Conference Digest

Sixth British Electromagnetic Measurements Conference

2 - 4 November 1993

National Physical Laboratory
Teddington, Middlesex

The cover design depicts the microwave planar near-field scanner being developed at NPL.

National Physical Laboratory
Teddington, Middlesex, United Kingdom, TW11 0LW

ISBN 0 946754 15 2

Additional copies of this digest are available
from the above address, price £50.00.

Dielectric Measurements in open resonators: some new corrections.

A C Lynch and R N Clarke, Division of Electrical Science, National Physical Laboratory, Teddington, Middx., TW11 0LW

1. Introduction

The microwave open resonator is generally recognised as an excellent apparatus for the measurement of the permittivity of dielectrics having small dielectric loss (eg. < 1 milliradian) [1-3]. It also provides almost the only reliable way of measuring their losses, but the results reported by various workers sometimes disagree, even when specimens are exchanged between one laboratory and another. This paper sets out to examine experimentally the sources of uncertainty in the various observations and theories that are used to deduce a value of dielectric loss. The work has become possible only by the use of an elaborate detector system for the microwave signals, combined with a computer-controlled routine which eliminates some sources of error by making a number of observations in quick succession. These developments are of some importance for those who require to minimise the dielectric loss in materials. One area, for example, where reduction of loss uncertainties has made an important contribution in recent years is in the measurement of loss in materials used as microwave windows in nuclear research [4]. When loss-tangents are required to be lower than 40 microradians (μrad.), as they are in this application, small imperfections in open resonators can give rise to errors of over 100% in the loss tangent, which can make all the difference to the suitability of a given material. The work described below is therefore addressed towards identifying, quantifying and reducing errors in the measurement of loss at the level of tens of microradians.

The theories suggested here are believed to be of general application, but they are illustrated by their application to a plano-concave resonator of the type normally used at NPL between 36 and 144 GHz, which has remained substantially unchanged for 20 years. It uses interchangeable upper reflectors, each with two coupling holes to suit a particular range of frequencies; all these reflectors are of 150-mm radius of curvature, and of aperture in the range 120 - 150 mm. The resonator length can be varied between about 150 mm ('hemispherical' configuration [1, 2]) and 90 mm (almost semi-confocal), corresponding to the range 50 - 200 half-wavelengths, depending on the frequency. Dielectric specimens are plane laminae approximately one-half to a few wavelengths thick. One modification to the resonators has has been used: the lower reflector, instead of being plane, is made slightly concave (radius of curvature 800 mm), thus providing a beam-waist not at the reflector surface but at a calculable distance about 5 mm above it. Other types of resonator (such as symmetrical 'short' resonators) are subject in some way to the effects described below, although the arguments used would need modifications in detail.

The symbols are substantially those used by Cullen & Yu [3] and by Jones [2], except that d_o is used instead of D for the resonator length, and w instead of ω for beam-width. Instead of the conventional Q for the magnification of the resonance, D is used to mean $1/Q$ and it is expressed here in *millionths*. D has the advantage over Q that its components are additive.

2. The detector system

Q-factor measurements are performed automatically. The details of this system have been published previously [5, 6]. Signal sources in the millimetre-wave region of the spectrum (36, 72, 97 and 144 GHz) are phase-locked to a stable 9-GHz local oscillator. Heterodyne detection is used to produce an intermediate frequency (IF) of 60 MHz. This signal is fed into a vector network analyser which, under computer control, processes the amplitude and phase information carried by the IF signal to compute resonance Q-factors. They usually lie in the range 200000 down to 100000 for the empty resonator and 150000 down to 2000 when specimens are in the resonator. Practical Q-factor measurement commences with the operator manually setting the length of the resonator to the desired resonance. The system then automatically measures the transmission coefficient through the resonator as a function of frequency (or length) and computes the Q-factor.

A recent development to improve sensitivity for very low-loss is to collect eight sets of data under computer control, covering a range of frequencies around the resonance, in the order: frequency decreasing, increasing, increasing, decreasing. The data take about a minute to collect. The sequence is such that if the temperature is varying, and thus causing thermal expansion or contraction, all first-order effects are averaged out. The increased number of data has reduced the random variations to about half of what they were.

3. The empty resonator: sources of loss

When the resonator is empty, or contains a low-loss specimen, the major contribution to the total loss is usually that caused by eddy currents in the reflectors. Ideally the contribution from the two reflectors would be $D_r = 2\delta/d$ where δ is the skin depth. In practice it is greater, because δ is of the order of $0.3\,\mu$m and the eddy currents are therefore sensitive to minor imperfections in the contours of the surfaces and to slight corrosion affecting the resistivity of the metal. Unfortunately, no direct measurement of the effective δ is possible.

A second important contribution comes from the coupling holes. For the work described here, there are two coupling holes in the upper reflector, symmetrically placed about its centre, at a separation of about a wavelength. Again the loss is not directly calculable. For modes for which the beam-width at the upper reflector is large compared with λ, the fraction of the total energy which reaches the coupling holes will be proportional to $1/w_1^2$. (When w_1 is smaller, the coupling holes can no longer be regarded as close to the centre of the beam, and they do not sample the maximum field; the $1/w_1^2$ relation then fails.)

Another possible source of losses is that the beam may be too large for one or other of the reflectors - the upper one when the resonator is too long, the lower when it is too short. It is easy to avoid these conditions. They can be predicted by calculation of w_1 and w_o; then the reflectors must have an aperture of at least 4 times the respective beam radii to keep the contribution to D below 0.1 millionths (in practice, some margin above 4 is necessary because even a slight error in centering the reflectors on the axis of the resonator causes a large increase in D).

The remaining contribution to D has received little or no published attention. It is often claimed that the open resonator has a sparse set of resonances, and that the various modes are orthogonal. Neither statement is as true as one would wish. For every wanted mode (that is, modes with the radial and azimuthal mode numbers $p = l = 0$) there are about ten others that can affect the value of D if the resonant length is nearly the same. They are not devoid of coupling to it, either through scattering by the coupling holes or because of mode conversion at imperfect reflections. There are two main types of interfering resonances, according to whether the unwanted mode is of small or large loss. In the first of these conditions, there are two superposed responses of comparable amplitude, producing a distorted response from which a wrong value of D will be deduced. The wrong value can sometimes be less than the true one. In the second condition there is always a transfer of energy from the wanted mode to the unwanted one, in which it is dissipated by loss of some of the beam at the upper resonator. The observed value of D is then increased by a factor which may sometimes reach 5 or even 10. These major effects are well known; they are of course obvious and an experimenter will work with another mode instead. But it now appears that every wanted mode is affected in this way to some small extent, and that what has previously been assumed to be random error in the observations is in fact repeatable and in principle systematic, although it may not be practicable to characterise it exactly.

4. Separating the losses

The loss in the empty resonator has been measured for a wide range of modes, and for frequencies 36 and 72 GHz. It can be plotted as $q'D$ against q'; since the resonator length is almost proportional to q', the points would lie on a horizontal line if the resistance of the reflectors were the main source of loss, and this line would lie at a calculable height if the surfaces of the reflectors were perfect. But the points do not lie on a straight line; their departures from it are roughly repeatable, and a line fitted to the points is not horizontal and is far above the calculated value. Similar behaviour is observed when the resonator contains a specimen, as shown in Figure 1.

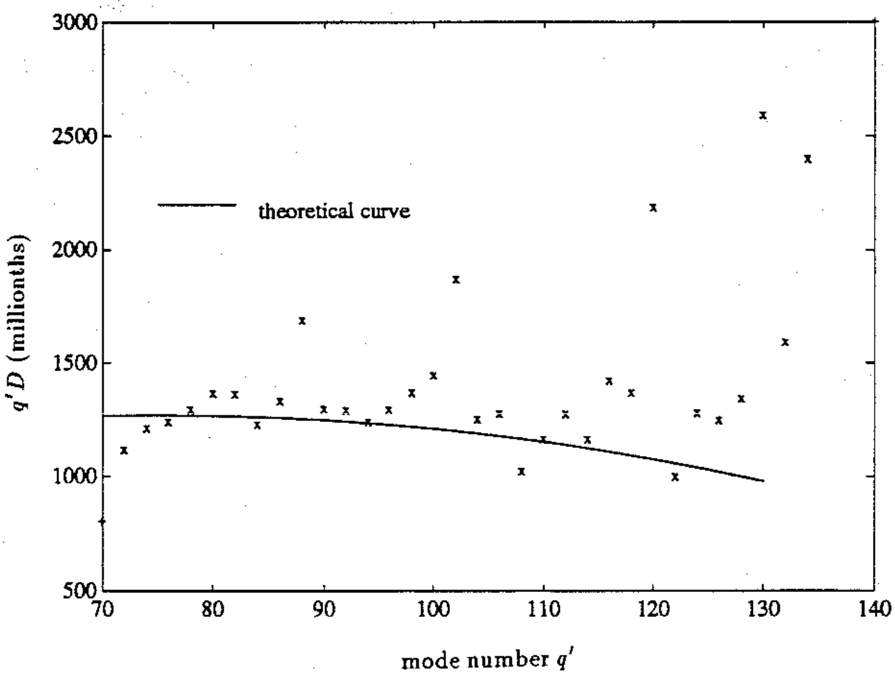

Figure 1 Measurements of $q'D$ at 72 GHz upon a quartz crystal specimen (ordinary ray: $\epsilon' = 4.43$, $\tan\delta \approx 45 \times 10^{-6}$, specimen thickness 7.99 mm). The trend in the curve marked 'theoretical' allows for coupling hole effects but ignores the effects of unwanted modes.

These departures from a smooth curve are thought to be caused by higher-order modes. The major departures are caused by easily identifiable resonances for which the calculated lengths are within $\pm 10\,\mu$m of that of the wanted mode. Most of the smaller departures are upward, although a few downward ones have been seen ($q' = 108$ in Figure 1). It is difficult to identify the best line through the points; it is certainly not the average. Without identifying it, we cannot make a precise separation of the two main sources of loss. Apossible calculated fit is shown on the graph. It appears that the reflector loss is large at 72 GHz; this is not surprising in view of the known effects on the attenuation of waveguides as the frequency is increased.

An important conclusion is that the coupling holes are contributing a loss of the order of one-third of the total. It can also be concluded that, while the major errors caused by higher-order modes can by experience be avoided, it is impossible to avoid the minor errors which are almost always present; the uncertainty they cause is of the order of 0.2 millionths in D.

Tentatively, we may also conclude that less trouble is caused by higher-order modes at 36 GHz than at 72 GHz, and that the reason is that the unwanted modes are further apart. If this is right, a resonator for 72 GHz probably ought to be a scaled-down version of that for 36 GHz, of half the length and with a more sharply-curved upper reflector. The loss D would be nearly doubled but might be more certainly measured.

5. The equivalent empty resonator

In measuring the dielectric loss of a specimen, we compare the resonator losses for specimen-in and specimen-out. The difficulty is that the resonators for the two conditions need to have equal coupling-hole losses and equal reflector losses, and these conditions are incompatible. The two should also have equal losses caused by higher-order modes, but this condition seems to be unattainable; all we can ask is that these losses should be small for both specimen-in and specimen-out.

For specimen-in, the metal loss depends upon the position of the specimen. If its upper and lower surfaces are not in fields of equal amplitude, the conditions are complicated but calculable (see below). The equal-field condition can be realised either (1) by using a specimen of thickness $\lambda'/2$, or some multiple of it, resting on a plane reflector (where λ' is the wavelength within the specimen), or (2) by supporting a specimen, of any thickness, clear of the lower reflector and at a position such that its central plane is at either a node or an antinode. The correct separation is easily identifiable; it is such that the length of the resonator for resonance is independent of small changes in the position of the specimen. If this equal-field condition is achieved, by either means, the reflector loss will be the same for specimen-in and specimen-out, when the mode-number q is the same. The resonator is shorter, however, for specimen-in.

For the condition just described, the coupling-hole loss, which depends on w_1, is greater for specimen-in. Three different formulae have been proposed for calculating it. For each of them we have: (refer to the quoted references for details)

$$w_1^2 = (2R_0/k)\sqrt{d'/(R_0 - d')} \qquad (1)$$

but d' takes three different values:

(1) $d' = d + (d + \delta t)/n^2$ where $\delta t = w_t^2/(4t(1 + (nkw_o^2/2t)^2))$ [7]

(2) $d' = d + t/n$ and a correction is made to the value of p', the displacement of the mirror on inserting the specimen [8].

(3) $d' = d + mt/n^2$ where $m = 1$ for node-at-surface, $(2n - 1)$ for antinode-at-surface [9].

For specimen-out d' becomes d_o. Fortunately the three formulae lead to closely similar values for w_1, and all are very close to $(d_o - p)$; that is, w_1 is nearly the same for specimen-in as for an empty resonator of the same overall length. Thus the 'equivalent' empty resonator is not physically realisable but the loss in it can be calculated by interpolating on a curve of D_r against d_o to find the value of D_r for the length of the resonator with specimen. If the uncertainties in the curve are thought small enough to justify it, a more accurate calculation can be made by using one or other of the three formulae for w_1.

Although this paper is not primarily concerned with the measurement of permittivity, it may be worth mentioning that the use of a resonator of the same overall length for specimen-in and specimen out also leads to slightly more consistent values of n and ϵ_r. The difference from values obtained by using the same mode-number q is however very small, within the limits of uncertainties from other causes.

6. Measurement of loss tangent of a specimen

There are well-known methods of evaluating $\tan\delta$ for the specimen from the data obtained when the specimen is either resting on a plane reflector or in a position such that its central plane is in either a node or an anti-node. We have previously published the outline of another method [6] which makes use of data obtained with the specimen at various spacing from the lower reflector. Ideally these data can be manipulated to lead to a straight-line graph, which provides a clear indication of whether some of the data are not to be relied on (as when they are affected by an unwanted mode of resonance, of the kind referred to in the last paragraph of Section 3, above). Experience of using this method shows that such effects occur often. By using several of the wanted modes, however, it is usually possible to find at least one graph for which the gradient of the line is well-defined. Figure 2 shows a fully satisfactory example.

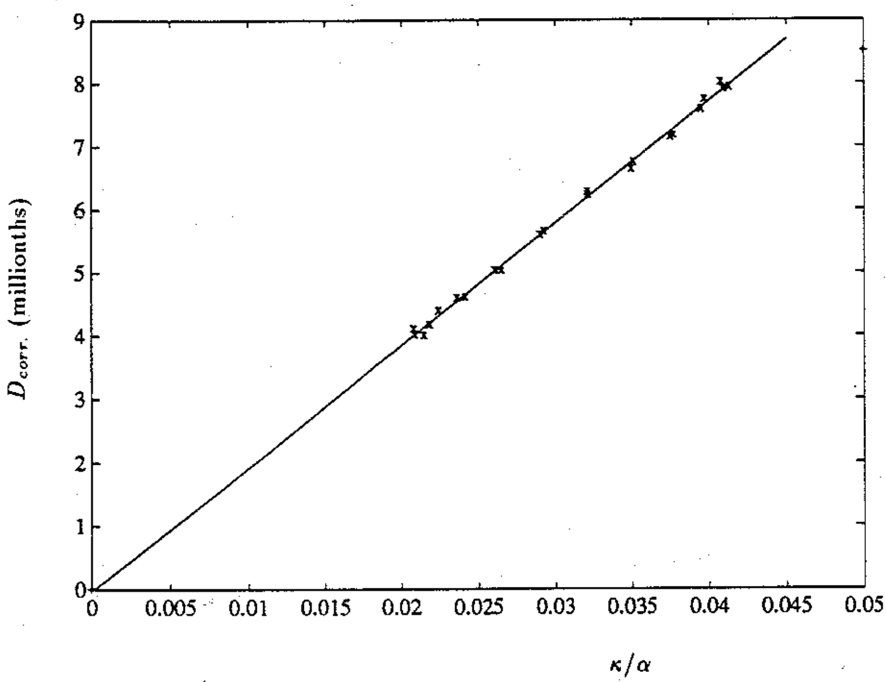

Figure 2 Corrected gradient plot for the loss tangent of a specimen of PTFE (specimen thickness 2.91 mm, $\epsilon' = 2.05$). The abscissa (κ/α) varies cyclically with the separation (s) of the specimen from the plane reflector (see [6] for details). The gradient gives $\tan\delta = 193 \times 10^{-6}$. The value of D has been corrected for the effects of coupling hole and reflector loss.

If each of the data is modified by subtracting not only the estimated reflector loss but also the estimated coupling-hole loss, the graph should pass through the origin, as shown in Figure 2. This procedure avoids any need for extrapolation, and can be used to show which points are unsatisfactory without the need to draw the graph at all.

7. Conclusions

This paper accounts for most of the unexpected effects which lead to errors with open resonator measurements if the operator does not recognise that something is wrong. It does not predict when these errors are to be expected, but it offers a procedure which will usually identify any such errors unmistakably.

When these errors can be avoided, as they often can, especially with a specimen of thickness equal to an integral multiple of $\lambda/2$, the loss tangent can be measured with an uncertainty of the order of 5% of itself $\pm 3\mu$radians.

8. Acknowledgements

The authors wish to thank The National Measurement System Policy Unit (NMSPU) of the Department of Trade and Industry for funding this and related work on dielectrics. We also wish to thank Prof. A L Cullen of University College, London and Dr R Heidinger of the Kernforschungszentrum Karlsruhe, Germany for valued theoretical discussions on open resonators.

References

1. Clarke, R.N. and Rosenberg, C. B., 1982, 'Fabry-Perot and open resonators at microwave and millimetre wave frequencies, 2-300 GHz', *J. Phys. E: Sci. Instrum.*, **15**, pp. 9-24.

2. Jones, R.G., 1976, 'Precise dielectric measurements at 35 GHz using an open microwave resonator', *Proc. IEE*, **123**, pp. 285-290.

3. Cullen, A.L. and Yu, P.K., 1971, 'The accurate measurement of permittivity by means of an open resonator', *Proc. Roy. Soc. Lond*, **A325**, pp. 493-509.

4. Heidinger, R., 1991, 'Design parameters of ceramic insulator materials for fusion reactors', *J. Nuclear Materials*, pp. 64-69.

5. Clarke, R.N. and Lynch, A.C., 1989, 'Developments in the measurement of dielectric and magnetic materials at millimetre wavelengths using open resonators', Proceedings of the BEMC 1989, paper no. 7.

6. Lynch, A.C. and Clarke, R.N., 1992, 'Open resonators: improvement of confidence in measurement of loss', *IEE Proc.-A*, **139**, pp. 221-5.

7. Cook, R.J. and Jones, R.G., 1976, 'Correction to open-resonator permittivity or loss measurements', *Electron. Lett.*, **12**, pp. 1-2.

8. Yu, P.K. and Cullen, A.L., 1982, 'Measurement of permittivity by means of an open resonator', *Proc. Roy. Soc. Lond.*, **A380**, pp. 49-71.

9. Lynch, A.C., 1983, 'Measurement of permittivity using an open resonator', *Proc. IEE, Part A*, **130**, pp. 365-368.