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Approximate Estimation of the Effects of Asymmetry on the Testing of Tensile Specimens

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APPROXIMATE ESTIMATION OF THE EFFECTS OF ASYMMETRY ON THE TESTING OF TENSILE SPECIMENS

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SUMMARY

An analysis is made of the plane stress problem representing the tensile testing of rectangular orthotropic material specimens. The effects of various geometric non-symmetries are considered and some approximate polynomial solutions are obtained and tabulated. Comparisons of the theory with specific finite element calculations show good general agreement.

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1 INTRODUCTION

Tensile testing techniques play a central role in the measurement of mechanical properties of materials, both in terms of stiffness and failure properties. Methods for the measurement of induced strain include extensometers, moiré techniques and the use of strain gauges. In the latter case, these are attached to the specimen at one or more positions in order to determine the effects of the loading. A proposed ASTM and ISO standard describes an alignment procedure based on strain gauges. Requirements on the position of these gauges is clearly part of the specification of a testing standard. Difficulties arise when the specimen is not mounted symmetrically within the machine; this changes the load conditions significantly with subsequent distortion of the strain field. Information on these effects is necessary in order to determine how much asymmetry can be tolerated in a useful test. The purpose of the work to be described here is to estimate the effects of various asymmetries on the stress, strain and displacement distributions over the specimen. It is hoped that this information will assist both those involved in the preparation of testing standards and those performing the experimental testing.

A relevant aspect of the problem concerns the local effects of the grips on the specimen. A constraint requiring all displacements and rotations to be zero is extremely difficult or impossible to obtain in practice. In many cases, the specimen tends to pull out of the clamp due to Poisson contractions in the thickness direction. Various grip designs are used to minimise this effect. These include geometric and hydraulic wedge action grips that apply increasing through-thickness loads as longitudinal load is increased. However the problem can persist. In addition, early local material failures due to stress concentrations can occur in corner regions, resulting in further local relaxation of constraints. That these effects are local follows from Saint-Venant's Principle [1]. Essentially, this states that a change in the distribution of an end load, without change in resultant, alters the stress significantly only near that end. Exactly how near is 'near' is not defined, although it is accepted that this implies a distance of the same order of magnitude as the dimensions of the loading area of the body. In the present context, a knowledge of the extent of the edge region is desirable in order that strain gauges can be located outside it. The point must be made that approximate methods of analysis of the whole specimen of the kind used here will <u>not</u> yield information on the extent of edge effects. Some knowledge of this effect can be obtained by finite element analysis of specific cases, but two difficulties arise. The first has been mentioned earlier, namely that detailed information on the form of the edge constraint provided by the grip is not available. Finite element analysis requires a full definition of the boundary conditions in terms of the type (eg one or more displacements or rotations) and on its distribution (ie applied to the whole edge or only part of it). Several combinations of restraints are clearly possible. The other difficulty concerns the treatment of possible singularities that can arise near a corner. The use of a high mesh density is necessary to resolve these regions thus making the calculations expensive. In addition it is likely that the shape functions used by the element do not properly represent the form of the singularity.

The present work, while not addressing the edge effect problem, is aimed at producing approximate formulae for the stress, strain and displacement distributions for various geometric asymmetries. In this way, the role of the material and geometric parameters can be identified.

The next section of the report describes the problems for treatment and Section 3 gives details of the mathematical analysis. Subsequent sections set out the approximate solutions and give some comparisons with finite element results.

2 LOADING CONFIGURATIONS

In this section we list the configurations to be analysed in later sections. We consider first the case in which the sample is incorrectly mounted as shown in Figure 1. Here the specimen centre line is offset from the grip centre line. On loading, an axial displacement is imposed on the specimen but the grips are allowed to rotate; this produces the situation shown in Figure 1(a) in which the specimen becomes curved. We shall call this Case 1 (Offset Placement with End Rotation).

A case related to the above is shown in Figure 2. Here the specimen is mounted correctly before loading, but during the extensional loading, the grips are made to rotate symmetrically as shown in Figure 2(a). This is described by a known gradient of the centre line at the end of the specimen.

We consider next the configuration indicated in Figure 3. Here the sample is positioned correctly before loading. During loading however, due to machine deficiencies, some lateral displacement of the grips takes place in addition to the main longitudinal displacement. This produces the result shown schematically in Figure 3(a). We point out here that for purposes of illustration, the figures show magnified effects; in reality the asymmetries may be quite small.

Finally we consider the case shown in Figure 4, in which the specimen is mounted skew to the grips before loading. The amount of skew is given by the angle θ as is also the relative displacement u_o of the grips after loading.

3 GOVERNING EQUATIONS

For purposes of the analysis, we shall assume that the specimens are of orthotropic material with the 1 direction aligned longitudinally with the x axis, and the 2 or transverse direction aligned with the y axis. We assume that a state of plane stress is applicable during loading and ignore the effect of any through-thickness compression applied by the action of the grip design. In the absence of body forces, the equations of equilibrium [1] are:-

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} &= 0 \end{aligned}\right\} \qquad (3.1)$$

where σ_x, σ_y denote the normal stresses and τ_{xy} is the shear stress. We introduce a stress function ϕ that satisfies these equations identically by defining

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} \;,\; \sigma_y = \frac{\partial^2 \phi}{\partial x^2} \;,\; \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} \;. \qquad (3.2)$$

The orthotropic stress-strain relationships [2] are

$$\varepsilon_x = \frac{\sigma_x}{E_1} - \nu_{21}\frac{\sigma_y}{E_2} , \tag{3.3}$$

$$\varepsilon_y = \frac{\sigma_y}{E_2} - \nu_{12}\frac{\sigma_x}{E_1} , \tag{3.4}$$

and

$$\gamma_{xy} = \tau_{xy}/G_{12} . \tag{3.5}$$

We have also the following relationship between the material constants

$$\nu_{12} E_2 = \nu_{21} E_1 . \tag{3.6}$$

In terms of the displacements, the strains are

$$\varepsilon_x = \frac{\partial u}{\partial x} , \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{3.7}$$

where u and v are the horizontal (x) and vertical (y) displacements respectively. In terms of the stress function ϕ the condition of compatibility becomes [1], [2]

$$\frac{1}{E_2}\frac{\partial^4\phi}{\partial x^4} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{1}{E_1}\frac{\partial^4\phi}{\partial y^4} = 0 . \tag{3.8}$$

For an isotropic material $E_1 = E_2 = E$, $\nu_{12} = \nu_{21} = \nu$, $G_{12} = E/2(1+\nu)$ and the above equation reduces to the standard biharmonic equation. Various solutions to (3.8) are required subject to boundary conditions in terms of applied forces and displacements. Certain boundary conditions are common to all the cases to be considered; these state that the longitudinal edges are stress-free. If c is the half-width of a specimen, then $y = \pm c$ denotes these edges; the common conditions are therefore

$$\sigma_y = \tau_{xy} = 0 \quad \text{on} \quad y = \pm c . \tag{3.9}$$

The conditions at the other edges are provided by the clamping and loading configurations; these will be detailed later as the individual cases are analysed.

4 ANALYSIS OF THE LOADING CONFIGURATIONS

4.1 Case 1. Offset Placement with End Rotation

This case is symmetric about $x = 0$; the co-ordinate system, dimensions and applied loading displacements are shown in Figure 1(b). We shall assume that known horizontal displacements of u_o are applied at $y = y_o$ on AC and BD as shown. In addition, we shall assume that the edge BD is free to turn about the point E, so that there is no applied moment about E. Clearly there are no shear forces applied at AC and BD; because of this and (3.9) we shall assume that the shear stress τ_{xy}, is zero throughout. We shall also assume that σ_y is identically zero throughout the specimen as no normal forces are applied at $y = \pm c$ due to (3.9). These conditions are satisfied by a stress function of the form

$$\phi = \frac{\alpha y^2}{2} + \frac{\beta y^3}{6} \tag{4.1.1}$$

where α and β are constants to be determined by the conditions on $x = \pm \ell$. This also satisfies the compatibility equation (3.8).

If t denotes the thickness of the specimen, the absence of any turning moment about E is expressed by

$$\int_{-c}^{c} (y-y_o)\, \sigma_x dy = 0 \ . \tag{4.1.2}$$

Substitution of (4.1.1) into (3.2) gives

$$\sigma_x = \alpha + \beta y \ , \qquad \sigma_y = 0 \ , \qquad \tau_{xy} = 0 \ .$$

Condition (4.1.2) then yields

$$\frac{\beta c^3}{3} = \alpha y_o c$$

or