Investigation of the Energy Distribution in a Re-entrant Cavity

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1 Abstract

This paper examines the 'filling factor' required to calculate the loss of materials measured in re-entrant cavities and discusses the key question of the correct method for determining the loss of the unloaded cavity. It has been usual in the past to determine this empirically. It is suggested that this leads to errors as the energy distribution in the unloaded cavity differs from that when the sample is present. To illustrate this issue and to help to resolve the question, experimental data are presented on low loss dielectric materials (HDPE and YAG). Values obtained with a re-entrant cavity are compared with those obtained with other methods in the frequency range 1.0 to 1800 MHz. It appears that modification of the current standard method is required for two reasons, but it is not yet clear what the preferred technique should be. Further studies are in progress.

2 Introduction

The re-entrant cavity originally developed by Parry [1] and Works [2] is widely used for dielectric measurements on low loss materials in the frequency range 0.1 to 1.0 GHz. The sample under test is supported on the lower of the two inner-conductor central posts, see Figure 1, and conventionally its dielectric loss is determined from the observed change in the resonant 'Q' when the sample is removed. Resonance is restored by adjustment of the length of the upper post using a precision micrometer, which is possible because a variable length bellows section forms part of the inner conductor post. The permittivity is determined from this change in length and the sample thickness. Re-entrant cavity measurements are generally the most accurate for determining the loss of low-loss dielectric materials in the frequency range 0.1 to 1.0 GHz [3]. Typical values for uncertainty (with coverage factor $k = 2$) for medium permittivity are ± 0.01 and for loss tangent ± 0.001. This paper outlines the key limitations of this type of cavity in order to decide whether an uncertainty in tan δ of ± 10 µrad can be achieved when measuring at the 100 µrad level on low permittivity materials such as polyethylene.

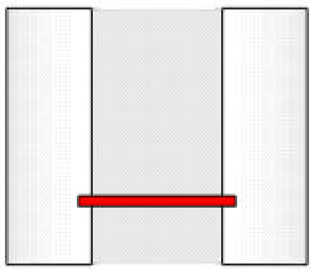

Figure 1. Simplified schematic cross-section through the re-entrant cavity showing the position of the laminar specimen in the capacitance gap between the upper and lower inner-conductor posts. The specimen shown has a diameter slightly larger than the inner conductor. The cavity has circular symmetry around the inner conductor axis.

It is often difficult to establish precise values of uncertainty when measuring very low loss materials, as any undetected source of error can make a significant contribution to the observed dielectric loss. For this reason it has become the practice at NPL to compare loss measurements from several different techniques across a wide frequency band and look for a regular progression of loss with frequency.

Use of re-entrant cavities is described in an IEC standard [4], which makes allowance for dilution of the sample's influence upon the Q-factor by the presence of an air gap above it, but does not otherwise correct for the fraction of the energy in the cavity that is contained within the specimen. This is an important parameter because it affects the sensitivity of the loaded cavity Q-factor to the loss of the specimen. This important information can only be obtained by modelling the electromagnetic fields in the cavity. In the past this has been carried out using lumped-circuit approaches [1,4,5] but more accurate techniques are nowadays available. Mode-matching models [6,7] can give a complete description of the fields in such a cavity but they are not available to all users and not always convenient to use. A convenient alternative that has been pursued here is to employ Finite Integration modelling, available in proprietary software packages such as CST's *'MAFIA'* or *'Microwave Studio'* [8]. In this case, allowance for the energy balance in the cavity is made by defining a *'filling factor'* for the specimen in the cavity, which is readily derived from energy density computations performed by the software packages. Assuming no air-gap above the specimen, the measured loaded Q-factor of the cavity, Q_L, the loss tangent of the specimen, tan d, and the filling factor F are all related by the following equation:

$$1/Q_L = 1/Q_1 + 1/Q_2 + 1/Q_3 + ... + \tan d \, / F$$

where Q_1 , Q_2 , Q_3, etc. account for cavity losses from all other causes, e.g. metal loss and coupling. A filling factor of $F = 1.0$ would imply that all of the energy in the cavity is in the specimen. In practice this is never the case, so F is always greater than 1.0. In practice there is also always an air-gap above the specimen, which gives rise to another filling-factor which multiplies F, but this can be readily computed with sufficient accuracy by a simple lumped circuit (series capacitance) model, or else it can also be included in the software model.

This paper briefly summarises work on assessing this *'filling factor'* but it also discusses a key question that has emerged during these studies regarding the correct method for determining the loss of the unloaded cavity. In the past it has been usual to determine this empirically as outlined above, by changing the length of the upper inner-conductor post to produce the same capacitance in the sample gap with and without the specimen. It is suggested here that this leads to errors because the energy distribution in the unloaded cavity differs from that when the sample is present.

3 Experimental.

Experimental data are presented on the low-loss dielectric high density polyethylene (HDPE). They compare loss measurements obtained in the re-entrant cavity with those obtained with other methods over the frequency range 1.0 to 1800 MHz.

The re-entrant cavity was tuned to 900 MHz for this study and the results were compared with those obtained by other methods. The three methods intercompared well. A regression analysis assuming that $\tan \delta = A + B.\sqrt{F}$ had a regression coefficient ($R^2 = 0.975$) with the interpolated values shown in Table 1.

Table 1. Measurements of the dielectric loss of HDPE. Uncertainties are given for a coverage factor of $k = 2$.

Frequency (MHz)	**Measured Dielectric Loss (tan d, mrad)**	**Interpolated value (calculated) (tan d = A + B.ÖF)**	**Ratio: (measured)/ (calculated)**	**Method of measurement**
1.0	47 ± 10	46	1.022	Hartshorn-Ward technique
6.0	53 ± 10	63	0.841	"
30.0	85 ± 18	79	1.076	"
65.0	95 ± 20	87	1.092	"
900.0	114 ± 24 *	113	1.009	Re-entrant cavity
1800	117 ± 30	120	0.975	Split Post cavity

* There is an additional uncertainty in the re-entrant cavity measurement, see Section 4 below.

The results, in this case, were calculated using the formalism given in IEC377-2 [4] with a correction for the fringing field around the electrodes derived form the Kirchoff Formula [9]. Comparison of the measured and interpolated values (Column 4) gives no indication that the data obtained with the re-entrant cavity are any less accurate than those obtained in other ways. Indeed, in common with Kaczkowski [7], a considerably better precision than that outlined by Baker-Jarvis [3] appears to be justified.

4 Modelling of the Re-entrant Cavity

This has been carried out using CST's *MAFIA* and *Microwave Studio* software packages. By making use of the cylindrical symmetry, a two dimensional (r, z) representation of the cavity was chosen for speed of processing in *MAFIA*. Allowance was made for a zero-error in the micrometer readings, which was found to correspond well with that measured mechanically. Experimental values of the 'Q' of the cavity at its lowest frequency (≈ 750 MHz) were used to normalise the *MAFIA* calculations to make allowance for the effects of cavity construction on the observed Q (e.g. effects relating to surface finish and the effect of the bellows and of resistive layers at metal-to-metal contacts). On this basis the ratio between observed and modelled values of Q ranged from 1.00 at 750 MHz to 1.08 at 1 GHz.

Computation of energy densities showed that the majority of the energy in the resonant cavity lay either in the specimen gap (approx. 90%, depending on specimen material and thickness) and in the fringing fields around the gap (approx. 9%). This is to be expected given that this is where the strongest electric fields are to be found, see Figure 2. For the fundamental mode, filling factors were therefore found to vary from approximately 1.1 for a specimen the same diameter as the inner conductor to about 1.01 for a specimen with a diameter just below that of the outer conductor. This is explained by the fact that most of the energy in the fringing field is inside the specimen in the latter case but only in air in the former case, making the loaded Q-factor higher.

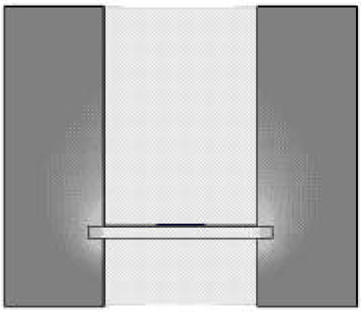

Figure 2. Illustration of a *Microwave Studio* computation showing the magnitudes of the electric fields in the air and sample regions of the cavity. They are shown in a grey scale with stronger fields in lighter grey. The strongest fields lie in the specimen in the capacitance gap. The fringing field region around the gap is the region with the strongest fields in air.

Problems arise when considering the measured sample losses. Typically, the modelled losses on the samples of HDPE were ~ 30 µrad higher than the observed values. This is believed to be due to the difference in distributions of magnetic fields on the metal surfaces of the cavity with and without the sample when the air-gap is adjusted to allow the cavity to resonate at the same frequency in both cases. This difference is significant, in the light of the target uncertainty (± 10 µrad), and so work continues to resolve the question. A lower loss, higher permittivity material, Yttrium Aluminium Garnet (YAG) with a nominal permittivity of 10.6 and typical loss of less than 50 µrad at this frequency is now being measured. Two questions are under consideration:

(a) Which method should be used for the determination of the loss of the unloaded cavity; the traditional empirical measurement, or should a modelled value for the cavity with a loss free sample be used?

(b) What is the preferred method for determining the permittivity; should a sample smaller than the electrodes be used, which is in the uniform field region (a practice which has long been standard in the USA) or a specimen which is larger in diameter than the inner conductor: this is easier to position and can take in virtually all of the fringing field (a practice which has been standard in Europe)?

5 Conclusions

It has been shown that the re-entrant cavity can reliably make dielectric loss measurements on polyethylene at the 100 µrad level to a repeatability of ± 10µrad. However, there exists a discrepancy of the order ± 30µrad between the results calculated using the conventional formalism and those obtained by modelling a loss-free sample with the same permittivity and size as the actual sample. Work is in progress to resolve this difference and decide upon the correct approach. It is, however, clear from this study that modification of the IEC standard [4] is required in any case to take account of specimen filling-factors, which can differ significantly from 1.0. Attention will also be given to the choice of the preferred specimen geometry.

6 Acknowledgements

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