SIMULATION OF PROGRESSIVE FIBRE FAILURE DURING THE TENSILE LOADING OF UNIDIRECTIONAL COMPOSITES

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ABSTRACT

A Monte Carlo model of a unidirectional composite subject to uniaxial tensile loading has been developed enabling the simulation progressive fibre fracture and ultimate failure. The model takes account of complex load sharing rules that must be operating when a fibre/matrix cell fails in the composite. The composite is represented as layers of hexagonal arrays of fibre/matrix cells which are stacked vertically. By making use of periodic boundary conditions in three orthogonal directions the simulation can in principle represent large samples of composite, and avoid the necessity for taking account of the effects of the sample edges during the simulation. The matrix failure strain is assumed to exceed that of the fibres.

Fibre failure is represented using weakest link statistics and the Weibull distribution. Experimental fibre strength data for a range of fibre lengths can be well represented by the two parameter Weibull distribution function. The effect of the matrix is accounted for through the use of fibre/matrix cells whose behaviour is governed by a micro-mechanical model that models stress transfer in the cell between fibre and matrix arising from both fibre fracture and fibre/matrix debonding. The micro-mechanical model is used to generate local stress-strain behaviour in the form of a look-up table that is accessed by the simulation procedure.

The effects of varying model parameters have been investigated. It has been found that in order to predict reliable mean failure strains in a time that is tolerable using a PC, each simulation needs to be repeated at least 10 times and the average taken, increasing significantly the amount of computing that is necessary. The mean failure strains predicted have shown significant dependence on the number of fibres used in the simulation. Preliminary results suggest that reasonable lower bound results can be obtained, in tolerable times using a PC, when the array size at least 10 x 10 (corresponding to 200 fibres) but larger sized model are preferable. The mean failure strains predicted are dependent upon the length of the sample; the failure strain decreasing as the sample length increases. The number of layers h of fibre elements used when assessing composite failure does not appear to have any significant influence on the mean failure strain.

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CONTENTS

LIST OF SYMBOLS

M, N	parameters defining array of fibres
L	number of layers of fibre/matrix cells used in the simulation
δ	length of fibre/matrix elements
h	number of layers of fibre/matrix cells considered when assessing failure $(2h + 1 < L)$
$F(\sigma)$	cumulative probability of failure of unit length of fibre
$P_F(\sigma)$	cumulative probability of failure of any length of fibre
λ	length of fibre
σ_0	Weibull scaling parameter
m	Weibull exponent
d	size of fibre/matrix cell such that d^2 is its cross-sectional area
σ	axial applied stress
ε	effective axial applied strain for damaged composite
$\tilde{\varepsilon}$	effective axial applied strain for undamaged composite
σ_{ij}^{r}	axial stress in fibre (i, j) at element r along its length
ε_{ij}^{r}	axial stress in fibre (i, j) at element r along its length
ΔT	temperature difference between simulation temperature and the stress-free temperature
E_A	axial Young's modulus of the composite when undamaged
α_A	axial thermal expansion coefficient of the composite when undamaged
E(I,J,r)	effective axial modulus of fibre/matrix cell at (I,J,r)
τ	critical interfacial shear stress for debonded region of fibre/matrix interface

1. Introduction

The principal objective of this report is to develop a methodology for predicting the tensile failure of a unidirectional composite taking account of the effects of fibre/matrix interface behaviour. The micro-mechanisms of failure that occur in such composites are typically fibre failure, matrix cracking, and fibre/matrix debonding leading to the development of longitudinal splits. The principal difficulty encountered is that tensile failure is statistical in nature, and a proper statistical treatment is possible only if the stress transfer modelling associated with fibre failures is greatly simplified. For example, when attempting to develop a complete statistical theory where the probability of failure of a composite is calculated using analytical formulae that depend upon the statistical properties of the strength of the fibres in the composite, the effect of the matrix has often been ignored completely (i.e. the composite has been represented by a dry bundle of fibres). For this case statistical theories are available [1, 2, 3]. In order to take some account of the role of the matrix in promoting load sharing between fibres, the composite has been represented by a chain of bundles of fibres where the length of each bundle is chosen to be representative of the stress transfer distance associated with a fibre break in a composite [4, 5]. These bundle techniques, which are based on equal load sharing rules where the load carried by a breaking fibre in a bundle is shared equally between the surviving fibres of the same bundle, are amenable to complete statistical treatments. Recognising that the matrix has a role in localising the load sharing that occurs when a fibre breaks, statistical models have been developed based on the use of chains of bundles and on local load sharing rules [6, 7]. The local load sharing models are extremely complex and the prospects are bleak for extending them so that more realistic failure sequences can be handled, involving fibre/matrix interface behaviour. In order to overcome such limitations a Monte Carlo approach is sometimes employed.

The Monte Carlo approach does not attempt to formulate and solve the problem of failure prediction using probability distribution functions and analytical techniques. The approach is to set up a simulation of the failure of a unidirectional composite where the composite is regarded as an assembly of thin layers of equal thickness δ. Each layer is then regarded as an array of representative volume elements of the composite, each element comprising just one fibre element together with the matrix with which the fibre element is associated. Throughout this report the representative volume element will be called a fibre/matrix cell. The failure strain of any fibre element in a cell is randomly generated from some statistical distribution function that characterises their strength. Having attributed a failure strain to each fibre element, the model of the composite can be used to determine the failure strain of the composite by identifying the locations of fibre/matrix cells in the system where fibres fail, and redistributing the load carried by a failing cell to surrounding fibre/matrix cells. For a given set of failure strains of the fibre elements in the system, the model will predict a single failure strain for the composite. In order to assess the statistical variation of the composite failure strain it is necessary to allocate another random set of failure strains to each fibre element and repeat the loading procedure to the point of failure. By repeating this procedure many times statistical data can be generated that characterises the failure strain of the composite. Clearly the Monte Carlo technique is generating statistical information for the failure strain of the composite by repeated numerical simulation. Monte Carlo simulation models have been considered in the literature, see for example [8, 9].

Manders et al [8] used a model comprising a chain of 100 bundles where each bundle contains a 10 x 10 array of fibre elements on a square lattice. Both equal (i.e. universal, where all fibres in the composite share the load as would be the case for a bundle of loose fibres) and local load sharing rules were considered where the latter shared the load on a failing fibre amongst its four nearest neighbours. The strengths of the fibre elements in each bundle were assumed to be specified by weakest link statistics and the Weibull two parameter distribution function. Curtis [9] developed a model that was designed to simulate the behaviour of the composite using the Monte Carlo technique. The Curtis model represents the composite by a set of stacked layers of rectangular arrays of fibres. For the Curtis model, failure is assumed to occur when all of the fibre elements have failed in just one of the layers. A consequence of this assumption is that the fracture surface will be very flat and normal to the direction of the applied load. This would be appropriate only for composites where the interface strength is high. Curtis assumes that the failure strains of fibre elements can be modelled by the normal distribution. Lienkamp and Schwartz [10] have carried out a Monte Carlo simulation of the failure of a seven fibre microcomposite using Weibull statistics to generate the strengths of fibre elements in each bundle of the chain, and using local load sharing rules. The approach to be used here will not attempt a proper statistical treatment, but will instead develop a methodology based on simulation techniques, i.e. a Monte Carlo approach is to be made.

Curtin and Takeda [11] have developed a Monte Carlo model for the prediction of the tensile strength of unidirectional fibre reinforced composites where stress transfer associated with fibre breaks is accounted for by a shear-lag model that imposes a uniform shear stress in the debonded region. Curtin and Takeda [12] have used the model to investigate the model by considering its application to polymer composites. Curtin et al [13] have further developed the model so that the effects of matrix cracking in brittle matrix composites can be taken into account.

Earlier work by the author (unpublished) specified the details of a model that was used to predict the progressive fibre failure that occurs when a unidirectional composite is loaded in tension to the point of failure. This model extended the ideas of Curtis [9] introducing cyclic edge conditions, and allowed for some interaction between layers when redistributing load, and when applying a failure criterion. As a consequence, fibre pull-out could be predicted. Thus the modified model has the potential to simulate realistically the effects of fibre/matrix interface strength. The earlier modelling did not account satisfactorily for matrix behaviour, as matrix effects were included only through the load sharing between neighbouring fibres in the event of a fibre failure. The objective of this report is to extend the earlier model so that the effects of the matrix and fibre/matrix debonding can be more realistically taken into account. The model will be formulated on the assumption that the matrix failure strain exceeds that of the fibres; a situation that is valid for polymer based composites. The approach to be taken regards the unidirectional composite as being a three dimensional array of fibre/matrix cells, whereas in the original model only the fibres in each cell were considered. In the event of a fibre failing in a particular cell, the load carried by the cell has to be redistributed to neighbouring cells according to a prescribed load-sharing rule involving cells in the same plane as the failed fibre element, together with cells in neighbouring planes above and below that containing the failed fibre. In this way the load transfer effect of the matrix can be modelled. Such load transfer effects are at the cell level and arise because the effective stiffness of any failed cell is changed when its fibre fails. However, when applying the micromechanical model to take account of the effect of the matrix on stress transfer, the debond length (and hence the stress-transfer distance) can vary significantly. This would result in having to vary the number of fibre layers involved with stress transfer for a single fibre break if the length δ for the layers is selected to be small, i.e. having the order of the fibre diameter. To avoid this difficulty, which would result in more complex software and a slowing down of the simulation, it was decided to choose δ to be large enough for the debond length to be contained within a single layer for all stages of loading. This strategy has been adopted in this report.

The effective stress-strain behaviour of the failed cell is an important input to the simulation technique. This is modelled through the use of a micro-mechanical model [14] that accounts for localised load transfer between fibre and matrix associated with a fibre failure. The model also takes account of the effects of debonding at the fibre matrix interface. Such debonding will have a significant effect on the local stress transfer between fibre and matrix leading to a significant effect on the effective axial stiffness of the fibre/matrix cell. The change of cell stiffness arising from a fibre fracture and associated fibre/matrix debonding leads then to stress transfer between neighbouring cells where the stress carried by the failed cell is reduced, and that carried by the neighbouring cells is increased, according to a specified load-sharing rule.

Design engineers utilise the strength of a composite as though it is a single valued function, although testing of composites reveals that strength is subject to statistical variability that can arise from the test method and from the material itself. The simulation technique to be described here is modelling the variability of the strength and strain to failure of a composite arising from attributing to the fibre element in each fibre/matrix cell a random value of fibre failure strain. If repeated simulations are performed using the same input values then the predicted strengths and failure strains of the unidirectional composite will clearly be variable. Thus the results of simulations, predicting parameters that describe composite failure, must be presented in terms of a statistical average and variance for these values rather than single point values.

2. Representation of the composite

Fig.1 shows a schematic diagram of the unidirectional composite where L rectangular layers of fibre/matrix cells having thickness δ are stacked vertically. The thickness of each layer is to be small enough for there to be just one fibre fracture in each element, and large enough for the stress transfer length to be included entirely within the layer for all states of loading.

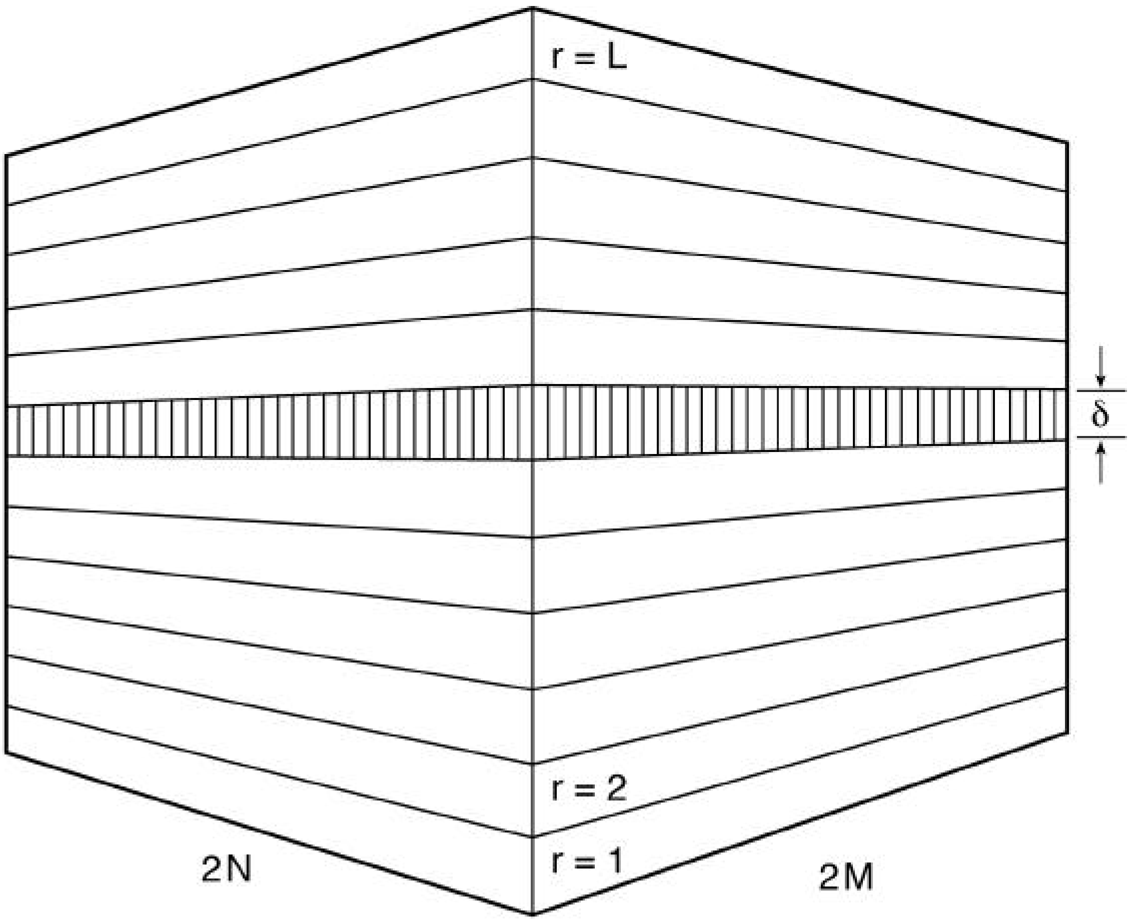

Fig.1 : Schematic diagram of a unidirectional composite that is divided into layers of equal length d

Each layer is divided into an array of individual fibre/matrix cells, each containing one fibre element of the composite together with the neighbouring associated matrix. The fibres are assumed to be hexagonally packed. The geometry of the distribution of fibre/matrix cells in layer r is shown in Fig.2. A rectangular lattice of squares is considered such that a single layer is represented by a 2M x 2N array of lattice points. Each lattice point in the composite is described by the integer values of the parameters (i, j, r). The fibre/matrix cells are located at the lattice points specified by :

if j is odd, i = 2, 4, 6, ... 2M
if j is even, i = 1, 3, 5, ... 2M-1.

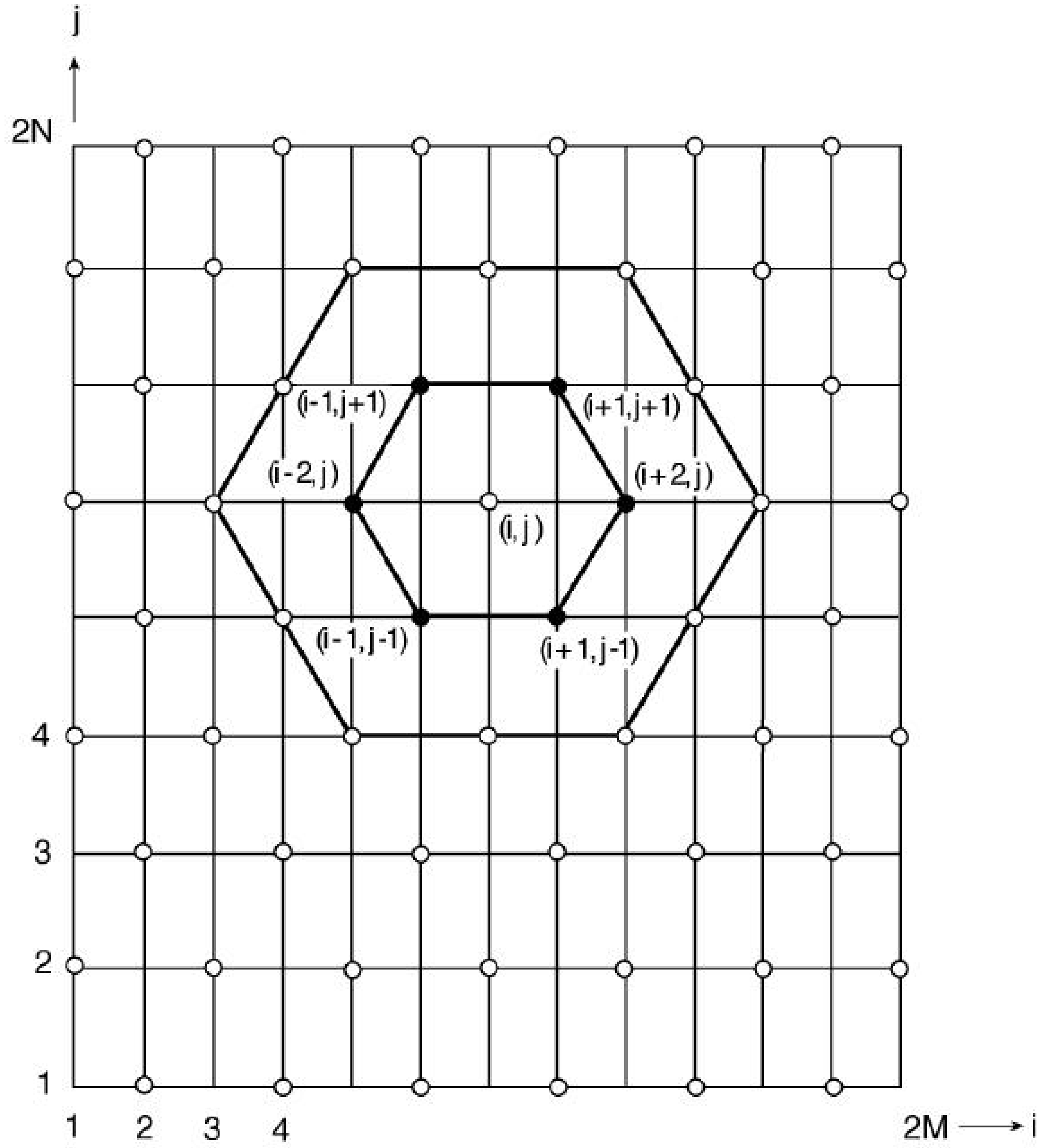

Fig.2 : Rectangular grid on which fibres are located having a hexagonal distribution

The total number of fibre/matrix cells present is clearly 2MN although there are 4MN lattice points in total. The hexagonal structure of fibre/matrix cell packing is also evident. It should be noted that for any fibre/matrix cell, there are six nearest neighbour cells, as seen in Fig. 2 from the fibre/matrix cell at (i, j) and its labelled neighbours. The fibre/matrix cells located at any array point (i, j) associated with a single fibre of the composite is considered as an assembly of L fibre/matrix cells each of length δ, each associated with a different layer in the composite. Thus a particular fibre/matrix cell can be identified uniquely by the numbers (i, j, r).

An important feature of the lattice structure for the fibre/matrix cell arrangement is that cyclic edge conditions can easily be implemented. This is an essential requirement when simulating composite behaviour because load redistribution must be considered whenever a fibre fails. By using cyclic edge conditions there is no need to investigate whether a fibre/matrix cell is at or near a free surface or corner where modified load sharing rules would otherwise have to be applied. The cyclic edge conditions are applied in the three orthogonal directions i, j and r where r = 1...L denotes the layer number. Cyclic boundary conditions are applied by allowing the parameters i, j, and r to have values outside the following ranges which relate only to the region of the composite being considered:

$$1 \le i \le 2M, \qquad 1 \le j \le 2N, \qquad 1 \le r \le L.$$

The values of i, j and r which are outside these ranges are replaced using the procedure specified by :

$$i \to (i + 2M - 1) \bmod (2M) + 1 ,$$

$$j \to (j + 2N - 1) \bmod (2N) + 1 ,$$

$$r \to (r + L - 1) \bmod (L) + 1 .$$

This procedure maps the effects of events at locations lying beyond the region of simulation into the region that is under consideration.

3. Allocation of fibre strains

3.1 Weakest link fibre failure statistics

The statistical failure of fibres in the composite will be modelled using weakest link methods [2]. It is well known that the strength of a fibre depends upon the fibre length. For the composite failure simulation under discussion this presents significant problems as the length of fibre δ involved in each fibre element of the simulation may be small. It is clearly not possible to carry out reliable experiments on fibres of small lengths so that some type of extrapolative approach is needed. In the simulations to be reported here δ can be as small as the order of 0.1 mm.

The basis of the weakest link technique is to assume that the strength of fibres is determined by a function $F(\sigma)$ such that the cumulative probability of failure of fibres of unit length during the stress increase $0 \to \sigma$ has the value $F(\sigma)$. The weakest link approach then implies that the probability of failure of a fibre element of length δ during the stress increase $0 \to \sigma$ is given by

$$P_F(\sigma) = 1 - [1 - F(\sigma)]^{\delta} . \qquad (1)$$

It is important to be able to relate the function $F(\sigma)$ to statistical data obtained from experiments on single fibres. For experiments carried out on fibres of length λ

$$P_F(\sigma) \equiv P_o(\sigma) = 1 - [1 - F(\sigma)]^{\lambda} . \qquad (2)$$

where $P_o(\sigma)$ is measured and represents the probability of failure of a fibre of length λ during the stress increase $0 \to \sigma$. The probability of failure of a fibre of length δ can then be expressed in terms of the measured function $P_o(\sigma)$ as follows

$$P_F(\sigma) \equiv 1 - [1 - P_o(\sigma)]^{\delta/\lambda} . \qquad (3)$$

When a set of fibre strength tests is carried out on fibres of length λ a distribution of fibre strengths is obtained. In a set of p tests on fibres of length λ let j denote the number of the test and let s_j denote the strength measured in the j^{th} test. The cumulative probability of failure is often calculated using the formula [15]

$$P_o(s_j) = \frac{j - 0.5}{n} , \quad j = 1 \ldots p . \qquad (4)$$

Langlois [16] has considered the validity of this approach and compared its performance relative to alternative formulae. The cumulative probability curve is obtained by plotting $P_o(s_j)$ as a function of s_j , j = 1...p . By using such data to represent the statistical variability of the strengths of fibres having length λ the failure probabilities of fibres of any other length can be estimated using (3).

In principle it is possible to use the results of experiments to define the $F(\sigma)$ through the use of (2) and (4). Such an approach would require the use of interpolation techniques, and perhaps extrapolation techniques which are far less reliable. The alternative, which is usually used in practice, is to assume a particular form for the function $F(\sigma)$ which is then fitted to the experimental data.

3.2 Use of the Weibull distribution function

The Weibull distribution [17] function is frequently used to model fibre strength data. The Weibull distribution has the important property, not often realised, that it is one of three limiting distributions predicted by extreme value statistics [18]. In fact it is the only limiting distribution that is defined only for positive values of the variable being considered. As strength is always positive, it is very appropriate to make use of the Weibull distribution to model the statistical variability of fibre strength. Its use leads to a reliable method of extrapolation of experimental data. The other two limiting distributions predicted by extreme value theory involve both positive and negative values of the variable which would not really be appropriate for modelling fibre strength measurements.

The cumulative form of the Weibull distribution [17] applied to the strength of fibres having unit length is given by

$$F(\sigma) = 1 - \exp[- (\sigma / \sigma_o)^m] \qquad (5)$$

where m and σ_o are parameters characterising the distribution. The mean value and variance of the distribution are given by

$$\text{mean} \;=\; \sigma_o \;\Gamma\left(1 \;+\; \frac{1}{m}\right) , \tag{6}$$

$$\text{variance} \;=\; \sigma_o^2 \left[\Gamma\left(1 \;+\; \frac{2}{m}\right) - \;\Gamma^2\left(1 \;+\; \frac{1}{m}\right)\right] , \tag{7}$$

where $\Gamma(x)$ is the Gamma function. On substituting (5) in (1) it follows that the cumulative Weibull strength distribution for fibres of length δ may be written

$$P_F(\sigma) \;=\; 1 - \exp[\, -\; \delta(\sigma / \sigma_o)^m] , \tag{8}$$

where δ is the length of fibre element. Strictly speaking δ is dimensionless, being the ratio of the fibre element length to the `unit' length (here selected to be 1 mm) that is the basis of the representation (5). Corresponding to (6) and (7) the mean and variance of the distribution (8) are given by

$$\text{mean} \;=\; \sigma_o \;\delta^{-1/m} \;\Gamma\left(1 \;+\; \frac{1}{m}\right) , \tag{9}$$

$$\text{variance} \;=\; \sigma_o^2 \;\delta^{-2/m} \left[\Gamma\left(1 \;+\; \frac{2}{m}\right) - \;\Gamma^2\left(1 \;+\; \frac{1}{m}\right)\right] . \tag{10}$$

The next stage is to construct a sequence of fibre strength levels that belong to the selected Weibull distribution. The method used is to generate a sequence of random numbers x_k , k = 1....n lying in the range $0 \leq x < 1$ such that n = 2MNL, the total number of fibre elements in the simulation. Values generated by this procedure are allocated in sequence to each fibre element of the system. One problem that arises from this procedure is that very large or rather small fibre strengths can be generated which are not really encountered in practice. To avoid the occurrence of this, if a fibre strength generated by the procedure exceeds its mean value by twice the standard deviation, then the value is rejected and a new value is calculated. In addition, if the fibre strength is smaller than the lowest value for fibre stress arising in the look-up table of the micromechanical model, then the fibre strength is again rejected and a new value is calculated.

On regarding the sequence x_k as representing values of the cumulative failure probability $P_F(\sigma)$, it is easily shown from (8) that the required random strength levels are given by

$$s_k \;=\; \sigma_o \left[\frac{1}{\delta} \ln \frac{1}{1 \;-\; x_k}\right]^{1/m} , \quad k \;=\; 1...n \; . \tag{11}$$

For carbon fibres, experimental fibre strength data [19] can be represented by the Weibull distribution if the parameter m = 7.4 and if σ_o = 5.928 GPa. Table 1 shows examples of the means and variances of samples of 10000 values generated by this procedure for the case when m = 7.4, σ_o = 5.928 GPa and δ = 1 mm, and provides values of the mean and variance given by the relations (9) and (10). It is clearly seen that the generating procedure is producing sequences of strength values that may be represented very well by the Weibull distribution. The sequences of values of strength values for fibre elements are converted into corresponding failure strains by dividing fibre strengths by the modulus E_f of the fibres

(208 GPa for the case of the carbon fibres being considered here).

MEAN (GPa)	VARIANCE $(GPa)^2$
5.5565	0.80075
5.5699	0.79354
5.5500	0.79988
5.5601	0.79434
5.5466	0.82941
5.5591	0.79102
5.5544	0.77250

TABLE 1 : Examples of the means and variances of random sequences of 10,000 fibre element strength values generated from a Weibull distribution such that m = 7.4, σ_o = 5.928 GPa and δ = 1 mm. The values of the mean and variance predicted by (9) and (10) are 5.56111 GPa and 0.78738 GPa respectively.

Table 2 shows values of the means and variances for the fibre strength distribution generated by (11) from a sequence of n = 10000 values, assuming that m =7.4 and σ_o = 5.928 GPa, for various values of the fibre length. The largest four values of δ correspond to fibre lengths used in fibre strength tests. Also given in Table 2 are the corresponding means and variances calculated using (9) and (10). It is seen that the random fibre strength generation procedure using (11) is producing sequences of values whose means and variances correspond to those of a Weibull distribution as required.

	n = 10000	n = 10000	$n \to \infty$	$n \to \infty$
δ (mm)	MEAN (GPa)	VARIANCE $(GPa)^2$	MEAN (GPa)	VARIANCE $(GPa)^2$
0.01	10.3751	2.7548	10.3617	2.7335
0.1	7.5891	1.4502	7.5910	1.4671
1	5.5666	0.7824	5.5611	0.7874
5	4.4608	0.5052	4.4741	0.5097
12	3.9750	0.4073	3.9749	0.4023
62	3.1832	0.2527	3.1838	0.2581
75	3.1007	0.2466	3.1029	0.2451

TABLE 2 : Values of the means and variances of the Weibull fibre strength distribution for various values of the fibre length δ when m = 7.4 and σ_o = 5.928 GPa. The columns two and three are calculated from random sequences of n = 10000 values generated using the procedure (11), while the columns four and five are calculated using (9) and (10) which are valid when $N \to \infty$.

	n = 30	n = 30	EXPERIMENTAL
δ (mm)	MEAN (GPa)	VARIANCE $(GPa)^2$	MEAN (GPa)
0.01	10.3003	2.7487	-
0.1	7.2852	1.5616	-
1	5.5990	0.9658	-
5	4.6287	0.5385	4.40
12	3.9990	0.4647	3.92
62	3.2941	0.1423	3.12
75	3.1753	0.2487	3.05

TABLE 3 : Values of the means and variances of the Weibull fibre strength distribution for various values of the fibre length d when m = 7.4 and s_o = 5.928 GPa. The columns two and three are calculated from random sequences of n = 30 values generated using the procedure (11), while column four shows experimental values [19].

Table 3 shows values of the means and variances for the fibre strength distribution generated by (11) from a sequence of n = 30 values, assuming that m = 7.4 and σ_o = 5.928 GPa, for various values of the fibre length. This is attempting to simulate the experimental procedure where normally approximately 30 strength tests are carried out at each fibre length. Also given in Table 3 are the experimental mean values which are seen to agree particularly well with the values given in Table 2. The predicted values in Table 3 agree reasonably well with those generated by (9) and (10) that are given in Table 2.

It is concluded that the Weibull distribution may be used to represent experimental data for measurements of the statistical variability of fibre strengths, and that the number of experimental measurements made at each fibre length is adequate for a good estimate to be made of the Weibull parameters. Predictions of strengths, at lengths that are smaller than those used for measurements, are easily predicted using the Weibull distribution.

4. Monitoring the state of fibre/matrix cells

When starting the simulation it is assumed that all the fibre/matrix cells in the system are intact, i.e. their fibre elements have not yet failed. The states of the fibre/matrix elements are stored in a logical three dimensional matrix $\hat{f}$ 1 such that the state 'true' denotes that the fibre/matrix cell is intact while the state 'false' indicates that the cell has failed. The convention will be adopted that parameters which have a logical value are denoted by symbols having a 'hat'. The use of a logical matrix has substantial benefits when considering a failure criterion that allows interaction between the various layers in the model (see section 8).

5. Mechanical equilibrium and stress-strain response

5.1 Effective stresses and strains

Let σ denote the effective stress that is applied to the composite which is defined as the total load F applied to the composite divided by the total cross-sectional area. Let σ_{ij}^{r} denote the effective stress in the fibre/matrix cell (i, j, r), whether its fibre element has failed or not, which is defined as the total load applied to the fibre and matrix in the cell divided by the area of cross section d^2 of an individual cell. For the situation where no fibre has failed the value of σ_{ij}^{r} will be the same for each fibre/matrix cell and will have the value $F/(2MNd^2)$. Whenever a fibre fails a local redistribution of load will occur that is governed by a specified load-sharing rule (to be discussed in section 7). Since each layer of the composite must support the applied load it follows that, for any state of fibre damage distribution within the composite before catastrophic failure has occurred,

$$2MN\sigma \;=\; \sum_{i,j}\sigma_{ij}^{r} \;, \quad r = 1...L \;. \tag{12}$$

The relation (12) thus ensures that mechanical equilibrium is preserved between the various layers of the composite. This property of the local effective axial stresses for each cell is exploited when prescribing load-sharing procedures following the failure of the fibre element in a fibre/matrix cell. The corresponding local effective axial strains for each cell will not necessarily satisfy such a relation as they will depend upon the distribution of failed fibre elements within the composite.

The corresponding effective strain in the fibre/matrix cell (i, j, r) is denoted by ε_{ij}^{r}. For a fibre/matrix cell where the fibre element has failed, the value of ε_{ij}^{r} will be determined for a given value of σ_{ij}^{r} by making use of a micro-mechanical model of stress transfer that will be described below (see section 5.2). For the case when the fibre element has not failed, the stress-strain relation for the cell is that of a perfect sample of unidirectional composite subject to uniaxial loading which is given by

$$\sigma_{ij}^{r} \;=\; E_A(\varepsilon_{ij}^{r} - \alpha_A \Delta T) \;, \tag{13}$$

where E_A and α_A denote the axial modulus and axial thermal expansion coefficient of the unidirectional composite, and where ΔT denotes the temperature difference between the current temperature and the stress-free temperature for the composite. The use of the relation (13) is an approximation that ignores any additional transverse stresses that exist in the composite arising from the presence of composite material around each cell where a fibre element has failed. Clearly when the value of σ_{ij}^{r} is specified, the corresponding value of ε_{ij}^{r} is given by

$$\varepsilon_{ij}^{r} \;=\; \frac{\sigma_{ij}^{r}}{E_A} + \alpha_A \Delta T \;. \tag{14}$$

Let ε denote the effective axial strain in the composite defined, for any state of fibre damage, by the average value

$$\varepsilon \;=\; \frac{1}{2MNL}\sum_{r=1}^{L}\sum_{i,j}\varepsilon_{ij}^{r} \;. \tag{15}$$

When the composite is undamaged the value of the effective axial strain is denoted by $\tilde{\varepsilon}$. It then follows from (12), (13) and (15) that the effective stress-strain relation for an undamaged composite as a whole is given by

$$\sigma = E_A(\tilde{\varepsilon} - \alpha_A \Delta T), \qquad (16)$$

which is valid only for composites subject to uniaxial loading in the axial direction.

5.2 Accounting for fibre failure

Assuming that the effective stress applied to a fibre/matrix cell having a broken fibre element is known, it is now required to show how the effective strain of the cell is estimated. It is necessary to make use of a micro-mechanical model of stress transfer between fibre and matrix, taking account of the nature of the interface following fibre fracture. A concentric two cylinder model is used [14] which was developed in an earlier task of Project CPD3. The model and associated software may be used to generate the effective axial stress-strain behaviour of the fibre/matrix cell, and other relevant quantities such as the debond length, the axial stress in the fibre remote from the fibre fracture, and similarly for the axial matrix stress. These values are tabulated in parametric form where the varying parameter is $\tilde{\varepsilon}$, the axial strain that would arise in an undamaged fibre/matrix cell subject to the same axial load and temperature. The approach to be taken is to generate look-up tables of stress-strain values for the following types of interface behaviour :

- perfect interface bonding,
- debonding having a uniform interfacial shear stress,
- debonding having an interfacial shear stress controlled by Coulomb friction.

The feasibility of the modelling approach will be demonstrated here only for the case of debonding having a uniform interfacial shear stress, a model which is also relevant when considering shear yielding of the matrix in the region of the interface near a fibre fracture. If the approach is successful it will be extended to the other types of interface behaviour.

When applying the concentric two cylinder micro-mechanical stress-transfer model [14] for fibre failure in a unidirectional composite, the effective axial applied stress corresponds to the value σ_{ij}^r found at the point (i, j, r) in the composite. The boundary condition that should be applied in the radial direction on the external surface of the cylinder representing the matrix is such that the radial displacement is identical to that found in an undamaged system subject to the same axial applied effective stress and temperature difference ΔT. This approach is a good method of taking account of the fact that the cell being represented by the micromechanical model is embedded in the interior of the composite and will be influenced by the surrounding composite material which is not part of the cell (i, j, r). This is an approximate approach as the application of such a boundary condition will lead to a residual radial stress arising from fibre fracture. Its magnitude will however be relatively small, particularly when it is realised that predictions of failure strain will have variability that swamps the effect of this small approximation. The difficulty of the approach is that the required applied radial displacement depends on the axial load and temperature difference ΔT which would have to determined for a very large number of situations that will arise during the simulation of progressive composite failure. An alternative simpler approach to be adopted here is to assume that the external surface of the cylinder representing the matrix is stress-free. This approach neglects the effect that surrounding composite material will have on the stress and deformation distributions within the cell (i, j, r). For this case the radial displacement on the external surface on the matrix cylinder will be non-uniform leading to the prospect of holes or overlaps occurring at the boundary between the external surface of the cell and the surrounding composite material. The errors arising from the simplified approach are expected to be small when compared to the variability of predicted failure strains. Software, related to the micromechanical model, has been developed that allows both types of boundary condition to be investigated, but the incorporation into the simulation of a displacement based external radial boundary condition has not yet been attempted.

The application of the micromechanical model to the cell (i, j, r) is achieved by regarding the loading as being characterised by a strain value $\tilde{\varepsilon}_{ij}^{r}$ which is the axial strain that would arise in an undamaged cell subject to the same axial loading. The effective stress and strain for a damaged cell are determined from the micromechanical model and written in the parametric form

$$\varepsilon_{ij}^{r} = \Lambda(\tilde{\varepsilon}_{ij}^{r}, \Delta T) \ , \qquad \sigma_{ij}^{r} = \Omega(\tilde{\varepsilon}_{ij}^{r}, \Delta T) \ , \tag{17}$$

where the functions Λ and Ω will be known only at a discrete set of points for a given value of ΔT. Such values can be tabulated as look-up tables for a range of values of the parameter $\tilde{\varepsilon}_{ij}^{r}$. The simulation technique will require values of ε_{ij}^{r} and σ_{ij}^{r} at other values which are not known, and which are estimated using a linear interpolation technique.

6. Load increases

During the simulation it is necessary to increase the applied load in order to promote the successive failure of fibre elements leading to catastrophic failure. The load is increased by applying to each fibre/matrix cell (i, j, r) an increased value of the parameter $\tilde{\varepsilon}_{ij}^{r}$ which is denoted by $\delta\tilde{\varepsilon}_{ij}^{r}$. If the fibre/matrix cell is undamaged then

$$\delta\varepsilon_{ij}^{r} = \delta\tilde{\varepsilon}_{ij}^{r} = \frac{\delta\sigma_{ij}^{r}}{E_A} \ , \tag{18}$$

where $\delta\sigma_{ij}^{r}$ is the corresponding increase in the axial stress applied to the fibre/matrix element. If the fibre/matrix cell is already damaged, i.e. the fibre element has fractured, then the stress increase $\delta\sigma_{ij}^{r}$ and strain increase $\delta\varepsilon_{ij}^{r}$ are obtained from the relations

$$\delta\sigma_{ij}^{r} = \Omega(\tilde{\varepsilon}_{ij}^{r} + \delta\tilde{\varepsilon}_{ij}^{r}, \Delta T) - \Omega(\tilde{\varepsilon}_{ij}^{r}, \Delta T) \ , \tag{19}$$

$$\delta\varepsilon_{ij}^{r} = \Lambda(\tilde{\varepsilon}_{ij}^{r} + \delta\tilde{\varepsilon}_{ij}^{r}, \Delta T) - \Lambda(\tilde{\varepsilon}_{ij}^{r}, \Delta T) \ . \tag{20}$$

From the nature of the micromechanical model it is expected that

$$\delta\sigma_{ij}^{r} = E_A \delta\tilde{\varepsilon}_{ij}^{r} \ , \tag{21}$$

which from (18) is a relation that may be applied to both damaged and undamaged fibre/matrix cells. The relation (19) may be used to check consistency, or check the magnitude of errors arising from interpolation. Strictly speaking, when assessing the effects of increased loading, the micromechanical model is needed only to estimate the increase in effective strain of a damaged fibre/matrix cell.

7. Load shedding

The next stage is to discuss how the load carried by failing fibre/matrix cells is transferred to neighbouring fibre/matrix cells in the composite. It is assumed that the load shed by a failing fibre/matrix cell is shared (in a manner to be determined below) between the failing fibre/matrix cell and its surviving nearest neighbours in the same layer as the fibre/matrix cell that has failed. This procedure specifies that the load-sharing region is a small part of the layer containing the failed fibre. The six cells shown in Fig.2, where the fibres are denoted by a solid rather than an open symbol, identify the nearest neighbour cells that are relevant for one hexagonal array of fibre/matrix cells surrounding a fibre that is about to fail. If the fibre elements in all these nearest neighbour cells have failed then load is assumed to be transferred equally between surviving fibre/matrix cells of the next ring of nearest neighbours, and so on. If just one fibre/matrix cell in the composite remains intact, albeit in an unstable situation, then its nearest surviving neighbour is its image lying beyond the representative volume element used in the simulation. At least one of these fibre/matrix cells will be detected by the use of cyclic boundary conditions and the complete failure of a layer will thus be possible. It should be noted that load sharing confined to a single layer is reasonable only if the thickness δ of each layer is greater than or equal to the maximum expected stress transfer length (often called ineffective length) associated with a single fibre break.

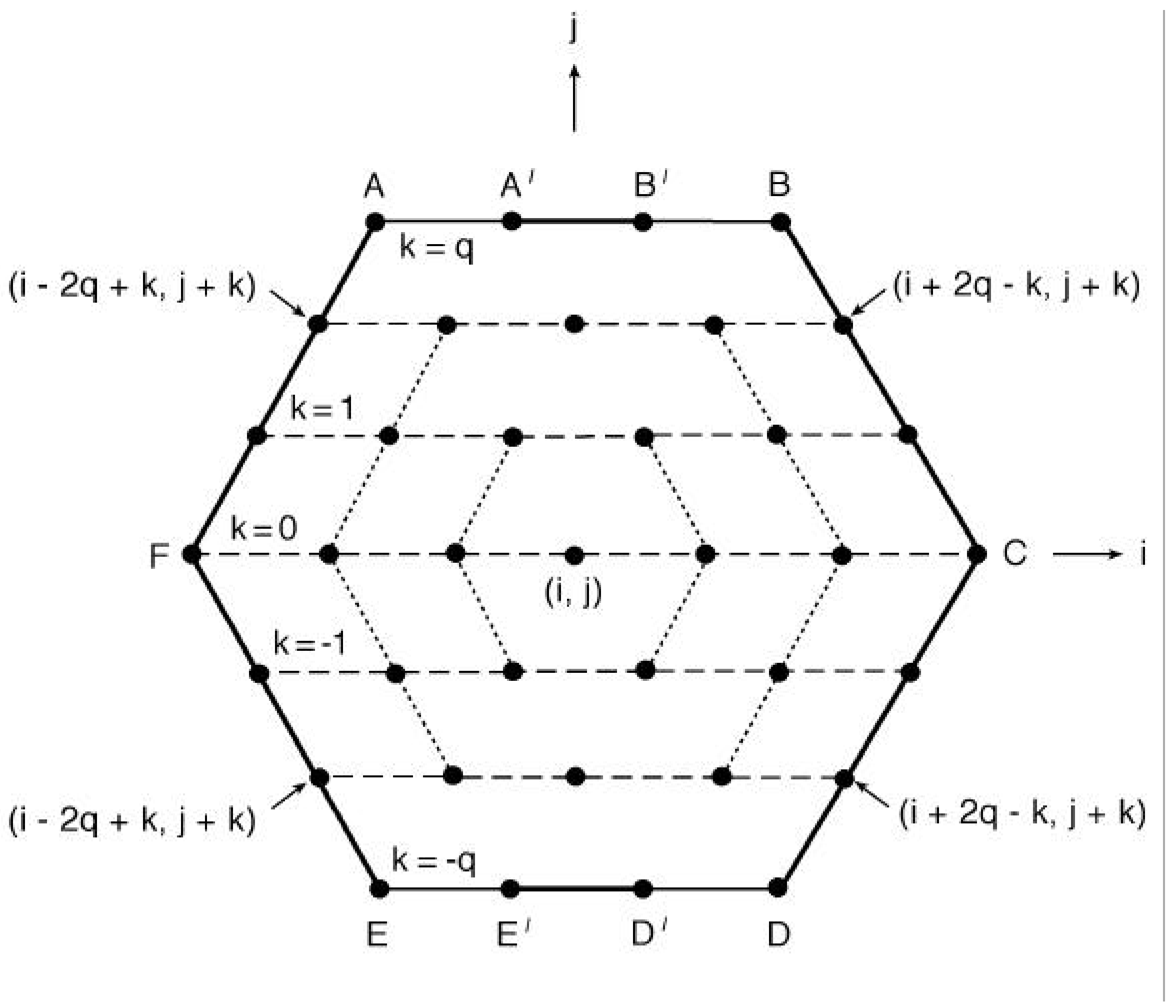

Fig.3 : Diagram defining the hexagonal rings around a fibre element located at (i, j)

When carrying out the procedure of load shedding the number of surviving fibre/matrix cells in a specified hexagonal ring around the broken fibre at location (i, j, r) needs to be calculated. If the q^{th} hexagonal ring in the r^{th} layer is being considered, as shown in Fig.3, then the number of surviving fibres N_q^r is calculated using the following method

$$N_q^r = \sum_{k=-q}^{q}\Big[f\big(i-2q+|k|,\ j+k,\ r\big) + f\big(i+2q-|k|,\ j+k,\ r\big)\Big]$$
$$+ \sum_{s=1}^{q-1}\Big[f\big(i-q+2s,\ j+q,\ r\big) + f\big(i-q+2s,\ j-q,\ r\big)\Big] \ , \qquad (22)$$

where the three dimensional matrix f(i, j, r) has the value one if there is a surviving fibre element at the point (i, j, r) and zero if the fibre element has failed.

The value of the function f at any location is easily derived from the corresponding logical matrix $\hat{f}$ 2 defining the state of all fibre elements in the system. The second sum in (22) is zero if q = 1. The total number of fibre elements that have failed in the load-sharing region is also required in order to operate the load sharing procedure, and to assess whether the composite has failed. This is easily calculated by subtracting the total number of surviving fibres N_q^r from the total number of fibre elements in the load-sharing region which, for q hexagonal rings surrounding a fibre, is given by $N_q = 1 + 3q(q + 1)$, including the central fibre which has just broken.

The next stage is to determine how much load is transferred to the nearest neighbour fibre/matrix cells when a fibre fails. It is assumed that within the failing cell the applied strain is held fixed while fibre failure occurs. Just before fibre failure the stress $\sigma_{ij,0}^r$ applied to the cell is known, as is the corresponding axial strain ε_{ij}^r which is to be fixed during fibre failure. The micromechanical model can be used to determine the effective stress σ_{ij}^r applied to the failed cell which is given by (17). As ε_{ij}^r is known, the value of $\tilde{\varepsilon}_{ij}^r$ is first calculated using the first of the relations (17). This value is then inserted into the second relation in order to calculate the corresponding effective applied stress σ_{ij}^r for the cell where the fibre has failed. The quantity $(\sigma_{ij,0}^r - \sigma_{ij}^r)d^2$ is the load that must be shared between the nearest neighbour fibre/matrix cells and the failed fibre/matrix cell itself.

When redistributing this load within the load-sharing region it is assumed that for each fibre/matrix cell, whether damaged or not, the increase in the value of the axial strain ε has the same value $\delta\varepsilon$, and the failing cell is included in this load sharing procedure. Most load will, however, be taken by undamaged fibre/matrix cells as their effective stiffness is significantly greater than that of the cells that have failed. The increase in the load applied to the N_q^r undamaged cells in the load-sharing region is simply $N_q^r\, d^2 E_A \delta\varepsilon$. The increase in the load applied to the $N_q - N_q^r$ failed cells of the load-sharing region is given by

$$d^2\,\delta\varepsilon \sum_{I,J}^{\{q\}} \{1 - f(I,J,r)\}\, E(I,J,r) \ ,$$

where E(I, J, r) denotes the effective axial modulus of the fibre/matrix cell (I, J, r) just before the fibre element fails and where the summation is over the I and J values lying within the load transfer zone which involves q hexagonal rings of fibre/matrix cells around the failed cell. As before, f(i, j, r) is zero if the fibre element at location (i, j, r) has failed, and is unity if it has survived. The value of the strain increment is determined from the following relationship

$$\left[N_q^r E_A + \sum_{I,J}^{\{q\}} \left\{ 1 - f(I,J,r) \right\} E(I,J,r) \right] \delta\varepsilon = \sigma_{ij,0}^r - \sigma_{ij}^r , \qquad (23)$$

which is obtained by equating the load shed when the fibre element fails to the increase in load carried by all the fibre/matrix cells in the load-sharing zone after the fibre has failed. The relation (23) should ensure that the relation (12) is satisfied in the r^{th} layer after the load redistribution has taken place. The strain increment $\delta\varepsilon$ is then added to each fibre/matrix cell in the load-sharing region associated with the failing fibre and the corresponding new effective applied stresses for each cell are calculated. For damaged fibre/matrix cells the addition of the strain increment means that the effective modulus of the damaged cell must be updated using the micromechanical model. The use of interpolation methods means that the load carried by the layer in which load-sharing is occurring may not match the other layers of the composite. To correct for this, the effective stress applied to the cell where the fibre has just broken is adjusted so that the correct load is applied to the load-sharing layer, and the corresponding strain for this failed cell is calculated. Throughout the simulation, a check is made, whenever load is added to the composite, to ensure that the load carried by each layer is very nearly identical; a warning being issued by the software if the discrepancy exceeds 0.1% of the effective stress applied to the composite.

The relation (23) is an approximation as the axial modulus E(I, J, r) for a damaged cell is that before the strain increment is added. Strictly speaking the moduli used for damaged cells in (23) should be those after the strain increment has been applied. To achieve this a numerical zero-finding routine would have to be included in the simulation procedure. The computing time penalties that result from the inclusion of such a routine would render the simulation technique impractical. Such a development will have to await future generations of PCs.

8. Failure criteria

There are two failure criteria that can easily be imposed. The first is that used by Curtis [9] where the composite is regarded as failing when just one layer has failed completely. This failure criterion results in fracture surfaces that would be flat and normal to the fibre direction, although some pull-out could be achieved by using large enough values of δ. This type of behaviour would be expected only in those composites for which there was strong fibre/matrix bonding. If the bonding is weaker then fibre element failures in more than one layer need to be considered, introducing some form of layer interaction into the model.

The second failure criterion is now described. When considering whether layer r has failed, the fibre/matrix cell at location (i, j) is regarded as having failed if a fibre element failure can be detected at location (i, j) in layers r - h , r - h + 1, ... r , ... r + h - 1, r + h , where $h \geq 0$ is a parameter that is selected and held fixed throughout the simulation. Assessing whether layer r has failed according to this failure criterion is particularly simple as the state of each fibre element has been stored in a three dimensional logical array where failed fibre elements are denoted by `false'. When allowing failures in the 2h neighbouring layers to contribute to the failure event, use is made of the following operation for the logical matrix $\hat{F}_r$ 3

$$\hat{F}_r(i,\ j,\ r+t) \ \rightarrow\ \hat{F}_r(i,\ j,\ r+t)\ \text{AND}\ \hat{f}(i,\ j,\ r) \qquad -h \le t \le h\,,\quad r = 1...L\,, \qquad (24)$$

where $\hat{f}(i,\ j,\ r)$ 4 is the logical array which is set to `false' whenever a fibre element has failed. Parameters $n_f^r$ , $r = 1...L$ , are calculated by counting the number of `false' entries in the logical matrices $\hat{F}_r$, $r = 1...L$ 5. The failure criterion is clearly

$$n_f^r \ = \ 2MN\ . \qquad (25)$$

The matrix $\hat{F}_r$ 6 can be regarded as the projection of the failures in the 2h + 1 logical matrices $\hat{f}$ 7 onto the r^{th} plane.

9. Simulation procedure

The simulation procedure is described by the following distinct calculation stages :

1. Set the failure flags (i.e. the elements of the logical matrices $\hat{f}$ 8 and $\hat{F}$ 9) to ‘true’, signifying that all fibre elements in the fibre/matrix cells are intact.

2. Allocate the failure strengths to each fibre element using Weibull statistics described in section 3. Calculate the corresponding failure strains of the fibre elements in each cell.

3. Scan all fibre/matrix cells in the system to determine the fibre element in the system that has the lowest failure strain.

4. Apply loading until the fibre element with least strength is just about to fail.

5. Allow the fibre element to fail and set the flag of the failed fibre element to ‘false’.

6. Assess whether composite has failed using criterion described in section 8.

7. Redistribute the load carried by the fibre/matrix cell containing the fibre element that has just failed using prescribed load-sharing methodology described in section 7. Update the strain values for all fibre/matrix cells, and the effective modulus of the damaged cells.

8. Scan all fibre matrix cells to determine the fibre element (if any) that has exceeded its failure strain by the greatest amount. If such a fibre element is found then go to stage 5 and proceed as before. If no such fibre elements are found then the state of the system, following a fibre element failure and resultant stress redistribution, has led to a stable configuration where mechanical equilibrium can be maintained.

9. Scan all fibre/matrix cells to determine the fibre element that is closest to its failure strain and then increase the applied load so that this fibre element is just about to fail, checking that each layer is supporting the same load.

10. Calculate the mean strain and the corresponding effective axial modulus for the composite. Go to stage 5.

The numerical simulations that perform the above procedure have been programmed using FORTRAN 77 for execution on a standard PC.

10. Parameters used for the simulations

It is useful to list the parameters that need to be specified before carrying out simulations, namely

M, N	parameters defining number of fibres in composite sample,
L	number of layers of fibre elements stacked vertically,
h	parameter characterising the number of layers considered when assessing failure (total number is $2h + 1$).

Various values will be investigated.

10.1 Selection of length of fibre elements

The length δ of the fibre elements in the system is determined by the nature of stress transfer between fibre and matrix in the neighbourhood of an isolated fibre break. The value of δ must be large enough for it to include all fibre/matrix debonding that occurs during loading following the failure of a fibre. Two values will be investigated; $\delta = 0.5$ mm and $\delta = 2$ mm.

10.2 Parameters for the Weibull distribution

The remaining parameters that need to be specified relate to the allocation of fibre strains using the Weibull distribution. These parameters are:

m	the dimensionless shape parameter for the distribution,
σ_o	a normalising parameter having the dimensions of stress (GPa),

In all simulations the following values for carbon fibres [19] relating to the Weibull distribution have been assumed :

$$m = 7.4 , \qquad \sigma_o = 5.928 \text{ GPa} .$$

10.3 Parameters for the micro-mechanical model

In order to test the methodology and the associated computer software, it is necessary to select values for fibre, matrix and interface properties that are required by the micromechanical model of stress transfer [14]. The material to be used for the micromechanical model is a carbon fibre reinforced epoxy composite where the radius of the fibres is 3.5 microns, the volume fraction of fibres is taken as 0.5, and the stress-free temperature is such that $\Delta T = -90^oC$. The debonding is modelled by the use of the constant interfacial shear stress model where the critical interfacial shear stress τ characterises interface debonding, or shear yielding. The values selected for the investigation of the feasibility of the simulation technique are : $\tau = 50$ MPa and $\tau = 200$ MPa. The properties for the carbon fibre and the epoxy matrix are :

	Fibre	*Matrix*
E_A (GPa)	208.0	3.89
E_T (GPa)	16.7	3.89
μ_A (GPa)	18.0	1.41971
ν_A	0.25	0.37
ν_T	0.35	0.37
α_A (/^{o}C x 10^6)	-1.1	55.0
α_T (/^{o}C x 10^6)	22.1	55.0

11. Performance of software

In order to test the performance of the software preliminary calculations were performed for the special cases where M = N = 5, 10, 15, 20, 25, 30, 35, 40 with L = 1 and h = 0 for the Weibull distribution of fibre strengths such that m = 7.4 and σ_0 = 5.928 GPa. Fig.4 shows the dependence of the computation times for a standard laptop PC as a function of the number of fibres used in the simulation. Significant computation times (> 6 min) result when the number of fibres exceeds 2500.

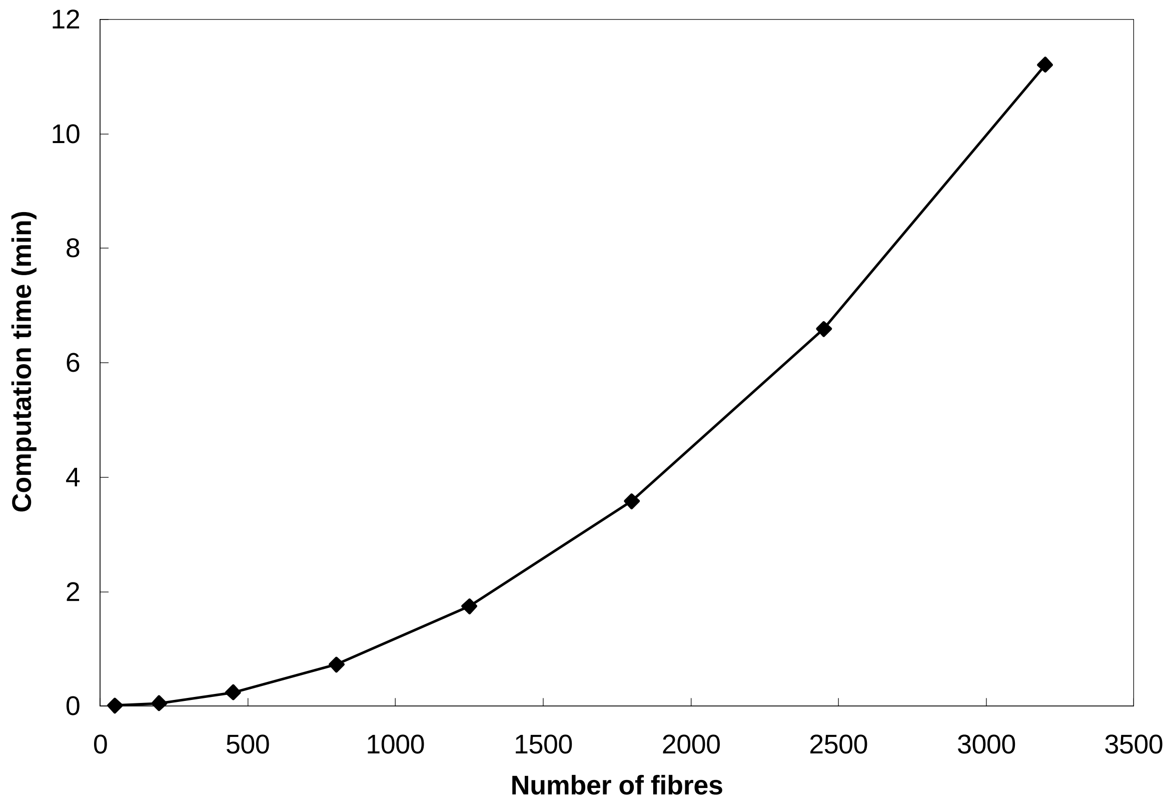

Fig.4 : Dependence of computation times as a function of the number of fibres in the simulation.

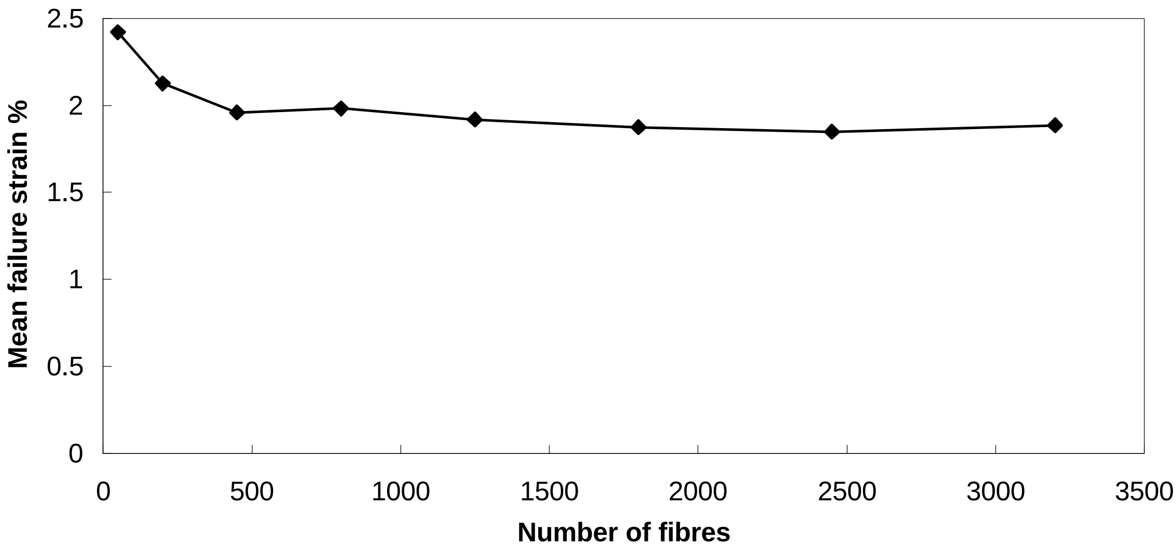

Fig.5 : Dependence of mean failure strain as a function of the number of fibres in the simulation.

Fig.5 shows the corresponding predicted mean failure strains (%) indicating that reasonable failure strains are obtained when the number of fibres in the simulation is greater than 500.

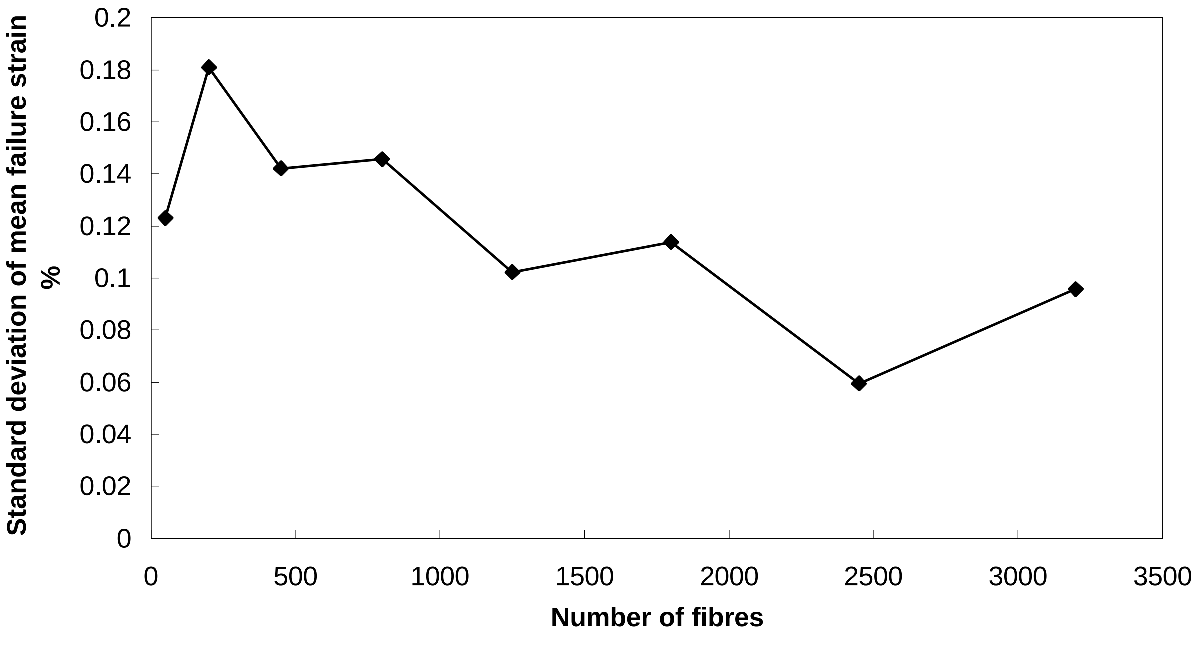

Fig.6 : Dependence of standard deviation of mean failure strain as a function of the number of fibres in the simulation.

Fig.6 shows the corresponding standard deviation of the mean failure strain (%) as a function of the number of fibres used in the simulation. It is seen that increasing the number of fibres tends to decrease the standard deviation.

12. Discussion of results

Having determined values for the parameters that define the type of simulation that is to be carried out, it is first necessary to decide how many repeat runs of the simulation are needed in order to be able to predict reliably the mean and variance of the results. Experience has shown that if the number of repeat calculations is selected to be at least 10 then the resulting means should be reasonably reliable. The larger this value the more reliable the results will be. Using large values for the number of repeat simulations can require considerable computing time when using a modern PC. For all calculations shown below the results for failure strains are the averages of 10 repeat simulations using the same parameters. It is necessary to investigate how the means of failure strains depend upon parameters selected for the problem. The preliminary results of such investigations are now presented.

The effect of sample size needs to be considered in two different ways. Firstly, the effect of increasing the parameters M and N needs to be investigated. Table 4 shows the results of simulations where the size dependence for the case M = N is investigated for the case of a single layer (i.e. L = 1, h = 0). It is seen that the mean failure strain increases as the value of M (= N) is increased.

M	N	L	Mean strain % $\delta = 0.5$ mm	Mean strain % $\delta = 2$ mm
10	10	1	2.096	1.911
20	20	1	1.894	1.839
30	30	1	1.884	1.753

TABLE 4 : Values of mean failure strain percents resulting from the simulation showing the dependence on the parameters M and N for the case M = N. The calculations assume that L = 1, h = 0, t = 50 MPa.

It appears from Table 4 that reasonable lower bound predictions of failure strains are obtained when M = N = 10 (i.e. 200 fibres) for the cases of $\delta = 0.5$ mm and $\delta = 2$ mm. These are the values used in the subsequent predictions for reasons of encountering tolerable computation times when using a PC. It is seen that increasing the size of the model lowers the predicted values for the mean strain. This behaviour is expected as increasing the number of fibres in the model will lead to an increased likelihood of low strength fibre elements.

Secondly, it is known that as the length of samples of individual fibres or fibre bundles increases, the mean strength decreases in a way that can be predicted using weakest link statistics for single fibres, or using equal load sharing rules for bundles. For a composite, where localised (rather than equal) loading occurs there is a possibility that the mean strengths of samples of the composite may also exhibit a length dependence. It is, therefore, essential to consider the effect of increasing the parameter L so that the length of the sample (in the loading direction) increases. Table 5 shows the dependence of means and variances of the failure strains on the values of L for the case when M = N = 10, h = 0 and $\delta = 0.5$ mm, for two values of the critical interfacial shear stress, namely $\tau = 50$ MPa and $\tau = 100$ MPa.

M	N	L	Mean strain % τ = 50 MPa	Mean strain % τ = 100 MPa
10	10	1	2.074	2.145
10	10	2	2.030	2.106
10	10	5	1.927	2.021
10	10	10	1.804	1.919

TABLE 5 : Values of mean failure strain percents and variances resulting from simulation showing the dependence on the number L of layers. The calculations assume that M = N = 10, d = 0.5 mm, and that h = 0 neighbouring layers are used when assessing failure.

It is seen from Table 5 that as the length of the sample increases the mean failure strain decreases. In addition the result of increasing the value of the imposed interfacial shear stress τ is to increase the failure strain. This behaviour is expected as a lower interfacial shear stress results in less stress being carried by the fibres thus leading to their failure at lower values of the applied stress.

The expected experimental values for the failure strain are in the range 1.56% - 1.74% [19]. The predicted values of mean failure strain are above this range. However, the results indicate that increasing the number of fibres in the system, and the number of layers L, could decrease the mean strains and close the gap between predictions and the experimental results.

13. Effect of choice of failure criterion

The failure criterion that is imposed by the simulation depends upon number h of fibre element layers that have been selected for the failure assessment (see section 8). Table 6 shows the values of the mean failure strain for the values h = 0, 1, 2, 3, 4 for the case when M = N = 10, L = 10 and δ = 0.5 mm.

No. of layers used for failure assessment (h)	Mean failure strain % τ = 50 MPa	Mean failure strain % τ = 100 MPa
0	1.875	1.934
1	1.898	1.911
2	1.819	1.940
3	1.911	1.910
4	1.897	1.936

TABLE 6 : Values of mean failure strain percents resulting from the simulation showing the dependence on the number of layers used when assessing failure for the case d = 0.5 mm, M = N = 10, L = 10.

It is seen that there is virtually no effect on the failure strain resulting from the imposition of the various failure criteria that have been examined. This is probably due to the load sharing rule favouring failure in fibre elements that are neighbours of the first fibre that fails. Visualisation tools are needed to check fibre fracture patterns during failure. It is virtually impossible to investigate this aspect of composite failure from numerical data, especially when the number of fibre elements in high.

14. Conclusions

1. A Monte Carlo model of a unidirectional composite subject to uniaxial tensile loading has been developed that can be used to simulate progressive fibre fracture and ultimate failure taking account of interface behaviour and complex load sharing rules that must be operated when a fibre fails.

2. By making use of periodic boundary conditions in three orthogonal directions the simulation can in principle represent large samples of composite, and avoid the necessity for taking account of the effects of the sample edges.

3. Fibre failure is represented using weakest link statistics and the Weibull distribution. On the basis of experimental fibre strength measurements, the extrapolation of behaviour to the small fibre element lengths needed by the simulation is reasonable.

4. It has been found that in order to predict reliable mean failure strains the simulation needs to be repeated at least 10 times. This results in significant computing times when using PCs.

5. The mean failure strains predicted have shown significant dependence on the number of fibres used in the simulation. Preliminary results suggest that the array size should be at least 10 x 10 (corresponding to 200 fibres) but larger values are preferable.

6. The mean failure strains predicted are dependent upon the length of the sample; the failure strain decreasing as the sample length increases.

7. The number of layers h of fibre elements used when assessing composite failure does not appear to have any significant influence on the mean failure strain.

8. Increased mean failure strains result when the value of the critical interfacial shear stress is increased.

9. The predicted failure strains are very similar to those for typical experimental measurements of the carbon fibre composite being analysed.

References

1. H E Daniels, 'The statistical theory of the strength of bundles of threads', Proc Roy. Soc. Lond. **A183**, (1945), 405-435.

2. B D Coleman, 'On the strength of classical fibres and fibre bundles', J Mech. Phys. Solids, **7**, (1958), 60-70.

3. L N McCartney and R L Smith, 'Statistical theory of the strength of fibre bundles', J. Appl. Mech. **105**, (1983), 601-608.

4. R L Smith and S L Phoenix, 'Asymptotic distributions for the failure of fibrous materials under series-parallel structure and equal load-sharing', J. Appl. Mech. **48**, (1981), 75-82.

5. R L Smith, 'The asymptotic distribution of the strength of a series parallel system with equal load-sharing', Annals of Probability, **10**, (1982), 137-171.

6. R L Smith, S L Pheonix, M R Greenfield, R B Henstenburg and R E Pitt, Proc. Roy. Soc. Lond. **A388**, (1983), 353-391.

7. S L Phoenix and R L Smith, 'A comparison of probabilistic techniques for the strength of fibrous materials under local load sharing among fibres', Int. J. Solids Struct. **19**, (1983), 479-496.

8. P W Manders, M G Bader and T-W Chou, Fibre Sci. & Tech. **17**, (1982), 183-204.

9. P T Curtis, 'A computer model of the tensile failure process in unidirectional fibre composites', Comp. Sci. Tech. **27**, (1986), 63-86.

10. M Lienkamp and P Schwartz, 'A Monte Carlo simulation of the failure of a seven fibre microcomposite', Comp. Sci. Tech. **46**, (1993), 139-146.

11. W A Curtin and N Takeda, 'Tensile strength of fibre-reinforced composites : I. Model and effects of local fibre geometry', J. Comp. Mater. **32**, (1998), 2042-2059.

12. W A Curtin and N Takeda, 'Tensile strength of fibre-reinforced composites : II. Application to polymer composites', J. Comp. Mater. **32**, (1998), 2060-2081.

13. W A Curtin, B K Ahn and N Takeda, 'Modelling brittle and tough stress-strain behaviour in unidirectional ceramic matrix composites', Acta Mater, **46**, (1998), 3409-3420.

14. L N McCartney, 'Analytical model for debonded interfaces associated with fibre fractures or matrix cracks', Proc. ICCM-12, Paris, 5-9 July 1999.

15. B Bergman, 'On estimation of Weibull modulus', J. Mat. Sci. Lett. **3**, (1984), 689-692.

16. R Langlois, 'Estimation of Weibull parameters', J. Mat. Sci. Lett. **10**, (1991), 1049-1051.

17. W Weibull, 'A statistical distribution function of wide applicability', J. Appl. Mech. **18**, (1951), 293-297.

18. E J Gumbel, ‘Statistical theory of extreme values and some practical applications’, Nat. Bur. Standards Appl. Maths. Series, **33**, (1954).

19. M G Bader, K L Pickering, A Buxton, A Rezaifard and P A Smith, Comp. Sci. & Tech. **48**, (1993) , 135-142.

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