm 9.2 \times 10^{-17}$ A, which gives an uncertainty of 2.9 parts in 10^5 for a reversible current of 1.6 pA. This uncertainty is about a factor of five larger than that obtained with the pick-up coil disconnected from the SQUID. Consequently, it may be necessary to control the temperature of the helium bath more carefully, to prevent temperature fluctuations causing induced currents generated from trapped flux in the pick-up coil.

By joining various combinations of windings in series opposition it has been demonstrated that the error of the ratio 1:100,000, for example, does not exceed 1 part in 10^8, considerably smaller than the resolution measured above.

Future work

A circuit proposed for the measurement of the single electron current generated in "turnstile" or "pump" devices is shown in figure (2). The current, I_Q, from a device operated at a radio-

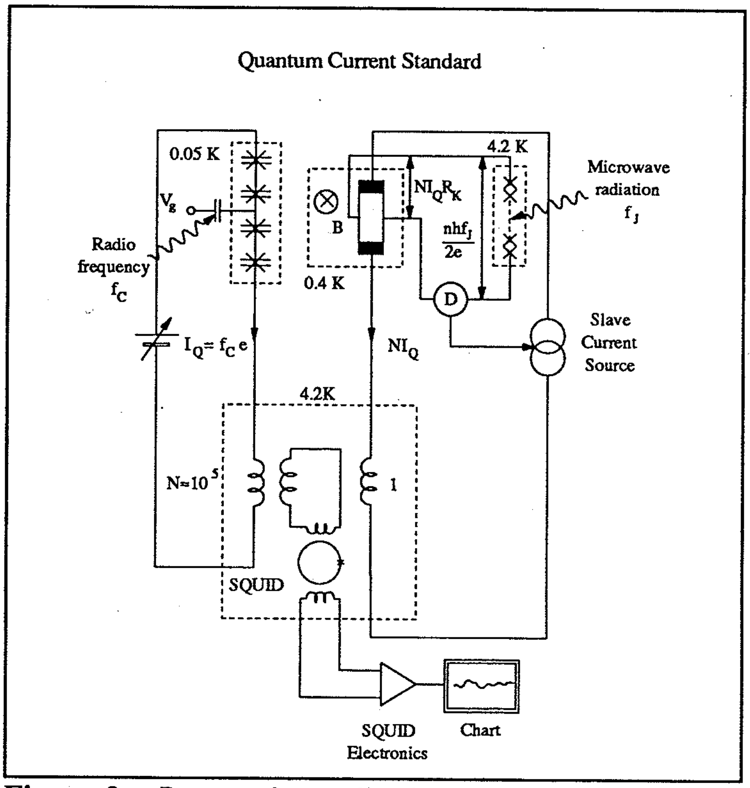

Figure 2. Proposed application of CCC for the measurement of single electron tunnelling.

frequency f_c and at a temperature of 50 mK, is compared in the CCC with the larger current

NI_Q generated from the combination of a voltage, $V_J = nf_J/K_J$, from a Josephson junction array irradiated at a microwave frequency f_J, applied to a quantised Hall resistance, R_K/i. This approach ensures that I_Q is measured in terms of the von Klitzing and Josephson constants, the known frequencies f_J and f_c, and integers N,n and i. It will also be possible, given sufficient accuracy in the measurements, to demonstrate the equality of R_K and h/e^2 from the relationship:

$$R_K = h/e^2.in/2Nf_J/f_c.$$

References

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