dti
Department of Trade and Industry

# THE COMPOSITE PERFORMANCE & DESIGN (CPD) PROGRAMME

## 1997 - 2000

A MATERIALS METROLOGY RESEARCH PROGRAMME FUNDED BY THE DEPARTMENT OF TRADE AND INDUSTRY

**Project CPD5**

**Dissemination, Standardisation and Reviews**

# Fatigue Performance of Polymer Matrix Composites

Theme Group Meeting

**May 2000**

**Centre for Materials Measurement and Technology**
**National Physical Laboratory**
**NPL Report CMMT(A) 265**

NPL

## DTI Criteria For Measurement Programmes

a. There must be a significant level of innovation

b. The work must be of high quality

c. The work should contribute to Standards and/or Codes of Practice

d. There should be a width of beneficiaries/users of the work (including by sector and company size)

e. The work should be pertinent to trade

f. The work should be pertinent to public policy (regulation, including safety and external policies)

## Composites Performance and Design Programme

CPD1: Structural Response and Integrity

CPD2: Life Assessment and Prediction

CPD3: Interface Characterisation and Behaviour

CPD4: Manufacture, Safety and Design: Specific Issues

CPD5: Dissemination, Standardisation and Reviews

This document has been produced as part of the Composites Performance and Design programme for the Department of Trade and Industry's 'Engineering Industries Directorate', under its Materials Measurement Programme.

# Fatigue Performance of Polymer Matrix Composites
## Theme Group Meeting

**Centre for Materials Measurement & Technology**
**National Physical Laboratory**
**Teddington**
**Middlesex, TW11 0LW, UK**

The report was prepared as part of the research undertaken at NPL for the Department of Trade and Industry funded project on "Composites Performance and Design - Dissemination, Standardisation and Reviews".

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National Physical Laboratory
Teddington, Middlesex, UK, TW11 0LW



Approved on behalf of Managing Director, NPL, by Dr C Lea,
Head of Centre for Materials Measurement and Technology.

# CONTENTS

## Objectives

As part of a series of theme meetings within the Composites Performance and Design programme, the objectives are:

- To present to UK industry details of the research results from the CPD projects on development of test and predictive methodologies for engineering data on the long-term performance of PMCs, trends in standardisation, etc.
- To enable UK industry to outline their own experiences and future requirements.
- To enable other Research Organisations/Universities to present related work.
- To discuss maximising the industrial relevance of the projects, developing a UK viewpoint and identifying generic future requirements.

## Programme

**Life Assessment and Prediction -** Dr Bill Broughton (NPL)

**Fatigue Research Wish List for Composite Wing -** Dr Martin Gaitonde (B Ae Systems)

**Cost Effective High Cycle Fatigue Testing -** Dr Rod Martin (MERL)

**Fatigue Performance of Composites with Terminating Plies -** Prof. Mike Wisnom (Univ. of Bristol)

**Standardisation Issues in Fatigue-** Dr Graham Sims (NPL)

**Some Composite Modelling Issues for the Prediction of Fatigue Performance -** Dr Neil McCartney (NPL)

**Fatigue Life Prediction - Requirements and Methodologies -** Dr. Andrew Davies (DERA)

**Analysis of Damage Mechanisms in Fatigue of Fibre-Reinforced Epoxy Laminates -** Dr Costas Soutis (Imperial College)

**Lifetime Prediction for FRP Components -** Prof. Frank Matthews (Imperial College)

**A Constant Life Model for the Prediction of Fatigue Life in Reinforced Plastics -** Prof. Bryan Harris (Univ. of Bath)

## Summary

The ten technical presentations, summarised below, covered a wide range of topics from the research and development work in CPD2, industrial applications and needs, cost effective testing, test methods and standardisation issues, structural testing and analysis and predictive modelling (including fracture mechanics, physically based modelling of micromechanisms, damage mechanics, 3-D finite element analysis, lifetime prediction and a constant life model). 31 delegates attended.

The presentations highlighted the relevance of both simple analytical formulations and frameworks to aid engineers and the need for longer term research into the modelling of physical mechanisms of damage initiation and progression in composite laminates and structures. There is an immediate requirement by designers/engineers for frameworks in order to provide reasonable confidence for use in high cycle applications. Physical based models should provide the fundamental understanding required for the improvement in the use of existing materials and development of new materials. These two approaches are complementary and with time should merge. If either of these approaches are to be used in confidence, experimental validation using consistent standardised test methods is necessary.

## 1. Project CPD2 - Life Assessment and Prediction

*Dr Bill Broughton* (NPL)

Dr. Bill Broughton's presentation focused on fatigue testing of composite rods/impregnated strands and laminates (unidirectional and cross-ply) as part of a building block approach to understanding fatigue of laminates. Methods of manufacturing and testing impregnated strands and small diameter composite rods were shown along with typical **S-N** data for E-glass fibre-reinforced polyester rods and glass fibre-reinforced epoxy laminates. It was shown that the normalised **S-N** response of composite rods and laminates can be approximated by the same simple algebraic expression. Preliminary work on a waisted laminate specimens showed improved performance over rectangular specimens (currently specified in the international standard ISO 527-5). Carbon fibre based unidirectional laminates showed excellent fatigue performance. Future research activities will include transverse crack measurement and loss of stiffness and strength for **0/90** laminates with loading cycles. The results are to be compared with physically based modelling of micromechanisms being developed at NPL by Dr Neil McCartney.

## 2. Fatigue Research Wish List for Composite Wing

*Dr Martin Gaitonde* (BAE SYSTEMS, Airbus)

Mr. Martin Gaitonde focused his presentation on research requirements for the Composite Wing programme at BAE Systems (Airbus). The work covered AMCAPS II programme (Affordable Manufacture of Composite Aircraft Primary Structures) with particular emphasis on the 'Fatigue Analysis' method developed. The presentation highlighted current practices, providing details on typical fatigue regime for large civil aircraft and the background and benefits to the use of "no growth" approach for design. The presentation covered compression fatigue tests conducted at DERA and material screening against fatigue. Finally, the speaker provided a wish list for both in-plane and through-thickness fatigue, bolted constructions, impact dent depth relaxation and evaluation of load/life factors for various structural details/materials.

## 3. Cost Effective High Cycle Fatigue Testing

*Dr Rod Martin* (MERL LTD)

Dr. Rod Martin spoke of the importance of fatigue testing where delamination was a primary damage mode, providing examples (Bell helicopter rotor system and GE fan blades) where high fatigue cycles were a performance issue. The presentation highlighted how automated crack growth modelling aids the use of fracture mechanics and the advantages of using electro-mechanical multi-station equipment as a cost effective validation method. The presentation highlighted the need for fatigue tests to be conducted to $10^8$ cycles and beyond with statistical accuracy if the data was to be used for design purposes. It was concluded that fracture mechanics should be used where delamination/cracking was a primary damage mode, where stress singularities exist and where damage tolerance is an issue. It was also stated that Global/Local modelling allows the complexity of fracture modelling to be achieved and that closed form solutions may give first order predictions.

## 4. Fatigue Performance of Composites with Terminating Plies

*Prof. Mike Wisnom* (University of Bristol)

Prof. Mike Wisnom presented results and the conclusions from tests conducted on tapered structures with terminating internal plies under tension fatigue loading. A number of factors were investigated (number of terminating plies, number of continuous plies, orientation of terminating plies, effect of taper, **R** ratio, stress rate and different materials) The strain energy release rate was determined for delamination along the interface between discontinuous plies and continuous plies for different lay-up combinations. It was observed that that failure mechanisms were similar for both static and tension fatigue with failure by delamination between continuous and discontinuous plies and that delamination starts immediately with no apparent threshold. Low **R** ratios were most damaging, whereas stress rate was not significant. It was concluded that the Paris law fits the data well, different specimen lay-ups have different crack growth rates with the delamination growth rate being similar for the same $\Delta G/G_c$ ratio. Also, the number of discontinuous plies was very important with terminating **0°** plies much worse than ± **45°** and that the taper is beneficial.

## 5. Standardisation Issues

*Dr Graham Sims* (NPL)

Dr. Graham Sims spoke on standardisation issues relating to fatigue testing. Highlighting some of the progress made in ISO 13003 (Fatigue - General Principles) and also briefed the delegates on the VAMAS fatigue round robin exercise. Dr Sims presented an overview of recent standards on mechanical testing (tension, compression, shear and flexure) of PMCs, mode I fracture toughness, plate-twist test (in-plane shear modulus), new work items relating to the open-hole tension, open- and filled-hole compression, pin-bearing and double notch shear tests, and ISO 1268 test panel manufacture. Uses of fatigue tests and fatigue test selection were also covered along with ASTM, AFNOR and CRAG related fatigue procedures/standards activities. A number of additional issues, such as the effects of rate/frequency, temperature, holes and notches and moisture, and damage monitoring and scatter bands were also included in the presentation. It was shown that by suitably normalising the data, all these factors reduced to a single normalised **S-N** curve for the GRP fabric standard. Dr. Sims concluded by saying that the progress made on static testing is being extended to fatigue testing and that review versions (CD, DIS) were now available for balloting via the national standards bodies.

## 6. Some Composite Modelling Issues for the Prediction of Fatigue Performance

*Dr Neil McCartney* (NPL)

Speaking on modelling issues, Dr. Neil McCartney reviewed some important issues relating to prediction of fatigue performance of PMCs and their progress. A description of progress of modelling fatigue damage development in composite laminates at the material (rather than structural) level was presented. It was stated that an empirical, such as the Tsai-Wu polynomial failure criteria was not based on any physical phenomena and was not easily applied to environmentally or fatigue damaged composites. It was emphasised that physically based models were needed and that failure modes must be modelled and understood. A brief summary of fatigue damage development was presented along with fundamentals of microstructural damage growth laws for composites (fibre and matrix) and metals. The mathematical analysis of degradation of properties of cross-ply laminates was shown. Plots were shown of predicted stress-strain behaviour, degradation of axial modulus and ply crack growth for a $[0/90]_s$ CFRP laminate. It was concluded that this area was still in its infancy and that there was a need to co-ordinate activity in the field within the UK in conjunction with industry. The way forward was to develop agreed design rules and validation of both models and design methods.

7. **Fatigue Life Prediction - Requirements and Methodologies**
*Dr. Andrew Davies* (DERA)

Dr. Andrew Davies spoke on fatigue life prediction research in progress at DERA in collaboration with Bath University, Cambridge University, Imperial College and FFA (Sweden). The roles of each organisation and interactions was presented along with brief overview on the background and objectives of the fatigue programme. Dr Davies emphasised the importance of developing damage initiation and growth predictive methodology and fatigue life prediction capability for PMCs and structures in order that tougher materials could be employed with confidence at high strains and stresses. The audience were briefed on the proposed work (including materials, test configurations, damage monitoring techniques and modelling approach) to be carried out by DERA and the partners within the programme. Details of work undertaken at DERA by Dr Emile Greenhalgh was also presented. Finally, Dr Davies concluded by discussing the deliverables for the research programme and their relevance to MoD interests both in current and future applications.

8. **Analysis of Damage Mechanisms in Fatigue of Fibre-Reinforced Epoxy Laminates**
*Dr Costas Soutis* (Imperial College)

Dr Costas Soutis presented an overview of the research carried out at Imperial College on modelling of layer damage of composites using a new approach based on the Equivalent Constraint Model (ECM) of the damaged lamina. This approach was shown to be applicable to determining multilayer matrix cracking and delaminations in glass/epoxy and carbon/epoxy laminates. The approach provides closed-form expressions for the reduced stiffness properties of the damaged laminae as well as the strain energy release rates associated with matrix cracking and delaminations. The parameters controlling the damage laminate behaviour were identified. It was shown that the predictions using the ECM approach compared favourably with the results obtained from other models and experimental data (see summary attached to presentations).

9. **Lifetime Prediction for FRP Components**
*Prof. Frank Matthews* (Imperial College)

Prof. Frank Matthews outlined the research at Imperial college on predicting fatigue life of composite structures and how current work is being aided by previous experimental work in the area. Previous experimental work (including objectives) were presented along with experimental results from four-point flexure loading of composite I-beams. It was shown that undamaged I-beams exhibited no detectable damage for tests conducted to $8 \times 10^6$ cycles. Fatigue failure was observed to occur after $4.78 \times 10^6$ cycles in I-beams containing 60 mm diameter circular notches which were located in the flange. Current research resulted in the development of a reliable method for the prediction strain energy release rate using finite element method and an automatic search technique for the detection of fatigue damage using an intelligent computer program. It was observed that coupon fatigue was not suitable for simulation of the **I**-beam, probably due to differences in the stress field between the two configurations. Prof. Matthews concluded with suggestions for future work in this area.

## 10. A Constant Life Model for the Prediction of Fatigue Life in Reinforced Plastics

*Prof. Bryan Harris (University of Bath)*

Prof. Bryan Harris presented a constant-life model for the prediction of fatigue life of PMCs, which was shown to apply for both CFRP and GRP materials under tension-tension, compression-compression and tension-compression fatigue loading regimes. The procedure, involves measuring tension $\sigma_t$ and compression, $\sigma_c$, strengths and obtaining **S/N** data at different stresses and **R** ratios. This information enables the solution of a simple algebraic relationship to obtain the **S/N** curve for any **R** ratio and enables constant-life plots to be constructed for any number of life cycles, $N_f$. It was shown that constant-life model was applicable to both virgin (i.e. undamaged) and impact damaged laminates with different stacking configurations. Constant-life parameter **f** (see attached presentation) for both CFRP and GRP laminates (virgin or impact damaged) for a constant life of $10^5$ cycles was shown to linearly decrease with log $\sigma_c/\sigma_t$ with straight-line of best fit passing through all the data at $N_f = 10^5$ cycles.

## Delegates

| | |
|---|---|
| **Adrian Hughes** | Dowty Propellers |
| **Andrew Davies** | DERA |
| **Arthur Webb** | Arthur Webb Associates |
| **Bob Lewin** | Rolls Royce plc |
| **Bob Machin** | Hunting Engineering |
| **Bill Broughton** | NPL |
| **Bryan Harris** | University of Bath |
| **Costas Soutis** | Imperial College |
| **David Payne** | British American Racing |
| **Deiter Weineke** | GEC |
| **Frank Matthews** | Imperial College |
| **George Hinopoulos** | NPL |
| **Graham Sims** | NPL |
| **Jan Niklewicz** | NPL |
| **John Cadei** | Maunsell Structural Plastics |
| **Keith Armstrong** | Consultant |
| **Maria Lodeiro** | NPL |
| **Mark De Cooper Jones** | Lucas Varity |
| **Martin Gaitonde** | B Ae Systems |
| **Michael Dale** | Consultant |
| **Mike Gower** | NPL |
| **Mike Wisnom** | University of Bristol |
| **Mike Jones** | B Ae AIRBUS |
| **Neil McCartney** | NPL |
| **Osama Attia** | Imperial College |
| **Richard Mera** | NPL |
| **Rod Martin** | MERL |
| **Roger Davidson** | AEAT |
| **Sam Gnaniah** | NPL |
| **Tony Kinloch** | Imperial College |
| **Wai Lee** | Matra B Ae Dynamics |

APPENDIX 1

# TECHNICAL PRESENTATIONS

Introduction

# Composites Performance and Design

*Dr. Bill Broughton (NPL)*

**NPL**

**National Physical Laboratory**



# COMPOSITES PERFORMANCE AND DESIGN (1997-2000)

- CPD1- STRUCTURAL RESPONSE AND INTEGRITY
- CPD2 - LIFE ASSESSMENT AND PREDICTION
- CPD3 - INTERFACE CHARACTERISATION AND BEHAVIOUR
- CPD4 - MANUFACTURE, SAFETY AND SPECIFIC PROJECTS
- CPD5 - DISSEMINATION, STANDARDISATION AND REVIEWS

FATIGUE PERFORMANCE OF POLYMER MATRIX COMPOSITES
NPL - 4 April 2000



## CPD1
## STRUCTURAL RESPONSE AND INTEGRITY

1. Critical review.
2. Development of existing structural tests under more complex environmental and stress histories.
3. Development of further structural test geometries.
4. Development of new tests for sandwich materials.
5. Investigation of scaling effects.
6. Developments related to damage tolerance.
7. Development of design procedure.



## CPD2
## LIFE ASSESSMENT AND PREDICTION

1. Critical reviews (PMCs/MMCs)
2. Development of accelerated test methods (PMCs)
3. Development of predictive models (PMCs)
4. Development of elevated temperature testing (MMCs)
5. Development of isothermal fatigue testing (MMCs)
6. Assessment of dimensional stability (MMCs)

## CPD3
## INTERFACE CHARACTERISATION AND BEHAVIOUR

1. Review of interface test and models
2. Evaluation of micro-mechanical test methods
3. Thermal fatigue of composites (fibre reinforced)
4. Test methods for chemical and physical characterisation
5. Modelling effects of interface properties on macroscopic behaviour



## CPD4
## MANUFACTURE, SAFETY AND DESIGN SPECIFIC ISSUES

A. Repair Technology
B. Manufacturing/Processing Measurements*
C. Implementation of Material Models in Design Procedures
D. Non-Destructive Evaluation
E. Fire Test Programme*
F. Machining and Specimen Preparation
G. Performance of Composite Friction Materials*

* Reviews only



## CPD5T3 - STANDARDISATION-1

- **Advancement of NPL Drafted Standards:**
  - Compression published, to add to earlier five standards published as BS EN ISOs (tensile x 2, flexure, shear x 2). Post-CRAG era established
  - Plate-twist shear test direct to IS Publication (no formal vote required)
  - ISO 1268-4 on test panel manufacture DIS ----> FDIS vote
  - ISO 10350-2 on test methods for technical datasheets DIS ----> FDIS vote
  - ISO 15024, Mode I, re-drafted for USA project leader DIS ----> FDIS vote
  - ISO 13003, Fatigue standard re-draft for French project leader DIS ----> FDIS vote



## LINKING ASPECT - 1. MODELLING

- Composite materials performance depends on many variables for which their effects cannot all be characterised experimentally,
  - fibre, matrix, interface, fibre format, laminate construction etc.
  - loading conditions, modes, environments and duration, etc.
- Understanding and interpreting the inter-relationship of variables demands reliable models to represent the relevant physics and engineering on which predictions can be based.
- The modelling can be at more than one level ,e.g. semi-empirical
- THUS models are able to:-
  - characterise materials behaviour quantitatively
  - predict performance (e.g. materials selection/material formulation)
  - be the basis of engineering design
  - identify important parameters that need to be measured
  - be the basis of new measurement methods for these parameters

**Project CPD2**

# Life Assessment and Prediction

*Dr Bill Broughton* (NPL)

# NPL National Physical Laboratory



## CPD2
## LIFE ASSESSMENT AND PREDICTION

1. Critical reviews (PMCs/MMCs)
2. Development of accelerated test methods (PMCs)
3. Development of predictive models (PMCs)
4. Development of elevated temperature testing (MMCs)
5. Development of isothermal fatigue testing (MMCs)
6. Assessment of dimensional stability (MMCs)

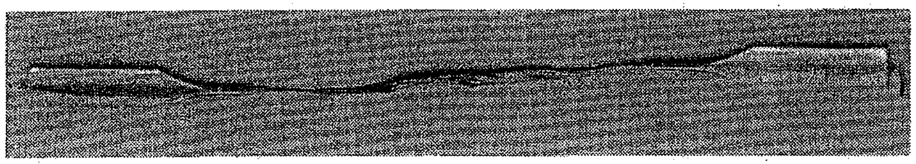


## Fatigue testing of fibre bundles

Research Activities/Conclusions

- E-glass and Grafil 34-700 carbon fibre bundle tests
- E-glass and carbon fibre bundles have excellent fatigue resistance

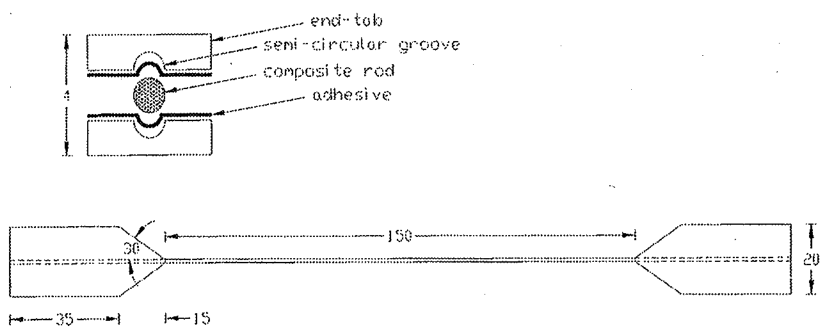
Endurance limit > $10^7$ cycles at 80-90% of ultimate tensile strength (UTS)



Carbon fibre tow test specimen

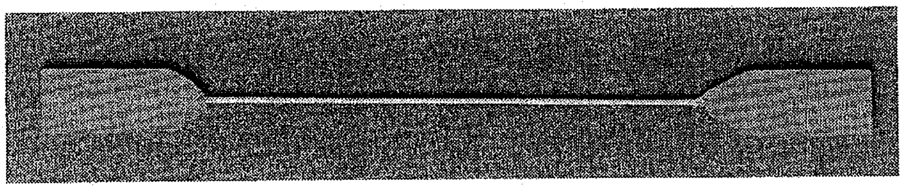


## Fatigue testing of impregnated strands/composite rods

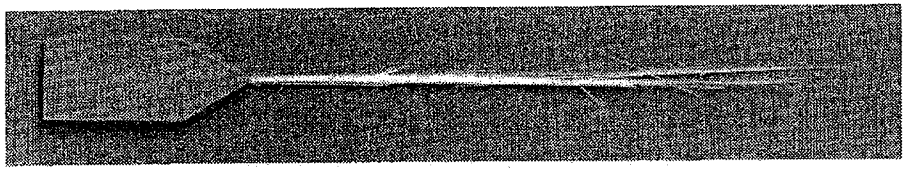



Schematic of composite rod specimen

## Fatigue testing of impregnated strands/composite rods



Composite rod specimen with bonded end tabs

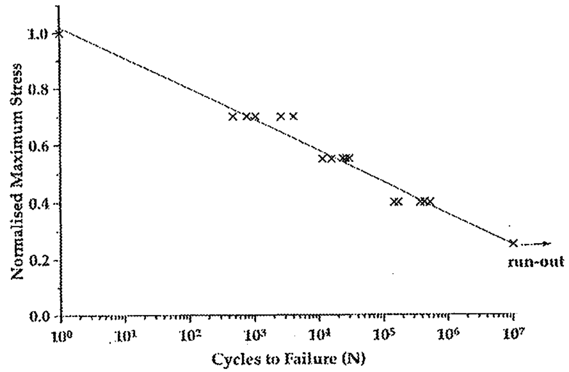


Failed composite rod specimen



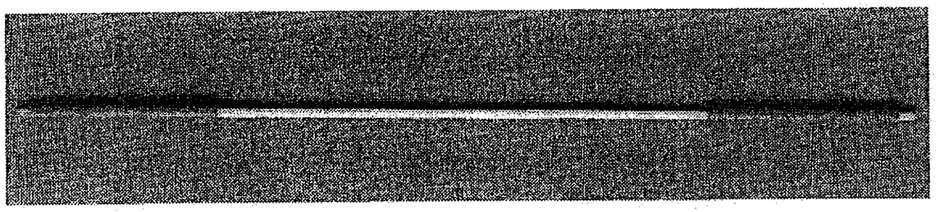
## Fatigue testing of impregnated strands/composite rods

Normalised S-N curve for E-glass/polyester rod specimens

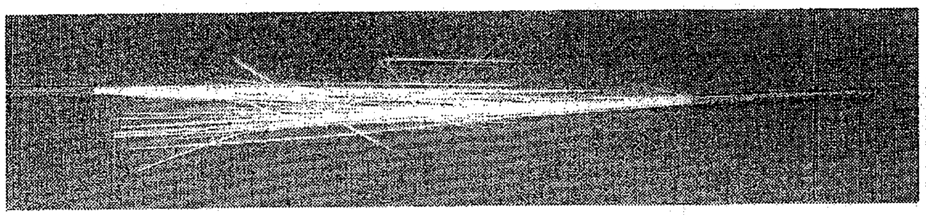





## Fatigue testing of impregnated strands/composite rods



Composite rod specimen with moulded end tabs



Failed composite rod specimen



## Fatigue testing of impregnated strands/composite rods

Conclusions

- Fatigue testing of composite rods is difficult.
- Cyclic fatigue tests in air (23 °C, 5Hz)

  S-N response of glass/polyester can be approximated by:

  $\sigma_{MAX}/\sigma_{UTS} = 1 - k \log N_f \quad (R = 0.1)$

  where the fractional loss in tensile strength per decade of cycles k = 0.1.
- S-N response of carbon/vinylester rods is relatively flat
- Current standard (Part 10 ISO 3597) for preparation and testing of composite rods requires revision to include tensile strength measurements.

## Fatigue testing of unidirectional laminates

Research Activities

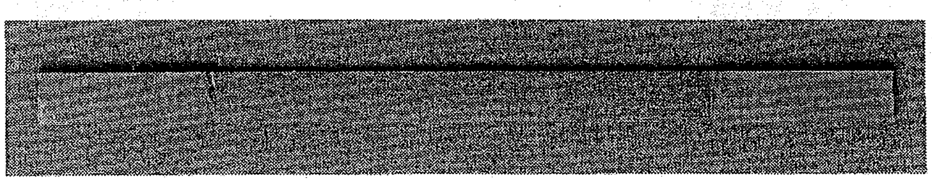
- Testing of E-glass/913, E-glass/F922, HTA/F922 and T300/924

  R = 0.1, 0.5 and 0.75 (sinusoidal/0.5 and 5Hz)

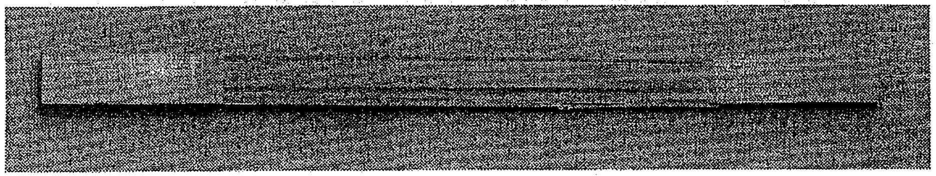
  S-N curves and residual strength
- Development and testing of specimen with integrated end tabs
- Testing of narrow strip specimens (3 mm wide)



## Fatigue testing of unidirectional laminates



Unidirectional laminate specimen with bonded end tabs

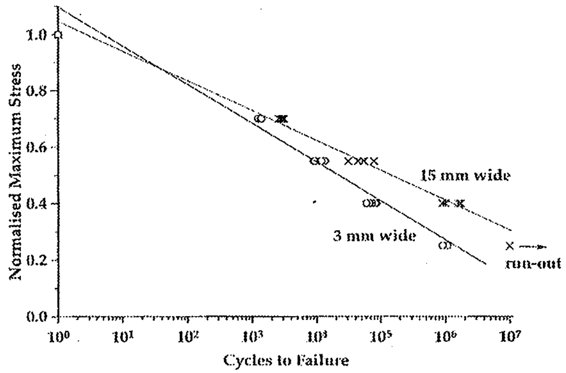


Failed unidirectional laminate specimen



## Fatigue testing of unidirectional laminates

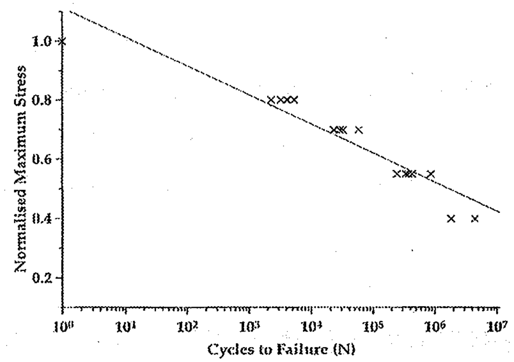
Normalised S-N curve for Unidirectional E-glass/F922 epoxy





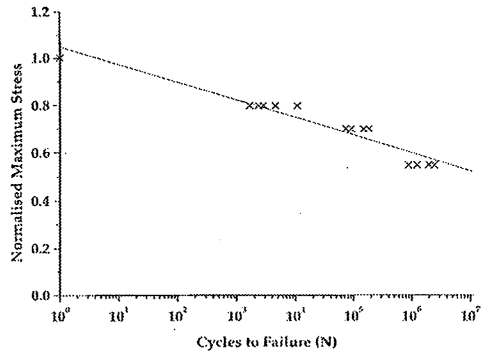

## Fatigue testing of unidirectional laminates

Normalised S-N curve for unidirectional E-glass/913 epoxy (R = 0.1)



## Fatigue testing of unidirectional laminates

Normalised S-N curve for unidirectional E-glass/913 epoxy (R = 0.5)






## Fatigue testing of unidirectional laminates

Conclusions

- Carbon-fibre based systems have excellent fatigue performance

  Endurance limit > $10^7$ cycles at 70-80% of UTS
- S-N response of continuous unidirectional glass-fibre based systems at ambient can be approximated by:

  $\sigma_{MAX}/\sigma_{UTS} = 1 - k \log N_f$

  where the fractional loss in tensile strength per decade of cycles k decreases as the R ratio increases (k = 0.1 for R = 0.1 and k = 0.075 for R = 0.5).
- Narrow strips (3 mm wide) show poor fatigue performance



## Fatigue testing of multidirectional laminates

Research Activities

- Static and cyclic fatigue testing of $[0/90_n]_s$ laminates

  E-glass/913, E-glass/F922 and HTA/F922

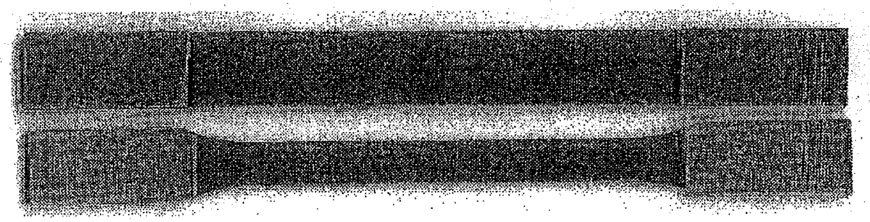
  R = 0.1 (sinusoidal/5Hz)

  S-N curves and residual strength

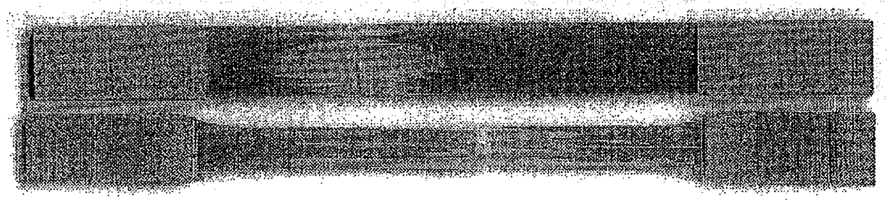
  Transverse crack measurement and stiffness monitoring with loading cycles
- Development and testing of straight edge and waisted specimens



## Fatigue testing of multidirectional laminates

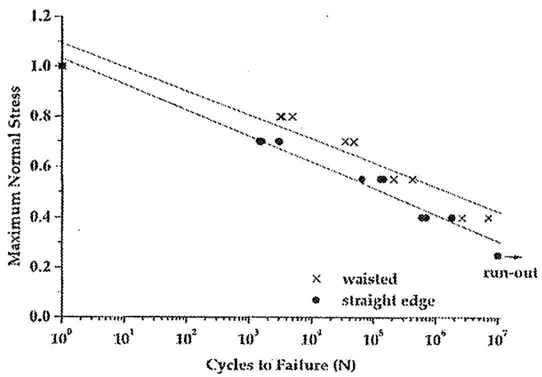


Multidirectional laminate specimens with bonded end tabs



Failed multidirectional laminate specimens

## Fatigue testing of multidirectional laminates






## Fatigue testing of multidirectional laminates

Conclusions

- Cyclic fatigue testing of E-glass/913 $[0/90]_s$ laminates

  Waisted specimens vs straight sided specimens

  - higher costs

  - superior fatigue performance

- S-N response of continuous unidirectional glass-fibre based systems at ambient can be approximated by:

  $\sigma_{MAX}/\sigma_{UTS} = 1 - k \log N_f$

  where k = 0.1 for R = 0.1.

## Fatigue Research Wish List for Composite Wing

*Dr Martin Gaitonde* (BAe SYSTEMS)

BAE SYSTEMS

# Fatigue Research Wish List for Composite Wing

**Martin Gaitonde**

**BAE SYSTEMS, Airbus**

*A presentation at CPD Theme Group Meeting 4th April 2000, NPL*

BAE SYSTEMS

## Contents:-

- Background
  - Typical fatigue exposure for large civil aircraft
  - "No growth" and its benefits
- AMCAPS II research
  - Development of method
  - Material screening
- Wish List



BAE SYSTEMS

## Background

- **Typical fatigue exposure for large civil aircraft**

  Typical once per flight (fatigue) load = 1g + 10fps

  Typical ultimate (static) load = 1.5 x 2.5g Manoeuvre = 3.75g

  Typical fatigue/static ratio = 30 - 40 %

  This represents a once per flight load

  Design Life ~ $10^5$ flights

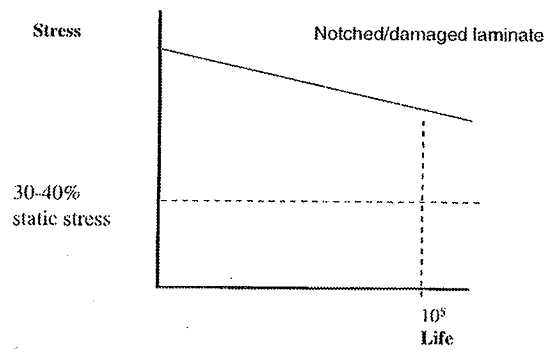
  (Additional cycles due to Gust & Ground Loading must also be included)



BAE SYSTEMS

## Background

- Typical fatigue exposure superimposed on SN graph






BAE SYSTEMS

## Background

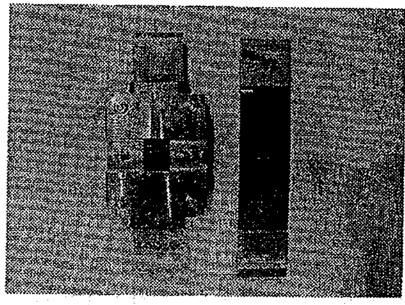
- **"No growth" and its benefits**
  - No growth of damage due to impacts or manufacturing defects
  - No growth ~ No loss of strength
  - CFRP designed for static strength meets requirement, in general
  - "No growth" avoids impractical inspection schedule
    - Impact damage difficult to detect
    - Visual inspection for damage growth impossible
    - Rapid damage growth/ frequent inspections
  - Potential inefficiency introduced through reduction of static CAI allowables needed to meet probabilistic damage tolerance objective
-

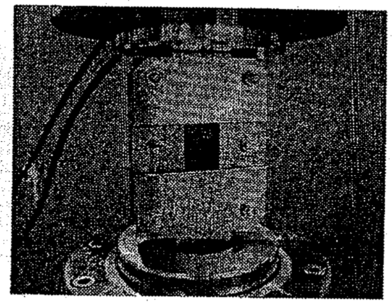
## AMCAPS II research

- **AMCAPS II Main Task 06: BAE SYSTEMS, DERA**
- **MT06 Testing**
  - Static Strength (RTD, CD, CW, HW, HD)
  - Fatigue Life (RTD) – SN Line R=-1, 10Hz
  - Residual Strength after Fatigue (RTD, CD, CW, HW, HD)
  - Residual Life after Fatigue - 2 block loading (RTD)
    - Non- linear damage accumulation
  - Spectrum Loading: Life and Residual Strength (RTD)
  - Low frequency cycling, 10 – 0.01 Hz



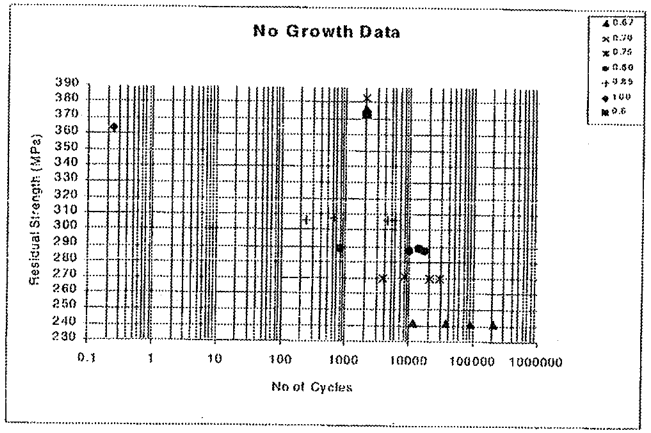
## AMCAPS II research







## AMCAPS II research






## AMCAPS II research

- **Overview Of MT06 Fatigue Analysis Method**

CONVERT SPECTRUM STRESS CYCLES TO R=-1 USING GOODMAN APPROXIMATION

SUM DAMAGE FOR EQUIVALENT (R=-1) STRESS CYCLES

**COMPARE DAMAGE AGAINST ALLOWABLE FOR "NO GROWTH"**

DERIVE MAX. ALLOWABLE DAMAGE FOR "NO GROWTH" BY RESIDUAL STRENGTH TESTING AFTER FATIGUE

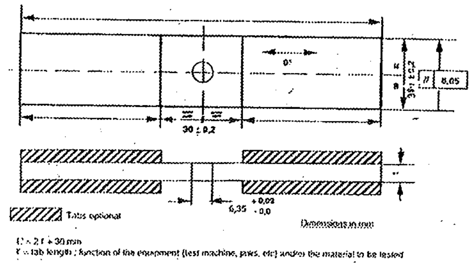
CARRY OUT FATIGUE TESTING: SN AND RESIDUAL STRENGTH AFTER FATIGUE

SELECT APPROPRIATE FATIGUE COUPON CONFIGURATION

BAE SYSTEMS

## AMCAPS II research

- **Material screening against fatigue (AMCAPS II Main Task 01)**
  - Residual strength test after "fatigue conditioning"
    - Applied stress = 50% of OHC static strength
    - R = -1 for $2x10^5$ cycles (5Hz)
  - Aim to show "no loss of strength" (cf. unconditioned specimens)
    -

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BAE SYSTEMS

## Wish List

- **In plane fatigue**
  - Impacted plain laminate (esp. low $0^0$ content), multiaxial
  - Impacted ply dropped laminate
  - Impacted ply jointed laminate
  - Defects in low cost materials & due to new processes
  - Bolt bearing (hole extension) in "low cost" laminates
  - Screening for further new materials
-



BAE SYSTEMS

## Wish List

- **Through thickness fatigue**
  - Bonded joints (eg. stiffener run-outs, pre-cured inserts)
  - Free edges
  - Buckled structure
  - Curved laminate
  - Laminate in bending
  - Tension bolting
-
-
-
-



BAE SYSTEMS

## Wish List

- **Also...**
  - Bolt fatigue
  - Impact dent depth relaxation
  - Evaluation of load/life factors for various structural details/materials (Lameris, NLR TP 90068 U)
-
-
-

## Cost Effective High Cycle Fatigue Testing

*Dr Rod Martin* (MERL)

# HIGH CYCLE FATIGUE CHARACTERISATION OF COMPOSITES

Roderick H. Martin

Materials Engineering Research Laboratory, Ltd.
Hertford, SG13 7DG UK

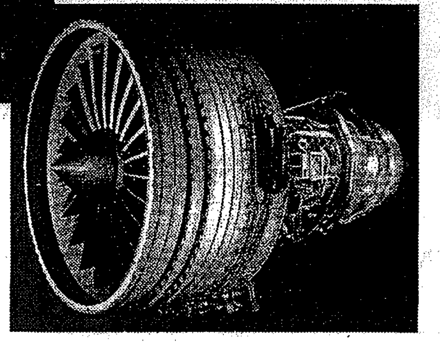
Tel. +44 (0)1992 500120
Fax +44 (0)1992 586439
rmartin@merl-ltd.co.uk
www.merl-ltd.co.uk

Fatigue Performance of Polymer Matrix Composites
CPD Theme Group Meeting
April 4th 2000



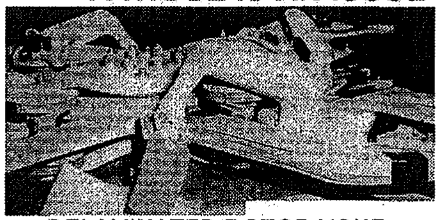
# OTHER HIGH CYCLE APPLICATIONS

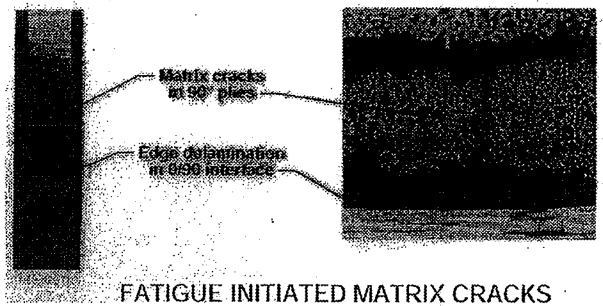

BELL HELICOPTER ROTOR SYSTEM


GE90 FAN BLADES



# FATIGUE IS AN ISSUE WITH COMPOSITES

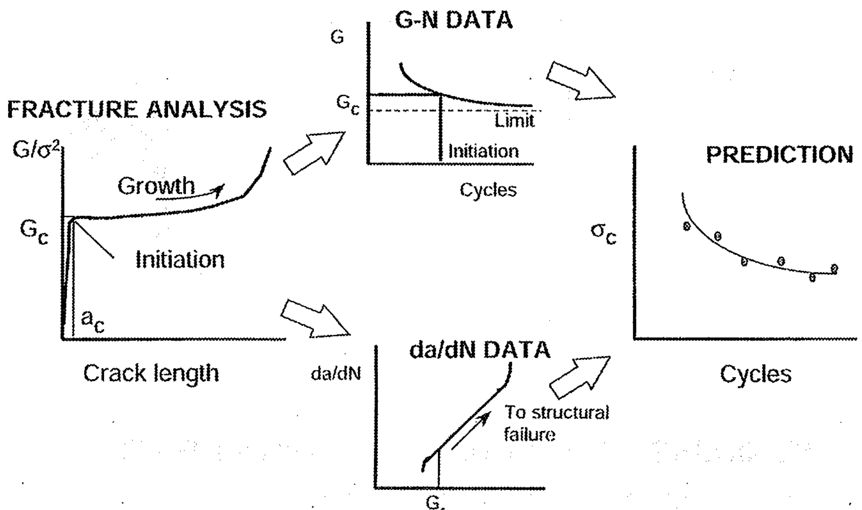

DELAMINATED ROTOR YOKE



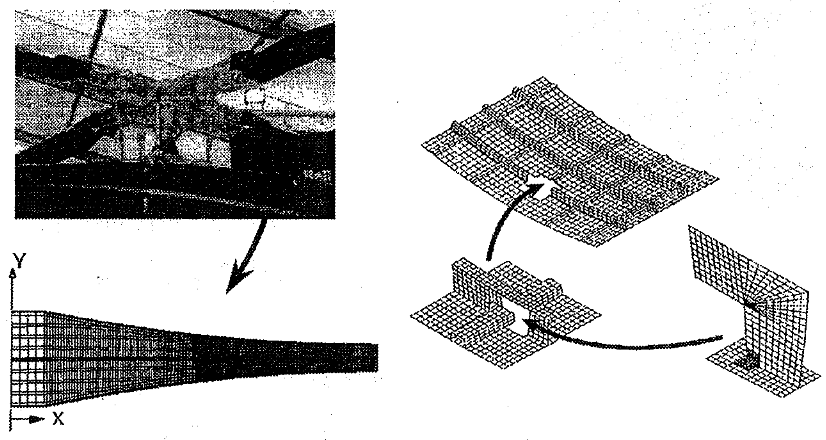
FATIGUE INITIATED MATRIX CRACKS AND DELAMINATION



# DELAMINATION PREDICTION



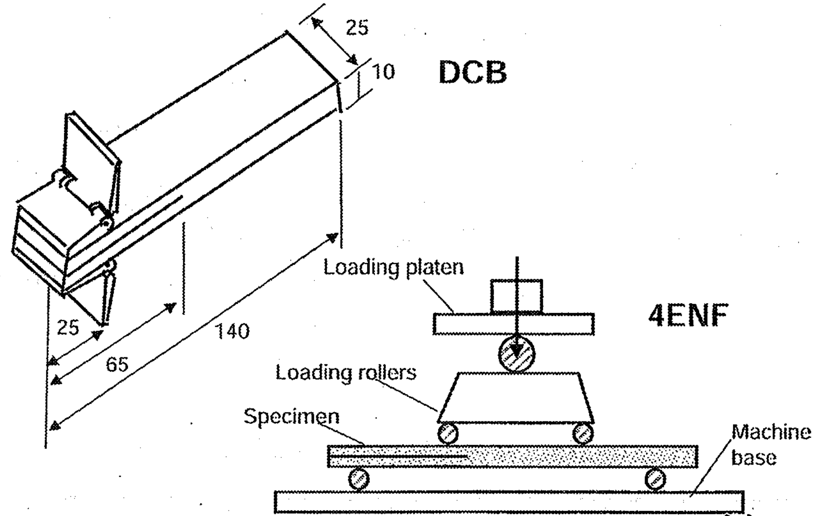
## GLOBAL/LOCAL MODELING




G.B. Murri et al "Fatigue Life Prediction of Tapered Composite Laminates" 53rd AHS Meeting, May 1997



## INTERLAMINAR FRACTURE TEST SPECIMENS



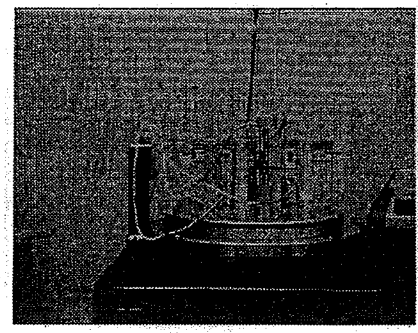


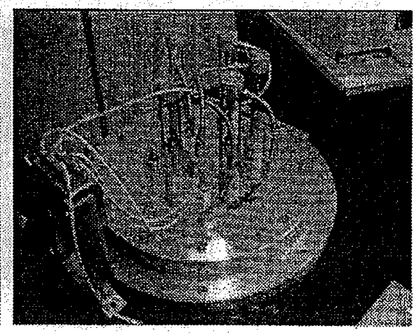

## COST EFFECTIVE LONG TERM FATIGUE TESTING

- Fatigue tests need to be conducted to $10^8$ cycles and beyond with statistical accuracy if to be used for design
- Methods to reduce fatigue test costs include:
  - Increase frequency
  - Multiple specimen testing
  - Electro-mechanical over servo-hydraulic
  - Automated data acquisition
- Typical G-N curve 4 specimens each at $10^3$, $10^4$, $10^5$, $10^6$, $10^7$, $10^8$ cycles
  Total of 4.4x $10^8$ cycles at 5Hz = 2 years 10 months
- Increase frequency to 20 Hz and test 4 specimens at a time
  Total of 1.1x $10^8$ cycles at 20Hz = 2 months
- Reduce labour costs through automated data acquisition
- Reduce operational costs through Electro-mechanical operation

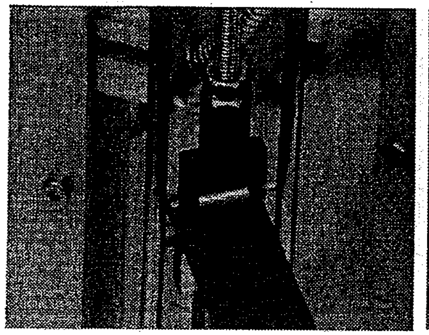


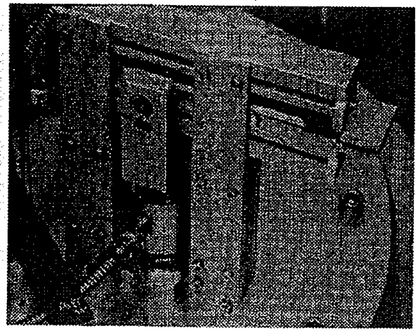
## ELECTRO-MECHANICAL MULTI-STATION MACHINE





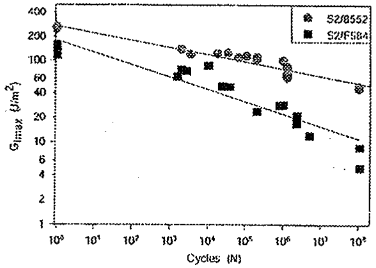
## DCB AND MMB ON MULTI-STATION MACHINE

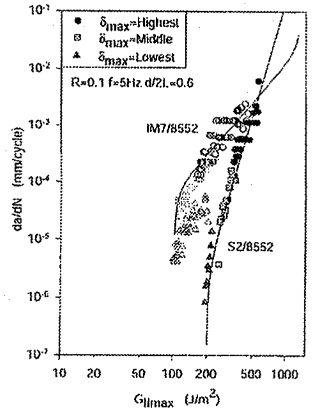






## FATIGUE DATA



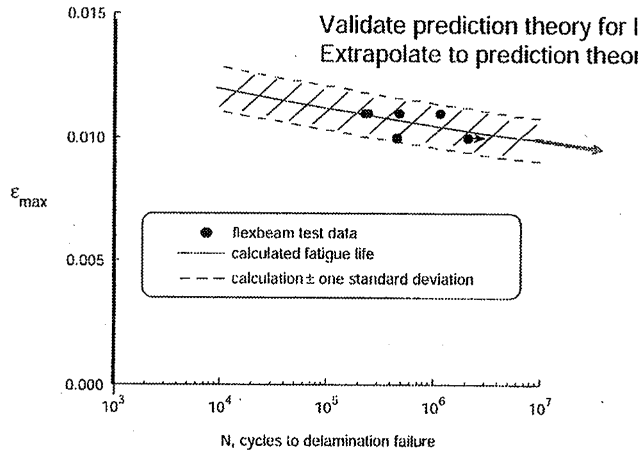

$$G_{\text{IImax}} = 10^{r}\left(N_{onset}\right)^{s}$$

$$\frac{da}{dN} = 10^{A}\left(G_{\text{IImax}}\right)^{B}\frac{\left[1-\left(\frac{G_{IIth}}{G_{\text{IImax}}}\right)^{D}\right]}{\left[1-\left(\frac{G_{\text{IImax}}}{G_{IIc}}\right)^{D}\right]}$$



## ROTORCRAFT APPROACH FOR HIGH CYCLE DELAMINATION

Validate prediction theory for low cycles
Extrapolate to prediction theory to high cycles




G.B. Murri et al "Fatigue Life Prediction of Tapered Composite Laminates" 53rd AHS Meeting, May 1997



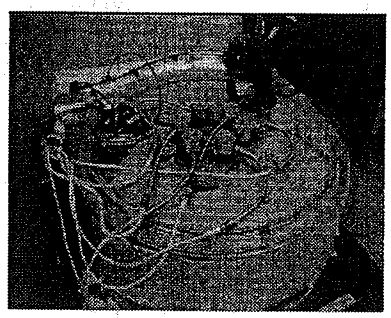
## EXTENDING THE APPROACH

**MATERIALS & FRACTURE MODES**
- 0/45 Interface
- Glass/Carbon hybrids

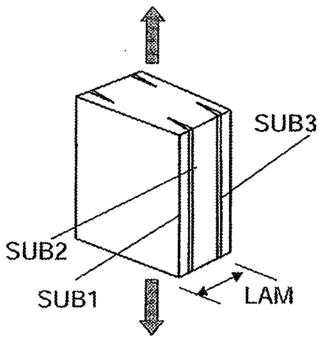
**EFFECT OF ENVIRONMENT**
- High humidity
- Salt spray
- Elevated temperature

**SPECTRUM LOADING**
- Effect of load history
- Variable loading
- Multi-directional loading

**AGEING**
- Accelerated ageing of material



## EDGE DELAMINATION UNDER TMF




$$G = t_{LAM}u_{LAM} - \int_1^{ns} t_{SUB}u_{SUB}$$

$$u = \text{strain energy density} = \frac{U}{blt}$$

$$U = \frac{1}{2}\int_V \varepsilon_{ij}\sigma_{ij}dV$$

$$U = \frac{bl}{2}\sum_k^N \int_{z_{k-1}}^{z_k} |\varepsilon|_k |\sigma|_k dz$$

$$|\varepsilon| = |\varepsilon^0|^M + |\varepsilon^0|^T + z\{|k|^M + |k|^T\}$$



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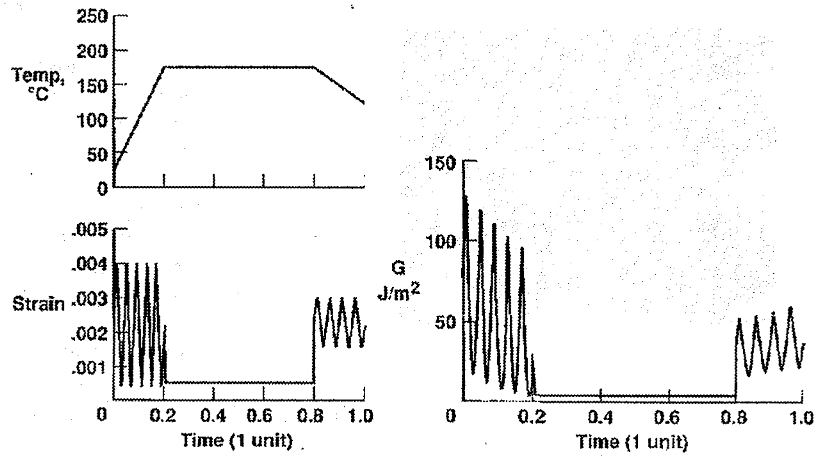
## G FOR A SIMULATED FLIGHT PROFILE



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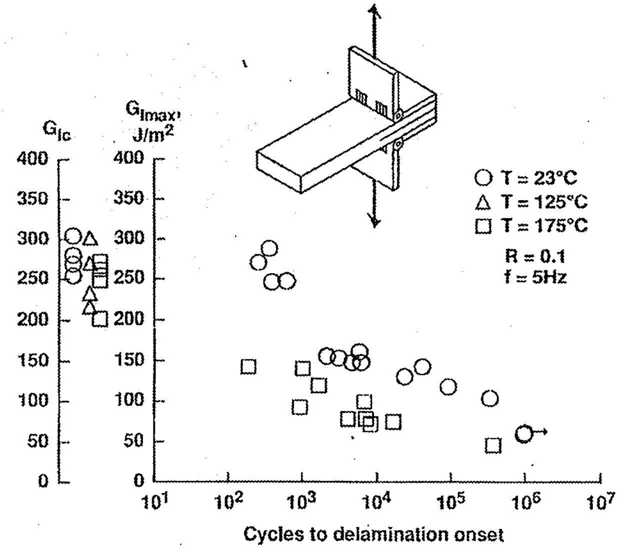
## DCB DATA AT HIGH MACH NUMBERS



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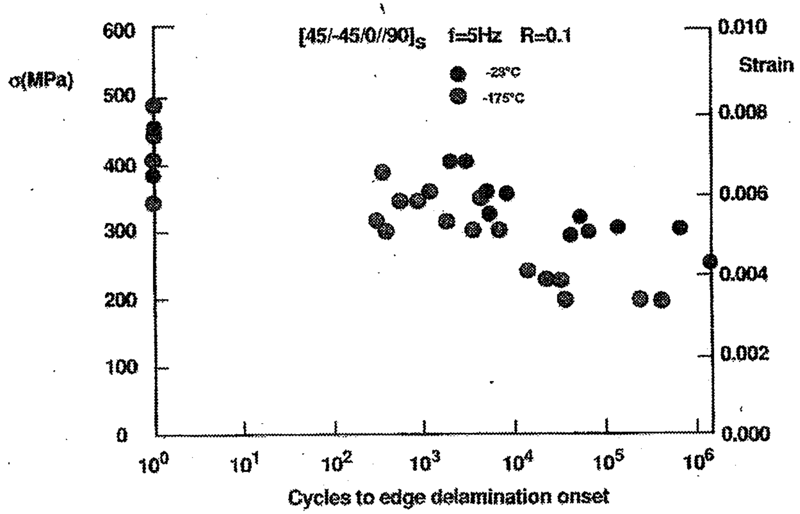
## LAMINATE DELAMINATION DATA



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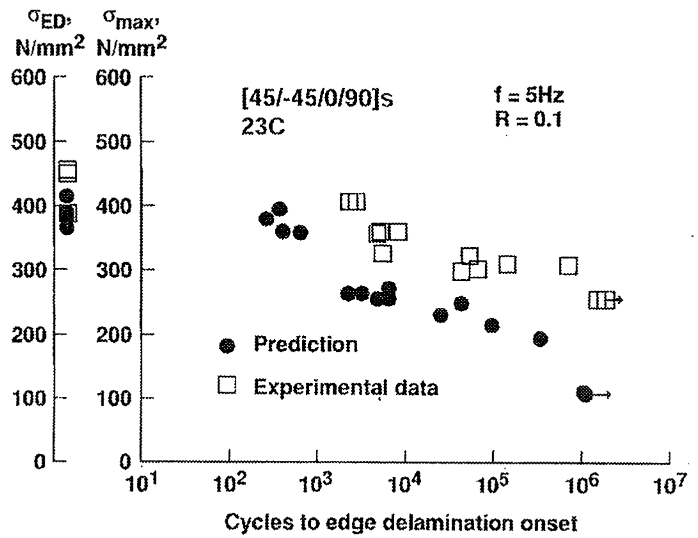
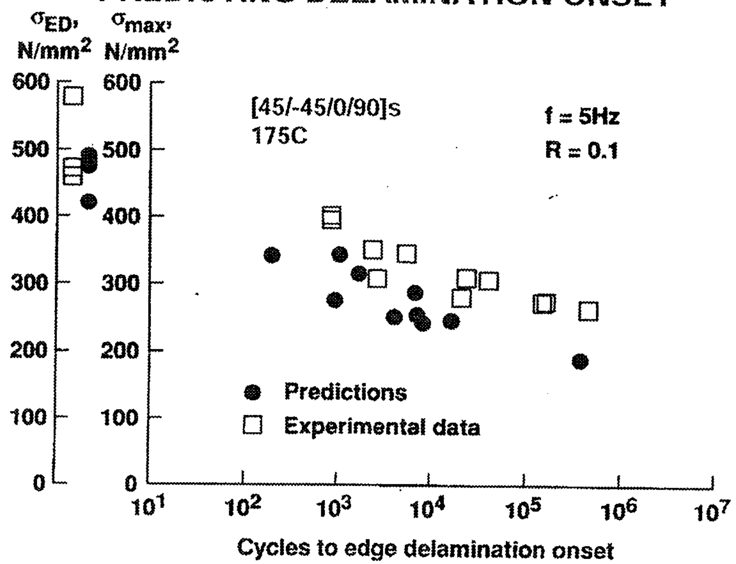
## PREDICTING DELAMINATION ONSET






## PREDICTING DELAMINATION ONSET






## CONCLUSIONS

Fracture mechanics should be used where
- Delamination/cracking is a primary damage mode
- Stress singularities exist
- Damage tolerance is an issue

Global/Local modeling allow the complexity of fracture modeling to be achieved

Closed form solutions may give first order predictions

Automated crack growth modelling aids use of fracture mechanics

Multi-station equipment used for validating safe life (damage tolerance) at reduced costs

# Fatigue Performance of Composites with Terminating Plies

*Prof. Mike Wisnom* (BRISTOL UNIVERSITY)

# DELAMINATION IN UNIDIRECTIONAL COMPOSITES WITH TERMINATING INTERNAL PLIES UNDER TENSION FATIGUE LOADING

**Michael R. Wisnom, M.I. Jones, Weicheng Cui**

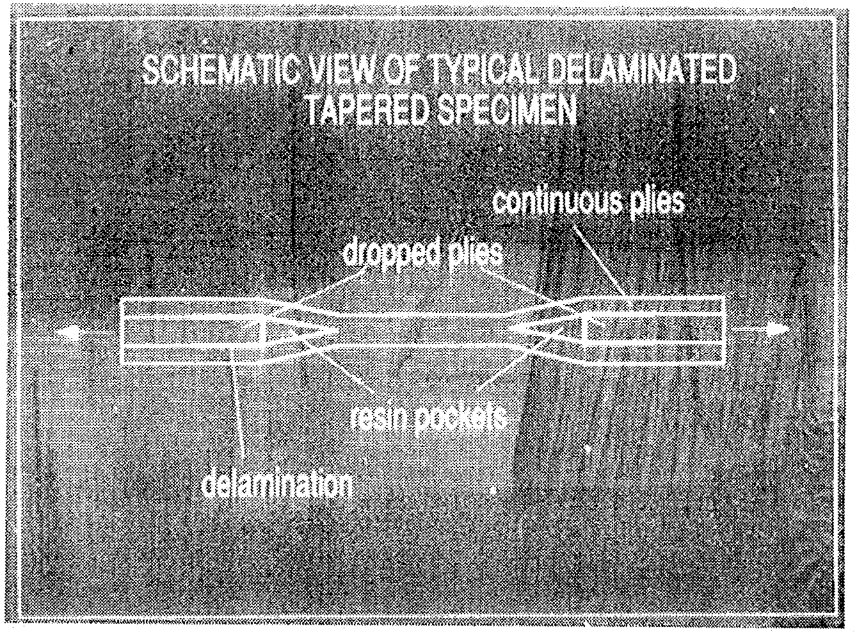
Department of Aerospace Engineering
University of Bristol

## Factors Investigated

- Number of terminating plies
- Number of continuous plies
- Orientation of terminating plies
- Effect of taper
- R ratio
- Stress rate
- Different materials

## Test Procedure

- Static tests at 0.5 mm/min
- Fatigue tests under load control
- $R = 0.1$ for most tests, some at $R = 0.5$
- Max stress 35-90% of static
- Constant stress rate, 5-20 Hz
- Most tests $10^3 - 10^6$ cycles, some to $10^7$



| Continuous 0° plies | Discontinuous Plies | Glass or Carbon | No of Specimens | Average Delamination Stress (MPa) | c.v. (%) | Fracture Energy (N/mm) |
|---|---|---|---|---|---|---|
| 4 | 1 Cut 0° | G | 4 | 1219 | 1.6 | 0.86 |
| 8 | 2 Cut 0° | G | 6 | 966 | 2.2 | 1.08 |
| 8 | 1 Cut 0° | G | 2 | 1298 | - | 1.08 |
| 8 | 4 Cut ±45° | G | 4 | 1356 | 1.9 | 1.57 |
| 8 | 2 Dropped 0° | G | 4 | 1274 | 1.4 | 1.88 |
| 8 | 2 Cut 0° | C | 4 | 1492 | 1.1 | 0.82 |

Table 2: Static Delamination Propagation Results

Calculation of energy release rate

After short delamination, constant G is reached

Simple equation to calculate G

$$G = P^2(k_o-k)/4k_ok$$

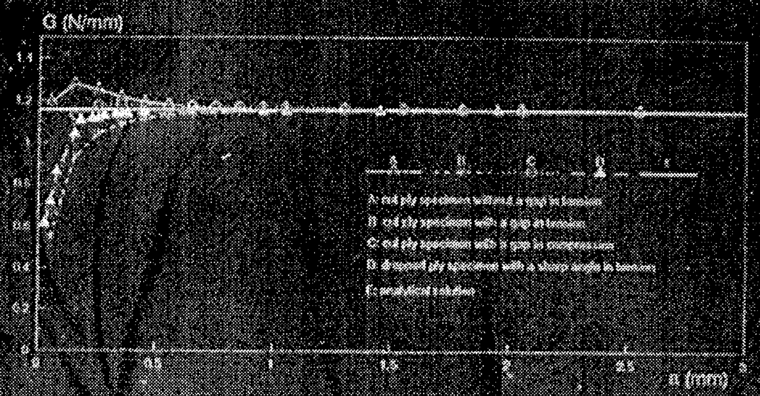



Fig.4 Total strain energy release rate for delamination along the interface between discontinuous plies and continuous plies for glass fibre-epoxy under 1000 MPa applied stress in the thin section

Fatigue Results

Similar failure mechanisms to static

Delamination starts immediately

No apparent threshold

Fibre fatigue for glass-epoxy

Typical results

- Few thousand cycles at 70% severity
- Little effect after $10^7$ cycles at 35%

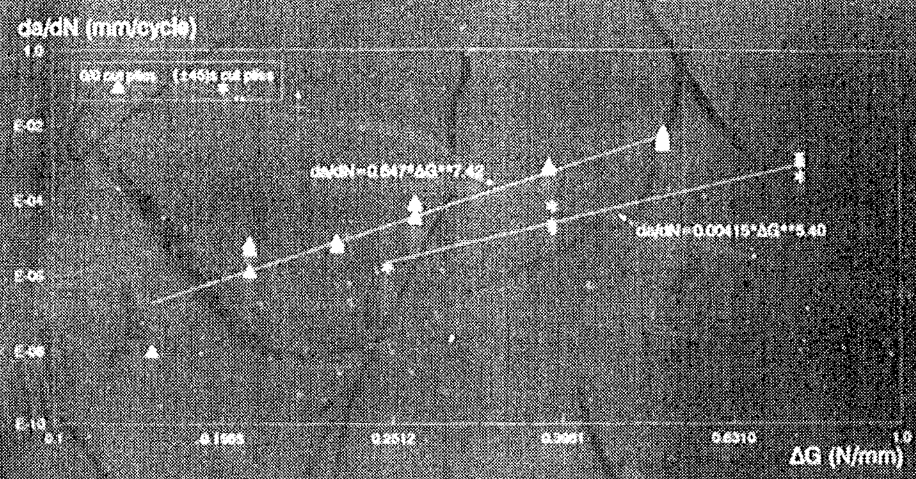



Fig 5 Effect of cut ply orientation, 8 continuous 0 degree plies, glass-epoxy, R=0.1

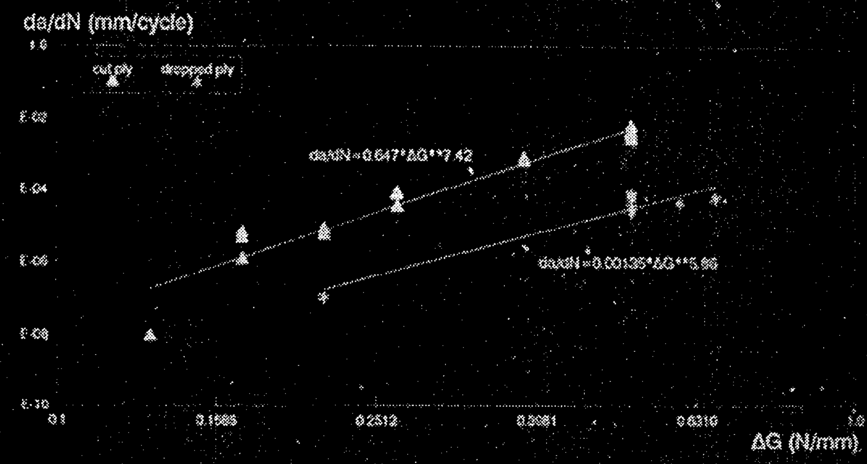



Fig 6 Effect of taper, 8 continuous/2 discontinuous 0 degree plies, glass-epoxy, R=0.1

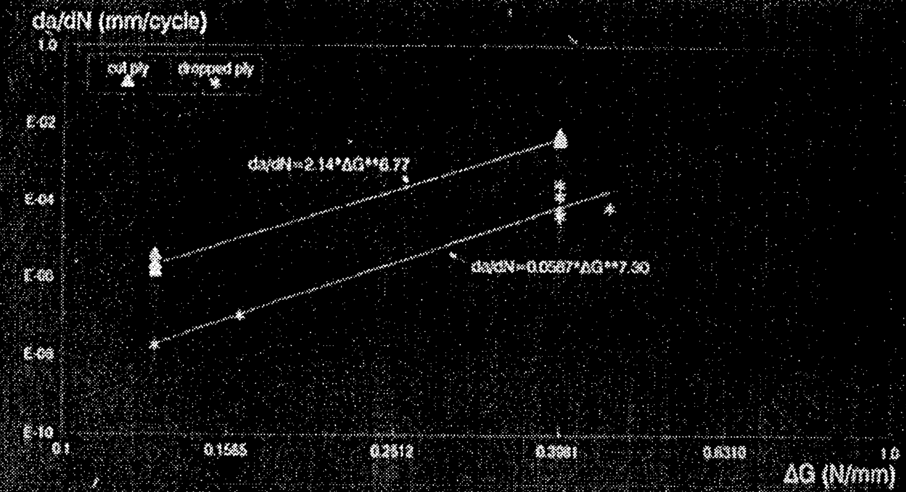



Fig 7 Delamination rates for carbon-epoxy, 8 continuous/2 discontinuous 0 degree plies, R=0.1

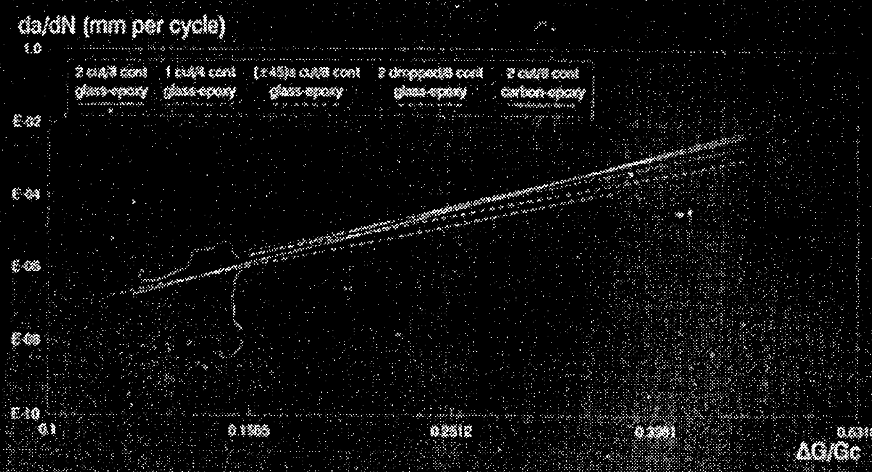



Fig 8 Delamination rates, energy release rate normalised by static value

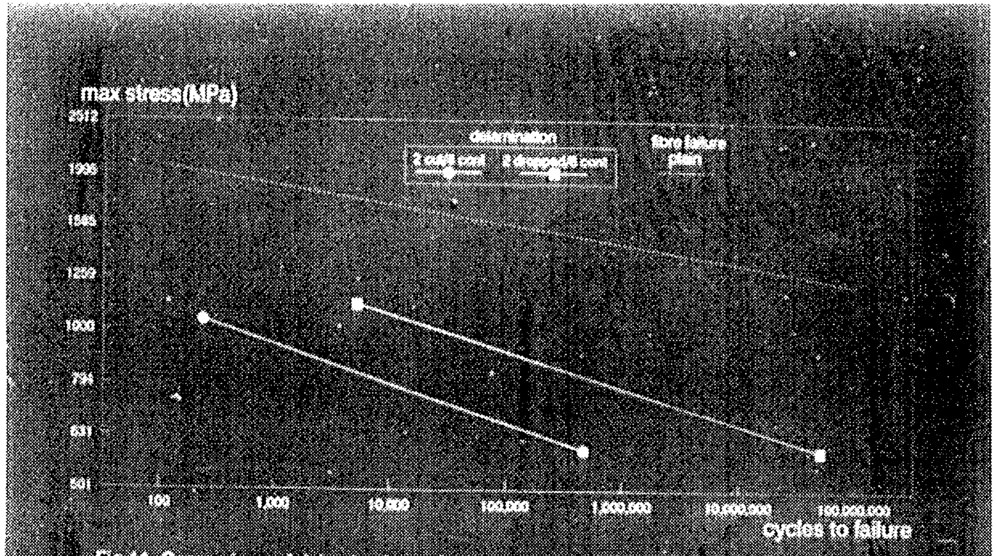



Fig 11 Comparison of delamination and fibre fatigue, carbon-epoxy, R=0.1

## Conclusions

Failure by delamination between continuous and discontinuous plies

Delamination starts immediately

No apparent threshold

Low R ratio most damaging

Stress rate not significant

## Conclusions (continued)

Paris law fits data well

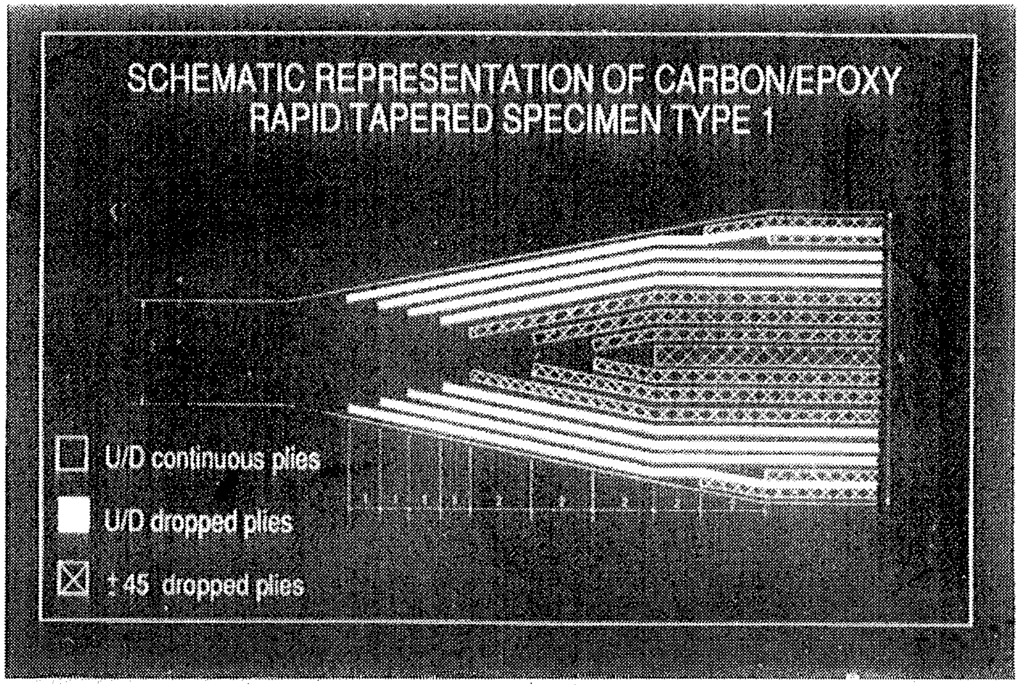
Different specimens have different da/dN

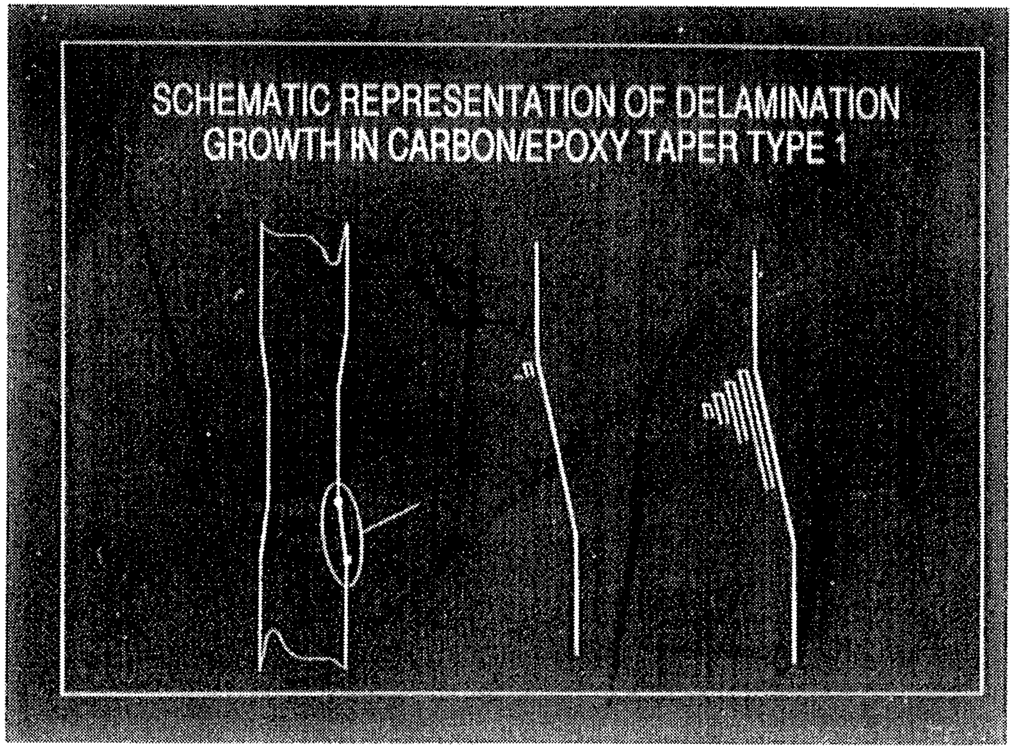
da/dN very similar for same $\Delta G/G_c$

Number of discontinuous plies very important

Terminating 0° plies much worse than ±45°

Taper is beneficial






## FATIGUE OF TAPERED CARBON FIBRE SPECIMENS

| Specimen type | Max stress (MPa) | No. of cycles | Predicted delam. (mm) | Longest crack(mm) |
|---|---|---|---|---|
| 1 | 1063 | 10,000 | 1.0 | 2.4 |
| 1 | 757 | 1,000,000 | 1.0 | 1.3 |
| 1 | 638 | 10,000,000 | 1.0 | 0.5 |
| 2 | 928 | 10,000 | 1.0 | 1.7 |
| 2 | 660 | 2,319,501 | 2.3 | 1.1 |

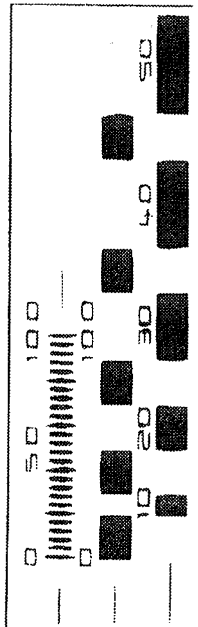
# Standardisation Issues

*Dr. Graham Sims* (NPL)

# STANDARDISATION ISSUES FOR FATIGUE TESTING

**Graham D Sims**

Centre for Materials Measurement and Technology
National Physical Laboratory



*4 April 2000*

NPL National Physical Laboratory

# OUTLINE

- 1. Introduction
- 2. Requirements for fatigue tests
- 3. Standards and test methods
- 4. Fatigue data frameworks
- 4. VAMAS round-robin
- 5. ISO fatigue standard
- 6. Conclusions



# ENs - EUROPEAN STANDARDISATION

- Mandatory to publish and withdraw other standards of same scope
- ISO test methods to be adopted if available
- Fitness for purpose
  - Lowest cost consistent with the provision of relevant and reliable data
  - Low scatter and consistent means
- Vienna agreement for shared working arrangements and joint parallel ballots between ISO and CEN, ASTM and JIS important
- CENs main interest is specifications for free trade and product liability
- Material standards - CEN TC249/SC2 (WG5 Test methods) (WG6 Pultrusions)
- Product standards - CEN TC 155(pipes), TC210(pressure vessels), TC 23(cylinders)



# HARMONISED STANDARDS

- **BS EN ISO 527 Part 4** (ISO 3268/ISO 527/ASTM D3039/CRAG 300) — Tension - "Isotropic materials"
- **BS EN ISO 527 Part 5** (ISO 3268/ISO 527/ASTM D3039/CRAG 300/EN2561) — Tension - "Unidirectional materials"
- **BS EN ISO 14125** (ISO 178/ASTM D 790/CRAG 200) — Three and four point flexure
- **BS EN ISO 14126** (ISO 8515/ASTM D3410/CRAG 400) — Compression
- **BS EN ISO 14129** (ISO 4585/CRAG 100) — Interlaminar (short beam) shear
- **BS EN ISO 14130** (ASTM 3518/CRAG 101) — In-plane shear by ± 45° tensile test

## INTERNATIONAL STANDARDS

- ISO 15586 Mode I Fracture Toughness
- ISO 15310 In-plane shear modulus by plate-twist
- ISO 13003 Fatigue - General principles
- ISO/FDIS 1268 Manufacture of test plates (ten parts)
- ISO NWI Open-hole tension
- ISO NWI Open/filled hole compression
- ISO NWI Pin-bearing test
- ISO NWI Double notch shear



## ISO 1268 - TEST PANEL MANUFACTURE

- Part 1 - General principles
- Part 2 - Contact and spray-up moulding
- Part 3 - Wet compression moulding
- Part 4 - Moulding of pre-impregnates
- Part 5 - Filament moulding
- Part 6 - Pultrusion moulding
- Part 7 - Resin transfer moulding
- Part 8 - Moulding of SMC/BMC
- Part 9 - Moulding of GMT/STC
- Part 10 - Injection moulding of BMC/DMC

Need for machining/tabbing guidance (DERA/NPL)



## USES OF FATIGUE TESTS

- to compare materials under standard conditions as a basis of materials development or selection procedures;
- to obtain design data relevant to service applications;
- to investigate the mechanisms of failure leading to degradation and failure.



## FATIGUE STANDARDS

- ASTM D3479 - Relates to tensile standard ASTM D 3039
- AFNOR T51-120 - Flexural fatigue (several modes)
- CRAG Recommendations - Related to static specimens
- ISO Proposal

## FATIGUE TEST SELECTION

- **The stress state to be applied**
  - multiaxial (biaxial, flexure), single (tension, compression, shear);
- **The control mode to be applied**
  - load, displacement.
- **The failure criteria to be applied**
  - fracture (micro, final), stiffness loss;



## TENSION - Load control

**• ADVANTAGES**

- Pure stress state,
- Clearly defined failure point,
- Suitable for non-homogeneous, laminated materials,
- Micromechanics analysis possible,
- Large stressed volume.

**• DISADVANTAGES**

- Complicated equipment,
- High load capacity needed,
- Tabs needed for aligned material,
- Stress concentrations at grips/tabs.



## FLEXURE - Displacement control

**• ADVANTAGES**

- Simple equipment
- Low load requirement
- Multiple stations possible

**• DISADVANTAGES**

- Complex stress state,
- Final failure not clearly defined-alternative failure criteria needed,
- Only suitable for homogeneous materials,
- Fretting can occur at support points,
- Compression, tensile, shear or mixed failure modes can occur,
- Small specimen volume at the maximum flexure stress



## VAMAS

- **VERSAILLES PROJECT ON ADVANCED MATERIALS AND STANDARDS**
- **International pre-standards research aimed at providing technical base for drafting codes of practice and specifications for advanced materials**
- **G7 (8) countries support + others**
- **Scope includes all enabling science and technology**
  - **Databases, design and test methods, materials technology**
  - **Research, inter-comparisons, recommended methods**

## VAMAS FATIGUE ROUND-ROBIN

- Tension and flexure modes
- Undirectionally reinforced
- Carbon and glass fibres reinforcements
- Epoxy and PEEK matrices
- UK, France and Japan material suppliers
- International participants
- 10mm wide specimens



## ISO/DIS 13003

## FIBRE REINFORCED PLASTICS: DETERMINATION OF FATIGUE PROPERTIES UNDER CYCLIC CONDITIONS

- Project leader: Dr Patricia Krawczak (France)
- Supporters:
  - Dr Gene Camponeschi (USA)
  - Dr Graham D Sims (UK)



## ISO 13003 - DETERMINATION OF FATIGUE PROPERTIES UNDER CYCLIC CONDITIONS - 1

- Constant frequency and amplitude
- Load or displacement control
- References "static" specimen conditions
  - Tension and flexure as examples
- Failure criteria
  - "rupture" for load,
  - "stiffness" for displacement
- Start-up conditions (transition)
  - Initial force and cycles to apply
- R ratios (stress/load, strain/displacement)



## ISO/DIS 13003 - DETERMINATION OF FATIGUE PROPERTIES UNDER CYCLIC CONDITIONS - 2

- ? Inertia effects on load measurement
- ? Recommended frequencies (Hz)
  - Frequency dependent materials
  - Autogeneous heating
- ? Waveform shape (sine, triangular, square)
- ? Recommended R ratios
  - eg 0.1 for tension-tension
- ? Recommended number of stress levels (4)
- ? Recommended replicates (5, as static)
- ? Residual strength and stiffness

## ISO/DIS 13003 - DETERMINATION OF FATIGUE PROPERTIES UNDER CYCLIC CONDITIONS - 3

- Microdamage v. macrocracks
  - delamination crack growth
- Continuous monitoring
  - themography/thermocouple
  - acoustic emission
  - dynamic mechanical properties
  - radiography
  - ultrasonics
  - translucency
  - optical/electron microscopy



## ISO/DIS 13003 - DETERMINATION OF FATIGUE PROPERTIES UNDER CYCLIC CONDITIONS - 4

- Cost effective and accurate testing
  - Reference to "static" specimens
  - Machine alignment
  - Initial material quality
  - Specimen manufacture quality
  - Machine and services reliability
  - Use ISO 3502 for non-ambient test temperatures
  - Check for autogeneous heating



## ISO 3502 TEMPERATURES (°C)

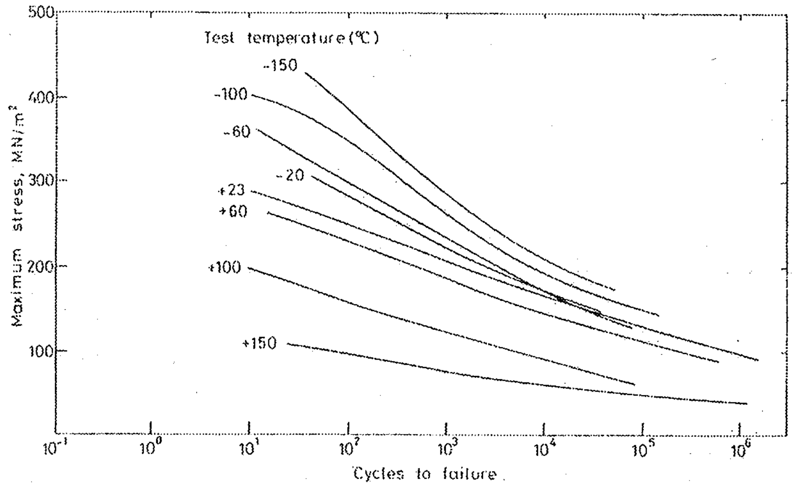
| | |
|---|---|
| - 269, - 196, - 161 | |
| - 70 to - 15 | every 15 |
| 0 , 5 | |
| 25 to 85 | every 15 |
| 100 , 105 | |
| 125 to 300 | every 25 |
| 350 to 450 | every 50 |
| 500 to 1000 | every 100 |



## FATIGUE DATA FRAMEWORK

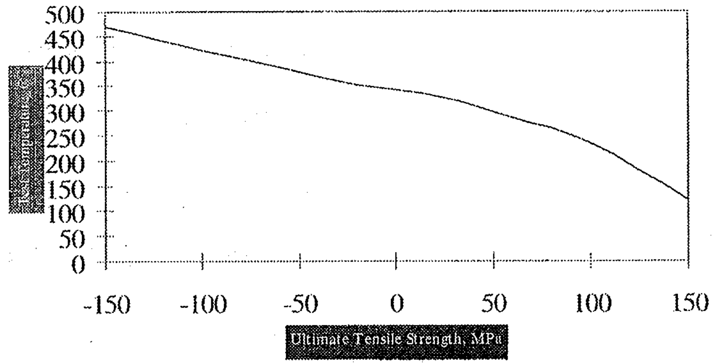
- Effect of rate/frequency
- Effect of temperature
- Effect of holes/notches
- Effect of water treatments
- Damage monitoring
- Scatter bands

## EFFECT OF TEST TEMPERATURE



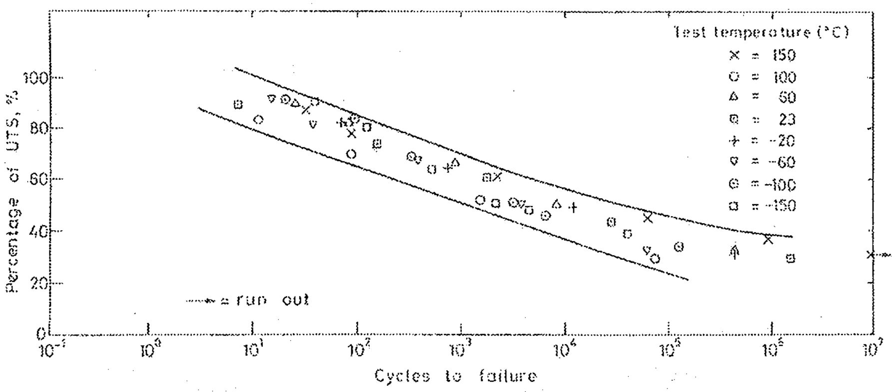



## UTS AS FUNCTION OF TEMPERATURE






## NORMALISED TEMPERATURE EFFECT






## CONCLUSIONS

- **The progress made on static test methods is being extended to fatigue testing**
- **Review versions available at "CD" and "DIS" level for ballot and vote via national standards bodies**
- **Information on composites standards in development and/or published will be posted on NPL composites web page**
  - **http://www.npl.co.uk/cog/index.html**

# Some Composite Modelling Issues for the Prediction of Fatigue Performance

*Dr. Neil McCartney* (NPL)

Fatigue performance of Polymer Composites

## Some composite modelling issues for the prediction of fatigue performance

**Neil McCartney**

**Centre for Materials Measurement and Technology**

**National Physical Laboratory, UK**

***4 April 2000***

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## Content

- Introduction & brief review of some important issues relating to prediction of fatigue performance
- Description of progress modelling fatigue damage development in composite laminates at the material (rather than structural) level
  - ➡ fatigue growth of ply cracks
  - ➡ predicting static failure
  - ➡ static fatigue
- Conclusions / future needs for modelling at the material level

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## Conventional failure criteria

$$f(\sigma_i) = F_i \sigma_i + F_{ij} \sigma_i \sigma_j + F_{ijk} \sigma_i \sigma_j \sigma_k \leq 1$$ Tsai - Wu (1971)

- phenomenological in nature - no physics !
  - ➡ based on invariance requirements
- applied to stress states for structures where no damage has been allowed for
- not easily applied to environmentally or fatigue damaged composites

Physically based models are needed !

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## Failure modes must be modelled & understood

- Static failure
  - ➡ **we cannot deal with this yet !**
- Static fatigue (especially GRP)
  - ➡ **introduces time dependence**
- Cyclic loading
  - ➡ **uniform amplitude loading**
  - ➡ **periodic random loading (overloads)**
- Combined static & cyclic fatigue
  - ➡ **time & cycle dependence for GRP**
- Complex loading modes
  - ➡ **combined in-plane multi-axial / bending**

***7/8 of Team Philips ?***

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## Stages of fatigue damage development

- Initiation of fatigue damage
  - very difficult to model as mechanisms of degradation are at the micro-level
  - problem has not been solved for metals !!
- Propagation of fatigue damage
  - need to assume the existence of well - defined damage whose growth can be predicted
  - need to predict lifetime and residual strengths
- Catastrophic failure
  - if lifetime is to be predicted then we need to predict when material can no longer support load, or has lost too much stiffness

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## Fundamentals of fatigue crack growth

Applies to :

homogeneous metals

composite fibres

composite matrix (including ply cracks)

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## Microstructural damage growth laws (fibre & matrix)

Static fatigue :

$$\frac{dc}{dt} = C K^n$$

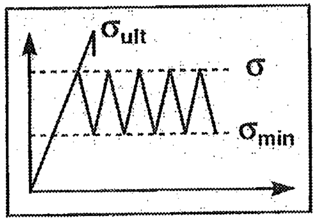
$$K = y\sigma\sqrt{\pi c}$$

Fatigue crack growth :

$$\frac{dc}{dN} = C \Delta K^n$$

$$\Delta K = y \Delta\sigma \sqrt{\pi c}$$

$$\Delta\sigma = \sigma - \sigma_{min}$$




**c** is defect size

**t** is elapsed time from first loading

**N** is no. of fatigue cycles

**y** accounts for defect geometry

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## Fatigue life prediction methodology for metals

- Select initial defect size $c_o$ - assume uniform amplitude loading
  - largest defect size to be missed on inspection
- Use fracture mechanics to predict critical size of defect at failure

$K = K_{Ic}$ or $G = G_{Ic}$ defines critical defect size $c_f$

- Integrate crack growth law to predict lifetime of initial defect size

$$\frac{dc}{dN} = C(\Delta K)^n \Rightarrow N_f = \frac{1}{C}\int_{c_o}^{c_f} \frac{dc}{(\Delta K)^n}$$

**Paris equation**

Can be used for growth of ply cracks / delaminations

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## Issues when using crack growth laws

More accurate growth law :

$$\frac{dc}{dN} = \frac{C\left(\Delta K - \Delta K_0\right)^n}{K_{Ic}^2 - K^2}$$

$\Delta K_0$ is a threshold value for crack growth

Crack growth accelerates to failure as $K \rightarrow K_{Ic}$

Growth law based on energy release rate ?

$$\frac{dc}{dN} = \frac{A\left(\Delta G - \Delta G_0\right)^n}{G_{Ic} - G}$$

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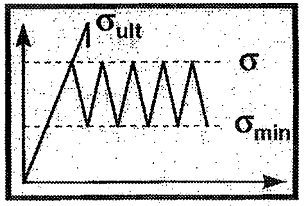
## Stress intensity or Energy release rate ?

Assume $K = y\sigma\sqrt{\pi c}$ $\Rightarrow$ $\Delta K = y\Delta\sigma\sqrt{\pi c}$

$\Delta\sigma = \sigma - \sigma_{min}$

or $G = \frac{\Omega}{E_A} K^2$ **for anisotropic materials**




But how do we define $\Delta G$ ?

$$\Delta G = \frac{\pi y^2 \Omega c}{E_A}\left(\sigma^2 - \sigma_{min}^2\right)$$

or $$\Delta G = \frac{\pi y^2 \Omega c}{E_A}\left(\sigma - \sigma_{min}\right)^2 \equiv \frac{\Omega}{E_A}\Delta K^2$$

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## Growth of ply cracks during fatigue loading

**Cross - ply laminates (multiple - ply)**

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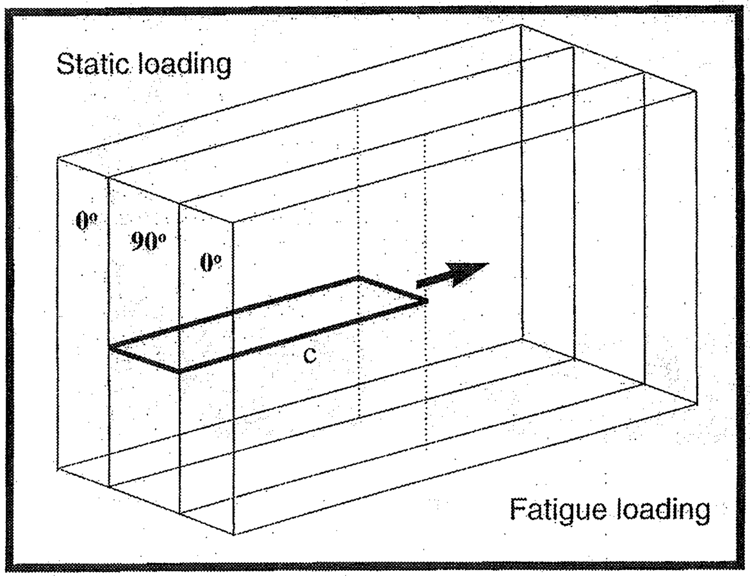
## Fracture mechanics for embedded ply crack




PROBLEM

How to calculate :

stress intensity factor K ?

stress range intensity factor $\Delta K$ ?

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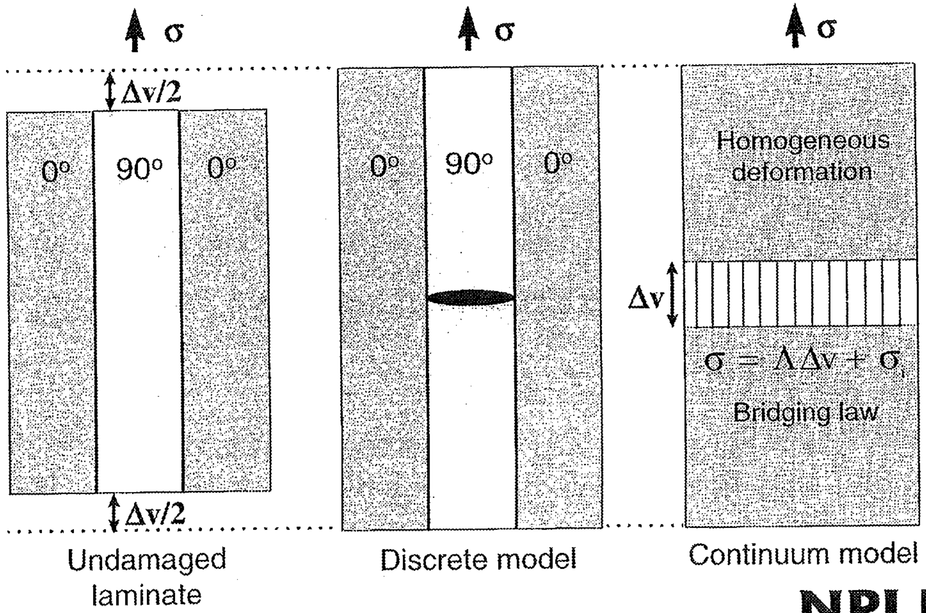
## Edge view of laminate with a 90° ply

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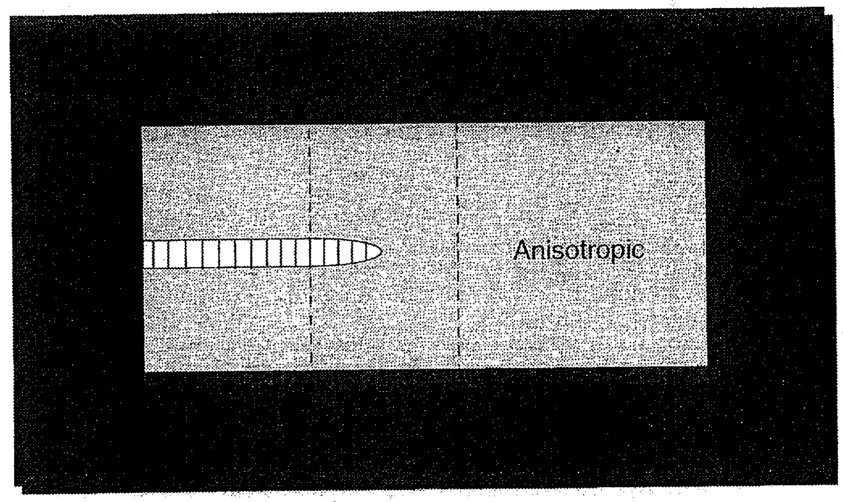
## Face view of crack growth using continuum model

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## Fracture mechanics parameters

Energy balance leads to :

**No crack length dependence**

$$\frac{1}{2\Lambda}(\sigma - \sigma_i)^2 = 2\gamma$$

Effective fracture energy :

$$2\gamma = \frac{2\gamma^{(90)}h^{(90)}}{h}$$

Stress intensity factors :

$$K = \sqrt{\frac{E_A}{2\Omega\Lambda}}(\sigma - \sigma_i)$$

$$K^{(90)} = \sqrt{\frac{hE_T^{(90)}}{2\Lambda h^{(90)}\Omega^{(90)}}}(\sigma - \sigma_i)$$

Stress range intensity factors :

$$\Delta K = \sqrt{\frac{E_A}{2\Omega\Lambda}}\Delta\sigma$$

$$\Delta K^{(90)} = \sqrt{\frac{hE_T^{(90)}}{2\Lambda h^{(90)}\Omega^{(90)}}}\Delta\sigma$$

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## Determination of $\Lambda$ and $\sigma_i$

$$\frac{1}{\Lambda} = \frac{1}{\rho}\left(\frac{1}{E_A(\rho)} - \frac{1}{E_A}\right)$$

$$\sigma_i = -\frac{\alpha_A(\rho) - \alpha_A}{\frac{1}{E_A(\rho)} - \frac{1}{E_A}}\Delta T$$

**ρ** - effective ply crack density

**ΔT** - temperature drop from stress - free temperature

Crack closure stress :

$$\sigma_i = -k_I \Delta T$$

$k_I$ is an undamaged laminate constant

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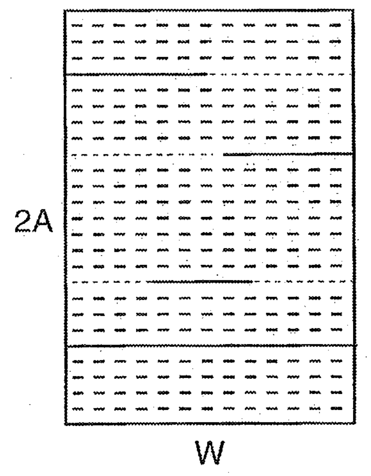
## Fatigue ply cracking methodology




- - - Potential cracking sites

—— Ply crack locations

Potential cracking sites are evenly spaced

Ply cracks are non - uniformly spaced having different lengths

Crack density $\rho = \frac{pc}{AW}$

p is number of original defects having the same initial size $2c_o$

2c is size of defects after N cycles

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## First order model

Defect growth law :

$$\frac{dc}{dN} = C\left[\Delta K^{(90)}\right]^n$$

where

$$\frac{dc}{dN} = \frac{WA}{p}\frac{d\rho}{dN} \qquad \Delta K^{(90)} = \alpha\,\Delta\sigma\sqrt{\frac{1}{\rho}\left(\frac{1}{E_A(\rho)} - \frac{1}{E_A}\right)} \qquad \alpha = \sqrt{\frac{hE_T^{(90)}}{2h^{(90)}\Omega^{(90)}}}$$

On integration :

$$\frac{pC}{WA}(\alpha\,\Delta\sigma)^n N = \int_{\rho_0}^{\rho}\left[\frac{1}{E_A(\rho)} - \frac{1}{E_A}\right]^{-\frac{n}{2}} \rho^{n/2}\, d\rho$$

Implies that : $\rho = \rho(N) \qquad E_A = E_A(\rho(N))$

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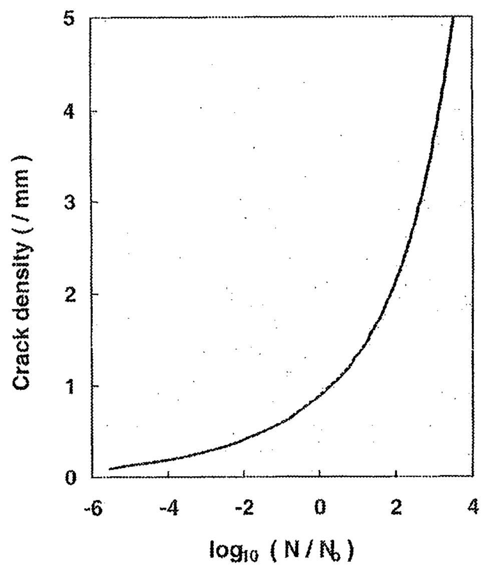


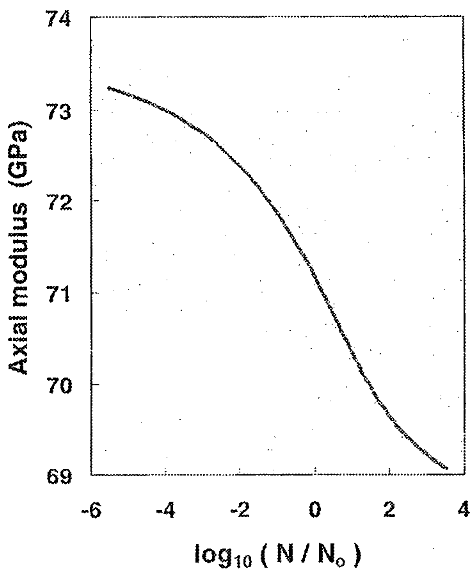


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## Degradation of properties of cross-ply laminates

Fatigue damage parameter : $D(N) = \frac{E_A}{E_A(N)} - 1$

$$\frac{1}{E_A(N)} = \frac{1}{E_A} + \frac{D(N)}{E_A} \qquad \frac{1}{E_T(N)} = \frac{1}{E_T} + \frac{k^2 D(N)}{E_A} \qquad \frac{1}{E_t(N)} = \frac{1}{E_t} + \frac{(k')^2 D(N)}{E_A}$$

$$\frac{\nu_A(N)}{E_A(N)} = \frac{\nu_A}{E_A} - \frac{kD(N)}{E_A} \qquad \frac{\nu_a(N)}{E_A(N)} = \frac{\nu_a}{E_A} - \frac{k' D(N)}{E_A} \qquad \frac{\nu_t(N)}{E_T(N)} = \frac{\nu_t}{E_T} - \frac{k k' D(N)}{E_A}$$

$$\alpha_A(N) = \alpha_A + \frac{k_1 D(N)}{E_A} \qquad \alpha_T(N) = \alpha_T + \frac{k k_1 D(N)}{E_A} \qquad \alpha_t(N) = \alpha_t + \frac{k' k_1 D(N)}{E_A}$$

k, k' and $k_1$ are easily calculated using CLT

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## Predicting static failure of cross - ply laminates

We must be able to do this if we are to predict lifetime

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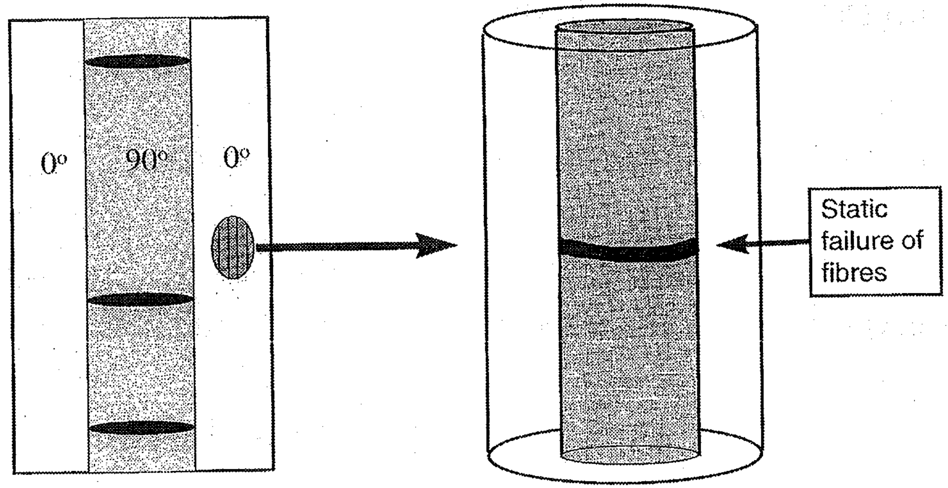
## Modelling failure of cross-ply laminates




Biaxial loading
Thermal stresses

Fibre matrix cell
Biaxial stresses
Thermal stresses

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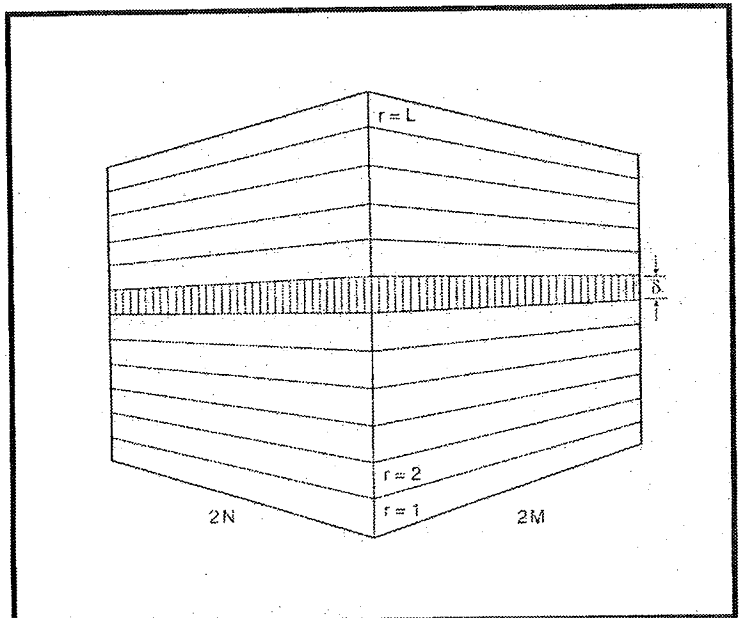
## Monte Carlo model for 0° - plies in laminate




**Parameters :**

**M, N, L**

**Element length δ**

$\delta = 1$ mm

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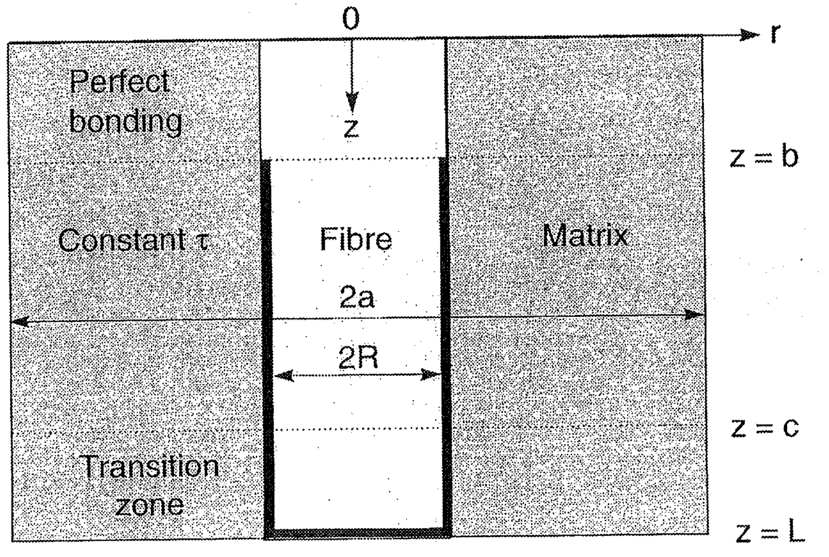


Plane of symmetry for fibre fracture

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## Weakest link statistics govern fibre failure

Use Weibull distribution for fibre strengths

- **m** dimensionless shape parameter
- $\sigma_o$ normalising parameter having dimensions of stress (GPa)

| | | |
|---|---|---|
| m = 7.4 | $\sigma_o$ = 5.928 GPa | Carbon fibre |
| m = 8.0 | $\sigma_o$ = 2.283 GPa | Glass fibre |

## Parameters for the micro-mechanical model

- Fibre radius = 3.5 microns CFRP ( = 10.0 microns GRP )
- Fibre volume fraction = 0.6
- Stress-free temperature $\Delta T$ = - 100°C
- Interfacial shear stress $\tau$ = 50 MPa

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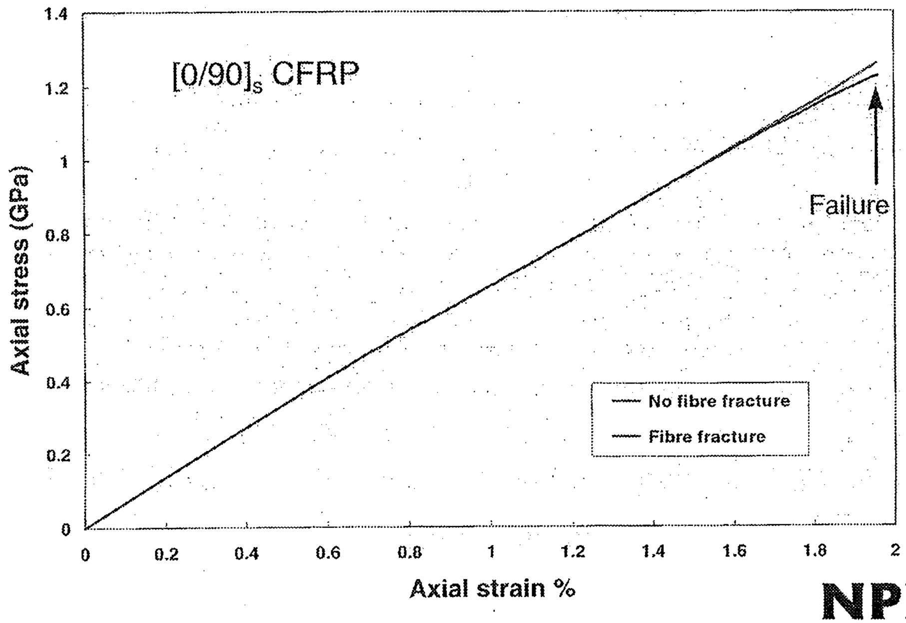
## Stress-strain behaviour

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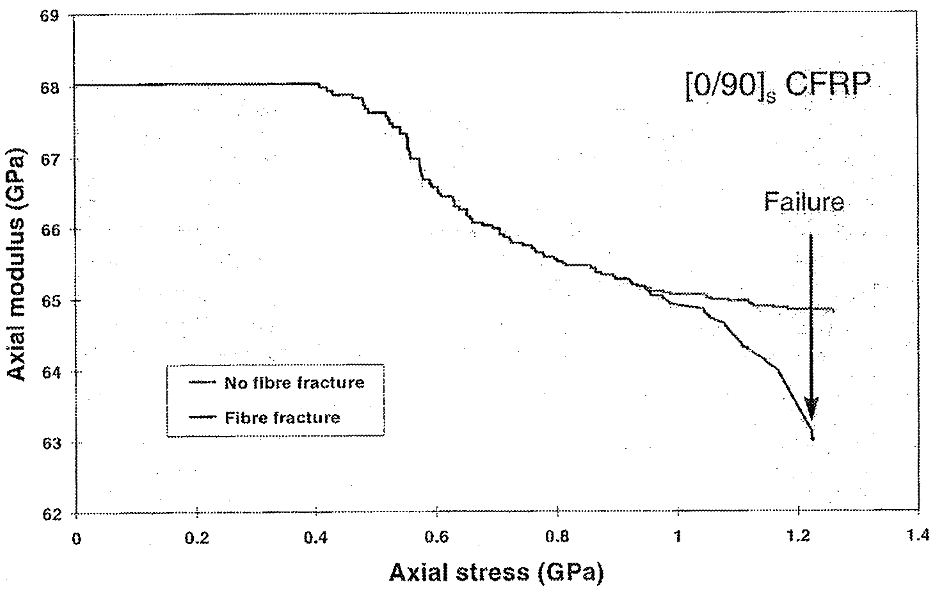
## Degradation of axial modulus

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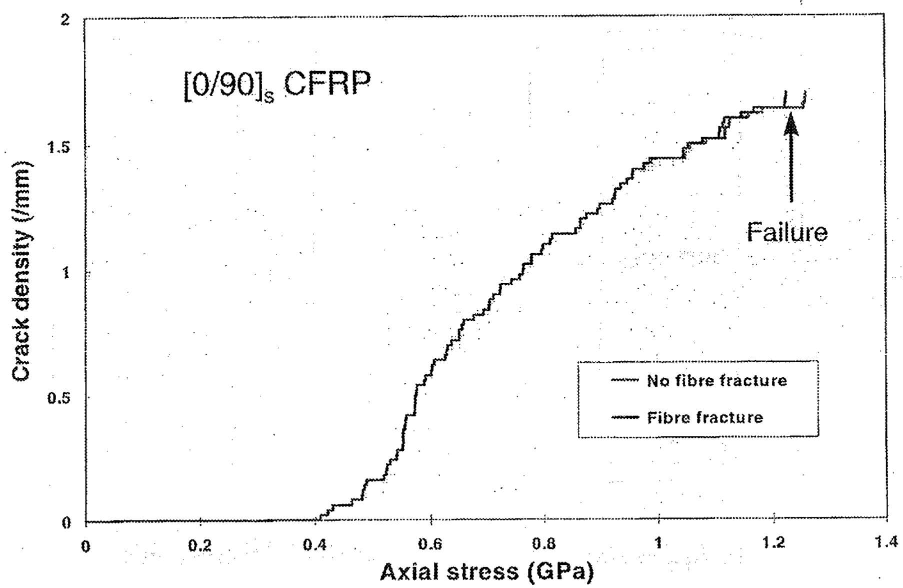
## Growth of ply cracks during axial loading

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### Modelling static fatigue

Ignore effects of ply cracking or matrix degradation

Damage is caused by stress corrosion cracking in fibres

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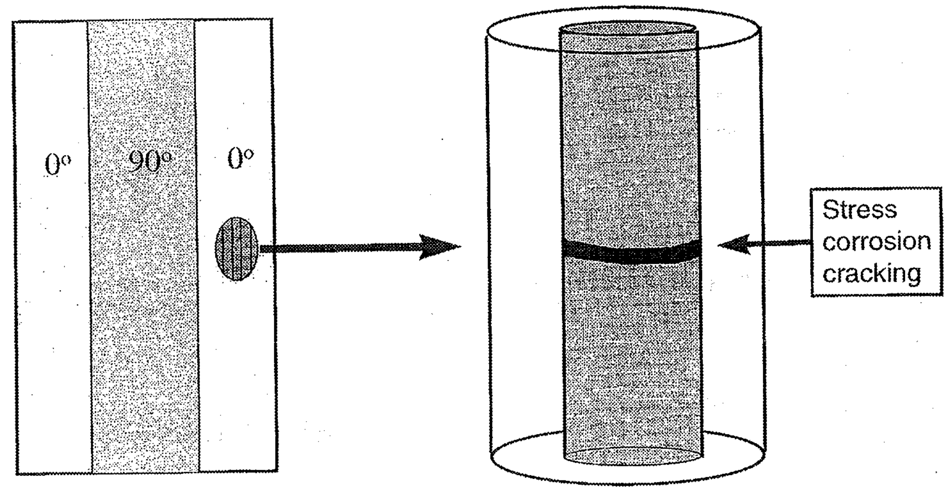
### Modelling of static fatigue in cross-ply laminates




Biaxial loading
Thermal stresses
No ply cracking

Fibre matrix cell
Biaxial stresses
Thermal stresses

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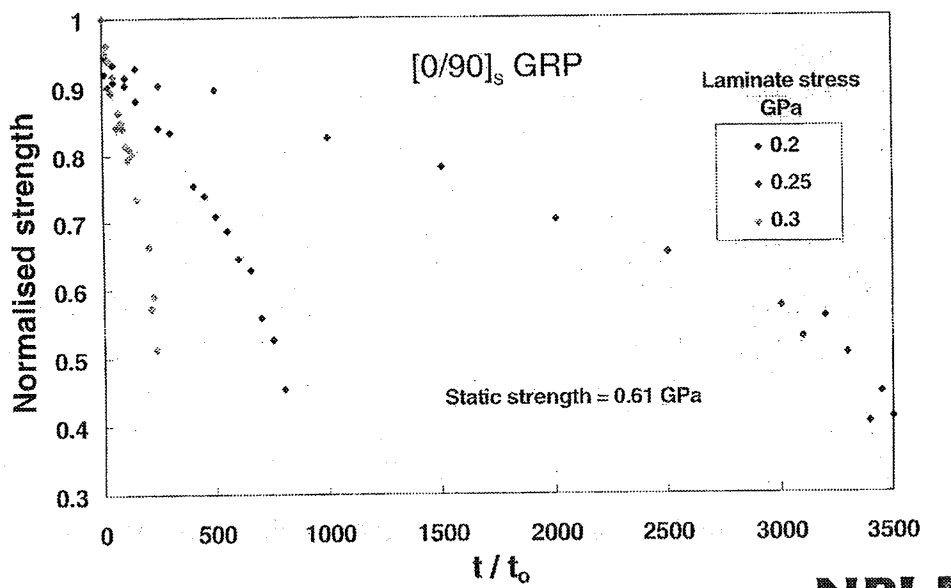
### Degradation of residual strength

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### Conclusions

- Prediction of fatigue life & residual properties is in its infancy
  - good progress being made for relatively simple X-ply laminates
  - future issues are general symmetric laminates / complex loading
  - delaminations / comprehensive validation of models
- Lifetime prediction must be based on understanding and modelling of micromechanisms that contribute to failure
  - many complex mechanisms operate
  - all mechanisms interact and are affected by loading which can be complex
- Within the UK there is a need to coordinate activity in the field in conjunction with industry
  - development of agreed design rules
  - validation of both models and design methods

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# Fatigue Life Prediction - Requirements and Methodologies

*Dr. Andrew Davies* (DERA)

## A Fatigue Life Methodology for Polymer Matrix Composites

Andrew J. Davies
Mechanical Sciences Sector

DERA

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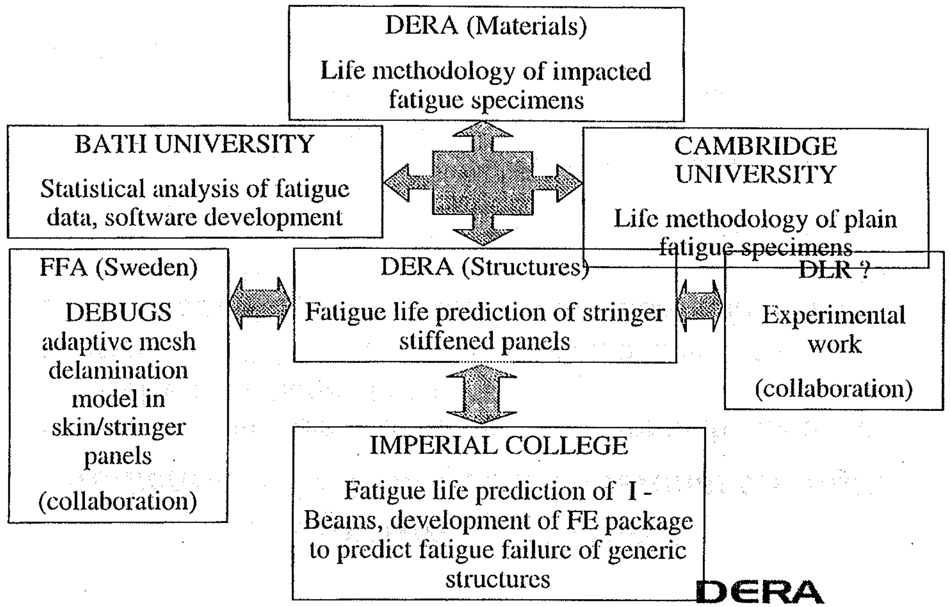
### Organisation of Fatigue Programme

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## *Background*

- Low design strains - fatigue not problem - static designs
  - Evidence - damage doesn't grow at working strains - 2700??
  - Thus is no fatigue lifing methodology, unlike metallics
    - Fracture mechanics may only work for delamination
    - Miner's rule unconservative and cannot cope with load sequence effects
- Situation is now reaching a crisis point
  - Increasing competition from advanced metallics
  - Pressure to reduce costs
  - Hence higher design strains must be used
- **Result - damage will grow in fatigue**
- Military airworthiness clearance based on safe life
  - In composites this is currently complied with by showing 'no-growth' and designing on static properties
  - As design strains increase, no-growth not an option
  - Safe life cannot be complied with if damage grows

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## New Airworthiness Procedures

- Based on fatigue and damage initiation/growth
  - Without lifing methodology need much more testing, thus higher costs for clearing composite structure.
- Also concluding that fatigue will be a major issue

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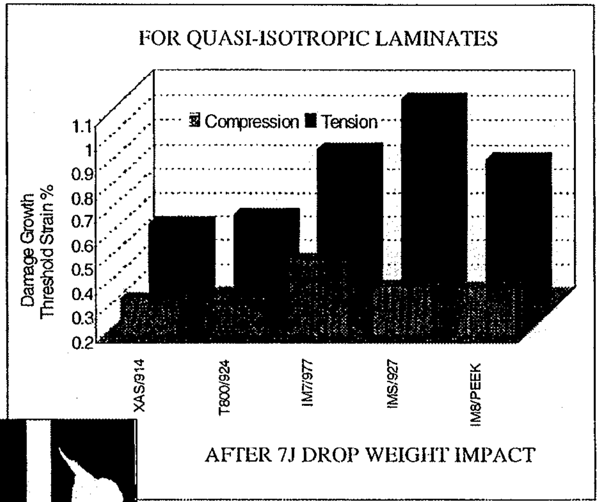
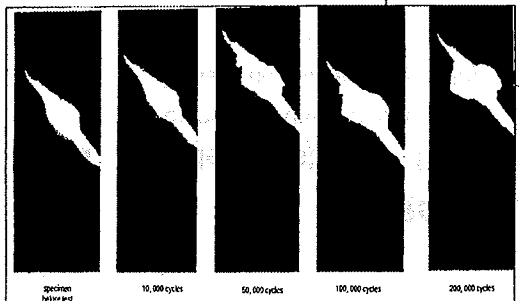
## Evidence for Damage Initiation & Growth

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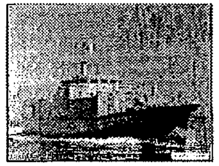
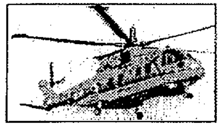
## Damage Growth Initiation Thresholds

- Conclude:-
  - If working strains reach 0.4% strain in compression, likely in 3-5 year timeframe, as these tougher materials are employed at higher design strains, damage initiation and growth in fatigue must be a serious design consideration.
  - Thus important, to start to develop a damage initiation and growth predictive methodology and fatigue life prediction capability for polymer composite materials and structures now if the advantages of these tough materials are to be exploited through designs at higher strains and stresses.





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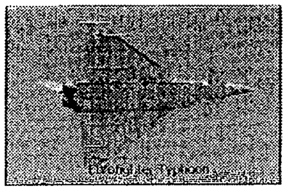
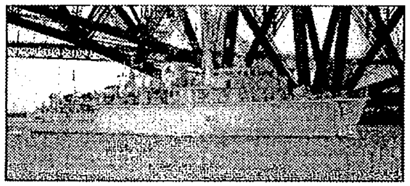
## Objectives

- To develop a lifing methodology for fatigue, damage initiation and growth in polymer composite structures
  - application in design, airworthiness clearance, re-lifing structures, lifing repairs, determining significance of damage through life etc.
- To move towards clearance for fatigue life based on calculation supported by test rather than the more expensive reverse option or full test as used at present.
- 





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## Programme - Phase 1

### *Prediction of bulk fatigue life/residual strength*

- Tackle materials, elements and through industry involvement, large structures
- Materials - CFRP, GRP to low cost fabrication routes
- Aim predict fatigue life based on ***physical principles***
  - include plain & notched materials,
  - Ideally inputs would be static properties and output fatigue life for any R ratio, block or spectrum loading.
  - Realistic first aim to deal with constant amplitude loading and predict fatigue life as function of R ratio.
  - Next step - extend to stress conc$^{n}$s & different layups etc
- Due note of past work - e.g Bath University

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# Fatigue Life Prediction of Composite Materials

By
Andrew Davies

Mechanical Sciences Sector

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## Introduction

- Many papers have been written on fatigue life prediction covering topics such as:-
- Damage accumulation models
- Data manipulation models
- Statistical accounts of static and fatigue failure distributions
- Very few models are **physically** based

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## Aim of materials fatigue programme

- "To investigate the behaviour of impacted composite specimens subjected to fatigue…and from this establish a lifing methodology for such specimens"

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## Proposed Work at DERA

- Thermographic techniques
- Initially plain ± 45° and Unidirectional specimens
- Plain quasi-isotropic specimens
- Different frequencies

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## Proposed Work at DERA

- Impacted quasi-isotropic specimens
- Different impact energies
- Similar tests with woven specimens

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## Proposed Work at DERA

- Use technique to establish damage initiation area in plain fatigue specimens and damage growth in impacted specimens

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## Proposed Work at DERA

- **Specimens Examination**
- De-ply
- Slicing
- Fractography

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- **Other Work**
- Examine the effect of specimen width on edge effects
- Continue evaluating the use of artificial damage to mimic impact damage as a way as simplifying response of impact damage to fatigue testing.

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- **Other Work**

- Provide Bath University with :-
- Fatigue data from established materials systems to fit to existing constant life model
- Fatigue data from new material systems, from both plain and impacted specimens
- Tensile and compressive data of new material systems
- NDE information from impacted specimens

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Fatigue Life Prediction of High Performance Composites

**Helen Carroll - University of Cambridge**

**DERA-Cambridge Gordon Laboratory**

- Damage mechanisms have been well documented but few are included in models
- Aim of project is to bring a mechanistic background to life prediction models such as the model developed by Harris

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Fatigue Life Prediction of High Performance Composites

- $((+45/-45/0/90)_2)_s$ Cytec 919HF/HTA
- Plain specimens
- Constant amplitude
- Variety of R ratios, tensile & compressive loading

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Fatigue Life Prediction of High Performance Composites

- Fatigue damage will be investigated using:-
  - Edge replication
  - Thermography
  - Slicing and Optical Microscopy
  - SEM

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Fatigue Life Prediction of High Performance Composites

**Dr Osama Attia, Imperial College**

- CFRP Fibre-Reinforced Polymer Matrix I - Beams
- Matrix Cracking / Fibre Breaking cause fatigue failure
- ABAQUS Finite Element Analysis to calculate stress/strain distribution in beam

Elements in which transverse ply strain exceeds

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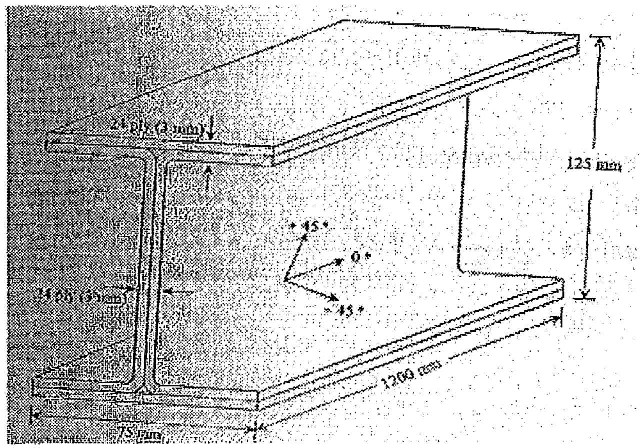
Fatigue Life Prediction of High Performance Composites

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Fatigue Life Prediction of High Performance Composites

- Results from Cambridge fatigue work will be used as input data for Bath model
- Different materials/fibre architecture and lay-ups will studied
- A link between failure mechanisms and fatigue life will be sought

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Fatigue Life Prediction of High Performance Composites

- Elements in which transverse ply strain exceeds a tensile threshold are considered damaged.
- Elements elastic properties are reduced according to material degradation data.
- Damage area and corresponding strain energy used to calculate strain energy release rate G.
- Paris Law is used to calculate fatigue life of beam

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*TG04 Fatigue Methodology Programme*
*Composite Structural Studies*

**Dr Emile S Greenhalgh**

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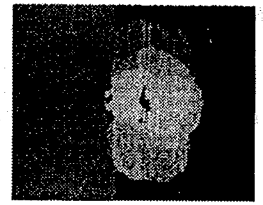
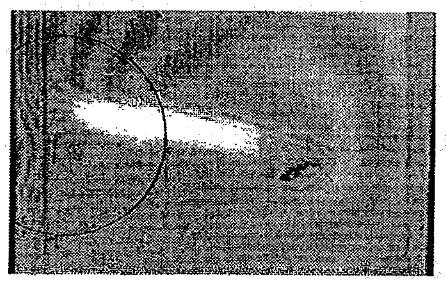
## Background - Previous Static Structural Studies

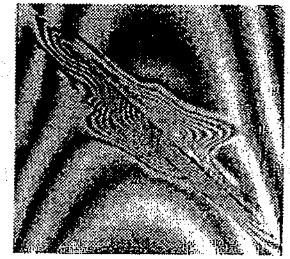
- Large amount of data on delamination and impact damage growth under static loading in composite structures
- Damage growth mechanisms well understood



*Impact damage over a stringe*



*Damage growth from a 50mm diameter embedded defect beneath a stiffener foot*



*Damage growth from an impact*

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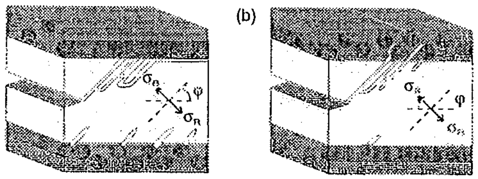
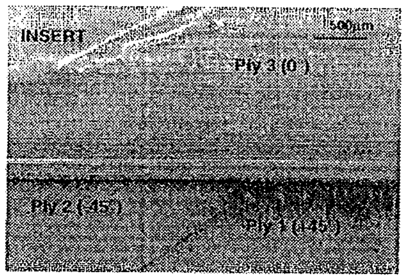
## Background - Damage Mechanisms

- Damage growth controlled by critical ply interfaces
- Delamination growth does not remain at one interface, but migrates through the layers
- Remains within an interface in which driving force and ply direction are coincident.
- Dominant factor in dictating delamination growth behaviour & key to damage tolerant design
- Does this behaviour occur under cyclic loading?






*Delamination migrating through plies*

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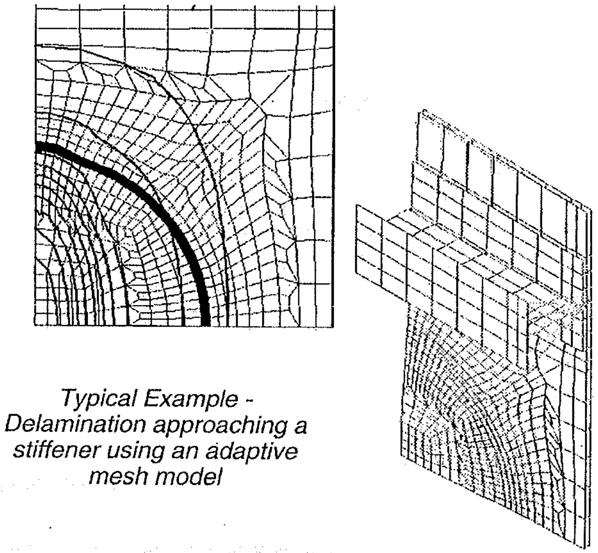
## Background - Predictive Models

- DEBUGS delamination growth model developed at FFA, Sweden
- Uses adaptive mesh to model delamination growth
- Being developed for impact damage
- Extended to model delamination growth in composite structures
- Scope to incorporate fatigue algorithms



*Typical Example - Delamination approaching a stiffener using an adaptive mesh model*

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## Aims of New Programme

- To generate realistic structural data of damage growth under cyclic loading in composite skin-stringer structures
- To develop predictive models of damage growth under cyclic loading in composite skin-stringer panels.

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TG04 Fatigue Methodology - DERA, Farnborough, November 1999

## Programme

Programme split into two parts:

- Experimental - Cyclic tests on stringer stiffened panels containing damage plus detailed fractography to deduce damage mechanisms
- Numerical - Extend DEBUGS model to cyclic loading, incorporating the fatigue models/algorithms developed for coupons.

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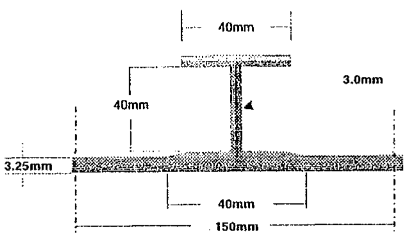
TG04 Fatigue Methodology - DERA, Farnborough, November 1999

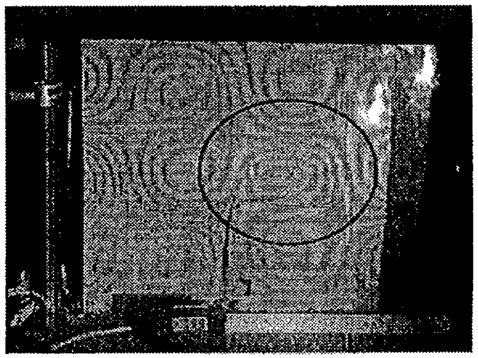
## Parameters - Panel Details

- Test configurations based on previous work on static loading
- Use DAMOCLES Baseline configuration - FFA/NLR/DERA collaboration on designing for damage tolerance




**Skin: $[+45/0/-45_2/0/+45/90/+45/0/-45_2/0/+45]_s$**
**Stringer: Each $[+45/-45/0/0/90/0]_s$**

*Baseline DAMOCLES design*



*Impact damage growth in a post-buckled stiffened panel*

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TG04 Fatigue Methodology - DERA, Farnborough, November 1999

## Parameters

- Skin-stringer panels: *DAMOCLES baseline design - 6 to 12 panels*
- Damage: *embedded defects (at a critical interface ) & impact damag*
- Static testing: *compression test to failure for panels containing embedded defects and impact damage.*
- Fatigue testing: *compression-compression fatigue, at a level predicte by the coupon models*
- Materials: *Hexcel IM7/8552 (and possibly T800/M21).*
- Damage location: *Within the panel bay and partly beneath a stringer.*
- Damage size: *35J impact & equivalent embedded defect size.*
- Instrumentation: *Strain gauges, LVDT, interrupted ANDSCAN, Moiré*

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## *Programme Sub-Tasks*

1. Literature review: *survey previous studies & explore collaboration*
2. Programme plan: *detailed test plan & instrumentation locations*
3. Fabrication: *Manufacture & instrument stiffened panels*
4. Static Tests: *Introduce impact damage & CAI tests*
5. Fatigue tests: *Cyclic tests, interrupted to monitor damage growth.*
6. Fractography: *Detailed analysis of the failed structures.*
7. Modelling: *Development of models, based DEBUGS*
8. Model/test comparison: *Compare and modify the models*
9. Reporting

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## Proposed Programme - Phase 2

### *Predict damage initiation & growth IN FATIGUE*

- Develop a simple approach to predicting initiation and growth of damage and its effects on fatigue life using a continuum approach, similar to that adopted in the equivalent hole model
- Must develop criteria for initiation as well as growth laws
  - initial airworthiness clearance based on safe life - initiation
  - relifing, repair etc will require growth laws
- Link to work on damage initiation and growth under static load
  - but cyclic loading will lead to initiation/growth at lower stresses
  - need to understand mechanisms for this in composites

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## Deliverables

- A continuum model for predicting fatigue life based on physical principles.
- By treating the more complex issue of damage initiation and growth in fatigue to deliver a full predictive model capable of predicting damage initiation and growth under any fatigue loading condition
- Interface these models with National Modelling tools project to make them widely available as design tools

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## MoD Relevance

- Wide range military platforms benefit, manned aircraft, UAVs, missiles, ships, submarines, and land vehicles.
  - all composite structures where fatigue loading is design requirement
- Particular benefits will be felt in especially fatigue demanding environments.
  - rotary wing applications, aero-engine components, transport airport, especially on low level flights, UAVs designed to higher strains than manned aircraft and marine structures such as minesweeper hulls and submersibles.
- Specific future platforms likely to benefit include
  - JSF in a harsh marine environment, FLA in low level flight
  - Composite armoured vehicles, submersible pressure hull

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# Analysis of Damage Mechanisms in Fatigue of Fibre Reinforced Epoxy Laminates

*Dr. Costas Soutis* (IMPERIAL COLLEGE)

# Analysis of damage mechanisms in fatigue of fibre-reinforced epoxy laminates

Costas Soutis and Maria Kashtalyan

*Department of Aeronautics, Imperial College of Science, Technology and Medicine*
*Prince Consort Road, London SW7 2BY, UK*

Resin-dominated damage modes, such as matrix cracking and delamination, are common failure mechanisms in composite laminates and are of primary concern in the current design with composites. Transverse cracking in the 90° ply has long been recognised as the first damage mode observed in composite laminates under static and fatigue tension as well as thermal loading. It reduces the laminate stiffness properties and is detrimental to the laminate strength. It also triggers the development of other harmful damage modes, such as delaminations at the free edges of the laminate and/or local delaminations, growing from the matrix crack tips.

Under multiaxial or general in-plane static or fatigue loading (mechanical or thermal), damage may affect more than one layer of the laminate, and different damage modes can interact with each other. Until now, multilayer damage of composite laminates has been very little modelled theoretically or simulated numerically (by means of the finite elements). In this study, a new approach based on the Equivalent Constraint Model (ECM) of the damaged lamina [1, 2] is applied to investigate multilayer matrix cracking and delaminations induced by it in glass/epoxy and carbon/epoxy laminates. Instead of considering a representative element defined by the intersecting pairs of cracks, intrinsic to the earlier models, inter-related problems for ECM laminates are solved. The approach provides closed-form expressions for the reduced stiffness properties of the damaged laminae as well as the strain energy release rates associated with matrix cracking and delaminations. The latter are used to predict damage initiation. The parameters controlling the damage laminate behaviour will be identified. It will be shown that predictions of the new approach compare favourably with the results obtained from other models as well as experimental data.

## REFERENCES

1. Zhang J, Fan J, Soutis C. Analysis of multiple matrix cracking in $[\pm\theta_m/90_n]_s$ composite laminates Part1: In-plane stiffness properties. Composites 1992;23(5): 291-298
2. Zhang J, Soutis C, Fan J. Strain energy release rate associated with local delamination in cracked composite laminates. Composites 1994; 25(9): 851-862

# ANALYSIS
# OF DAMAGE MECHANISMS IN FATIGUE OF FIBRE-REINFORCED EPOXY LAMINATES

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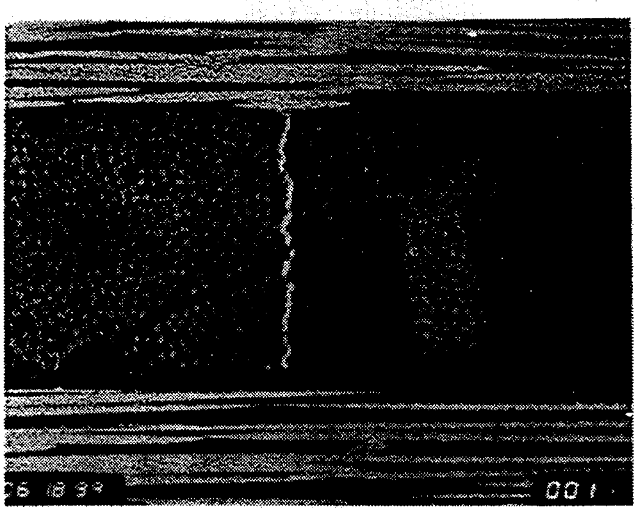
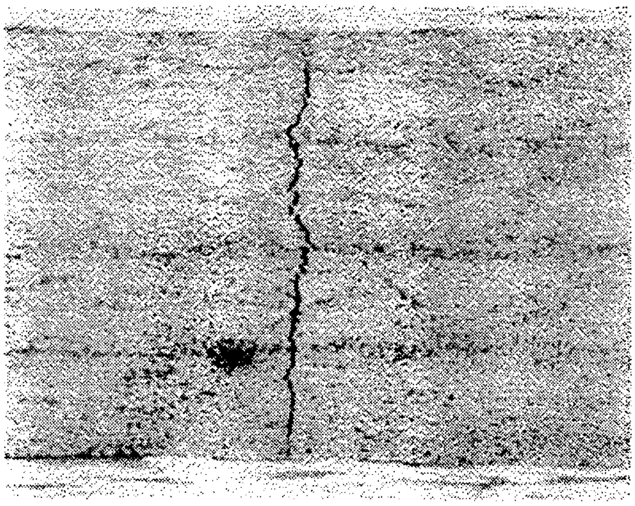
Prince Consort Road, London SW7 2BY

**Funded by EPSRC/MoD**

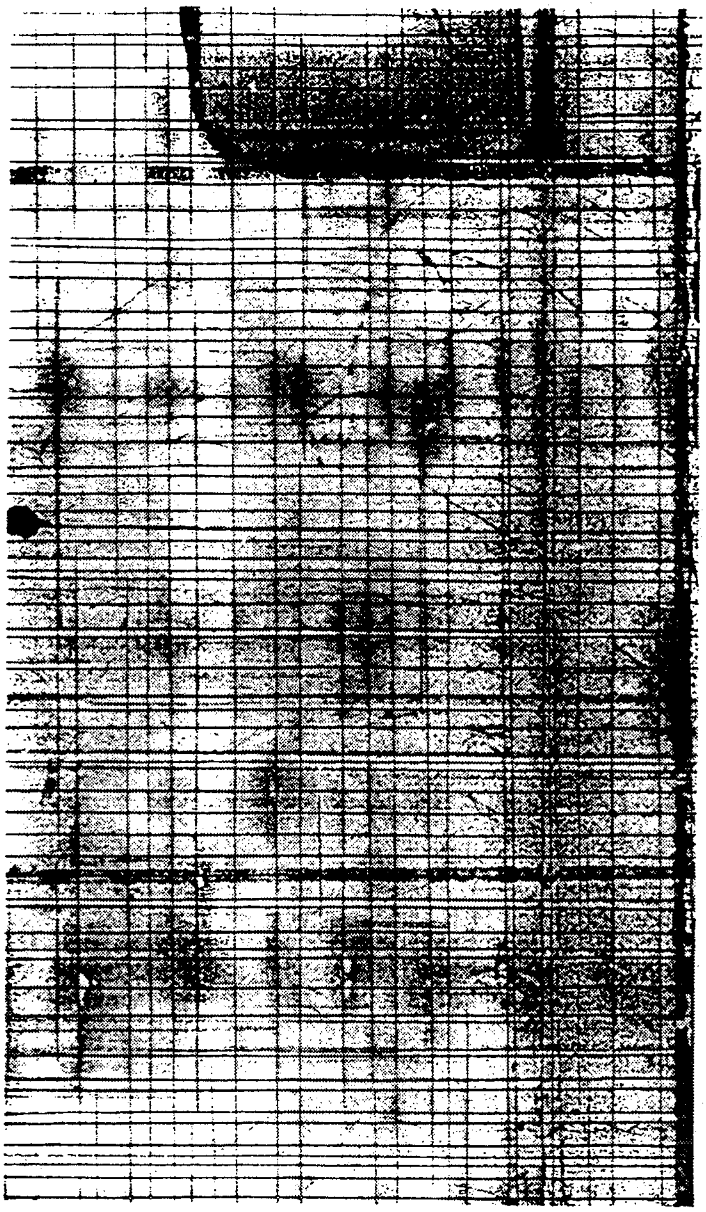


(a)



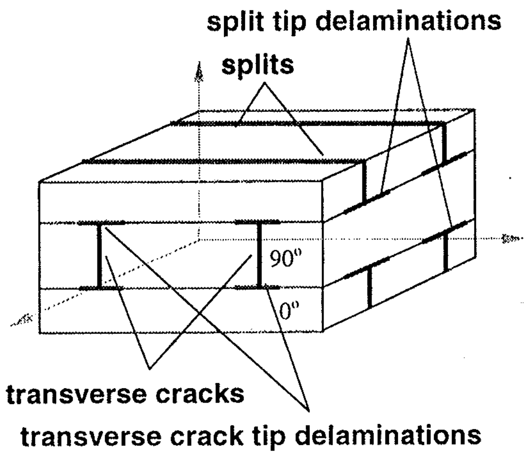
(b)

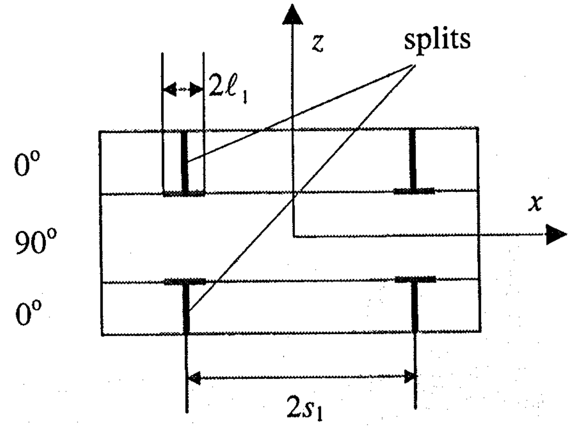
Transverse cracking in a) $[0/90]_s$ and b) $[0/90_2]_s$ laminates

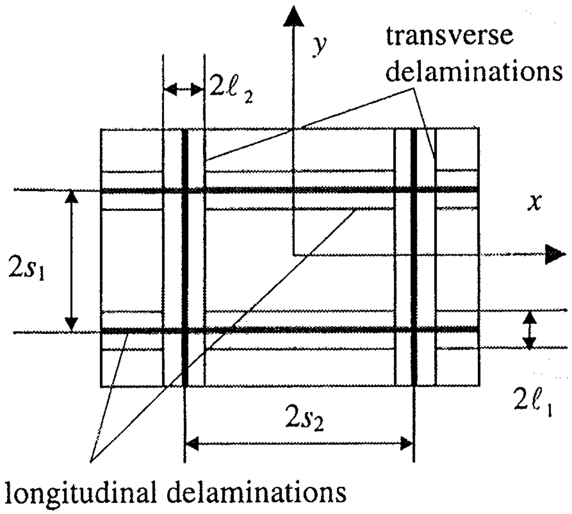


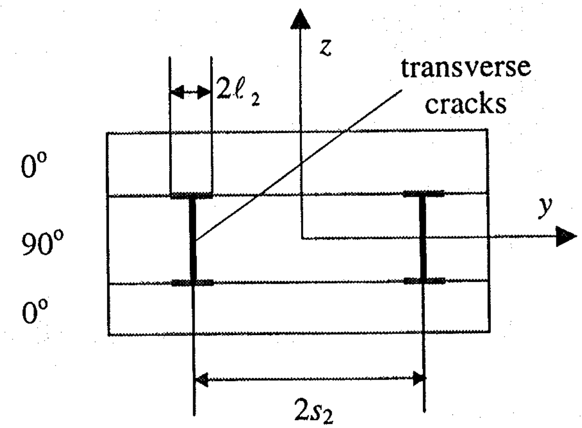
X-ray radiograph of a $[0/90_2]_s$ T300/5208 laminate at Stage II, $0.7S_{ult}$.
Jamison RD, Schulte K, Reifsnider KL, Stinchcomb WW (1984) Characterization and Analysis of Damage Mechanisms in Tension-Tension Fatigue of Graphite/Epoxy Laminates. In: Effect of Defects in Composite materials, ASTM STP 836, American Society for Testing and Materials, 21-55

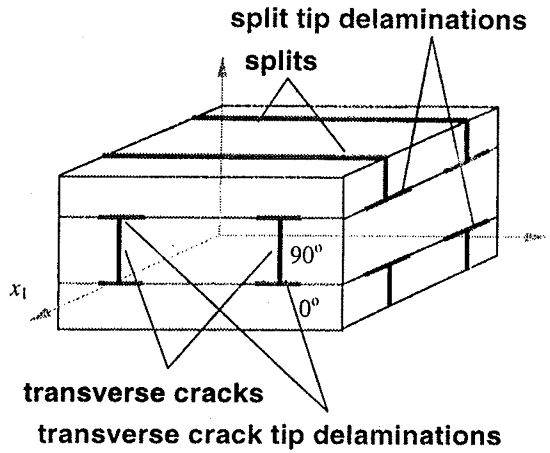
## DAMAGE PATTERN



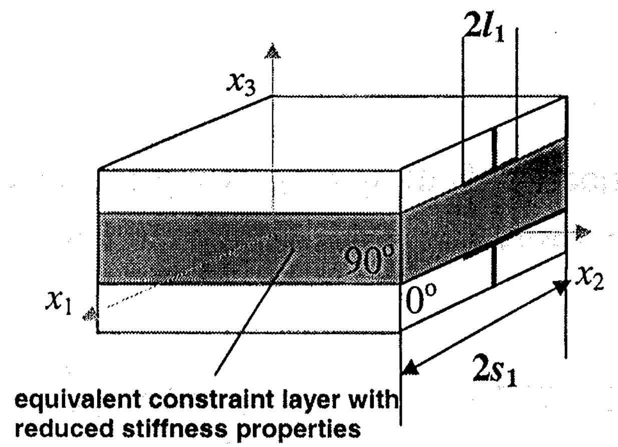





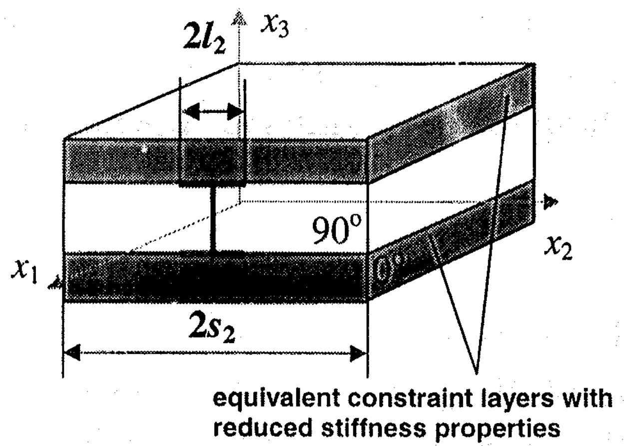

- Delaminations are strip-shaped and span the full width of the laminate
- Matrix cracks are spaced uniformly and span the full thickness and width of the ply
- The laminate is subjected to in-plane biaxial tension ($\overline{\sigma}_{11}$ and $\overline{\sigma}_{22}$) and shear loading ($\overline{\sigma}_{12}$).

Application of the **Equivalent Constraint Model** (ECM) of the damaged lamina to the problem means that the following two ECM laminates will be analysed simultaneously as a coupled problem. In ECM$\mu$ laminate, the $\mu^{th}$ layer contains damage explicitly while the other damaged plies are replaced with a homogeneous layers with reduced stiffness properties.

**ECM1**



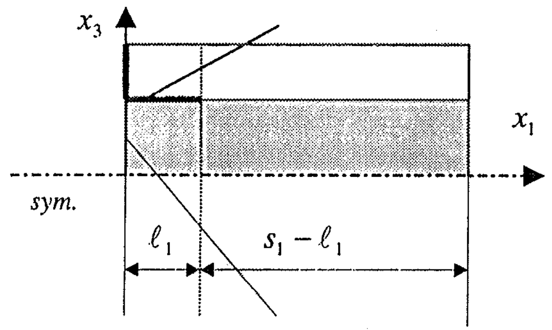

**ECM2**



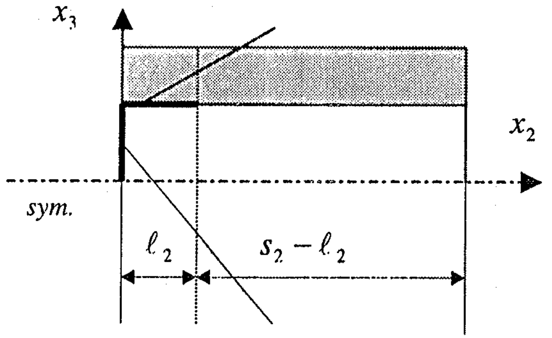
**The representative segment** of the ECM$\mu$ laminate is segregated into

- the locally delaminated ($|x_\mu| \leq \ell_\mu$) portion
- the laminated $\ell_\mu \leq |x_\mu| \leq s_\mu$ portions

**ECM1**




**ECM2**




**Micromechanical analysis of the ECM$\mu$ laminate**

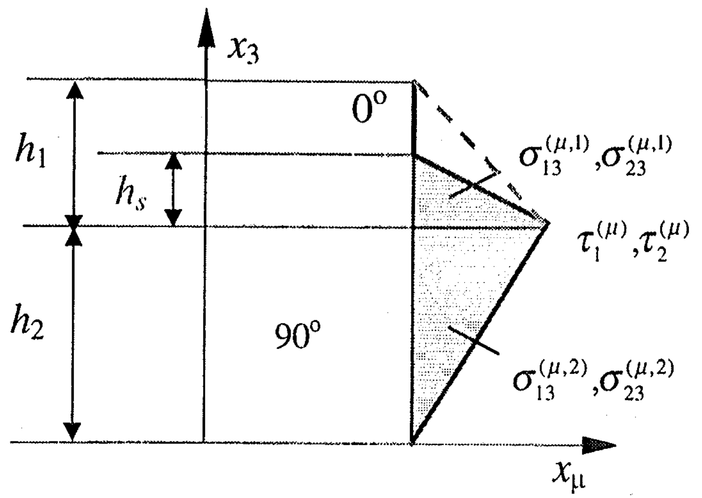
Let $\tilde{\sigma}_{ij}^{(\mu,k)}$, $\tilde{\varepsilon}_{ij}^{(\mu,k)}$ and $\tilde{u}_i^{(\mu,k)}$ denote respectively the in-plane microstresses, the in-plane microstrains and the in-plane displacements in the $k^{\text{th}}$ layer of the ECM$\mu$ laminate $(k, \mu = 1,2)$, averaged across the thickness of the ply and width of the laminate.

- **In the locally delaminated portion**, stresses in the explicitly damaged ply vanish, since crack and delaminations surfaces are stress-free

- **In the perfectly bonded portion**, stresses can be determined from the equilibrium equations

$$\frac{d}{dx_\mu}\tilde{\sigma}_{j\mu}^{(\mu,k)} + (-1)^k \frac{\tau_j^{(\mu)}}{h_k} = 0, \quad j, \mu, k = 1,2 \qquad (1)$$

$\tau_j^{(\mu)}$ are the interface shear stresses at the (0/90) interface of the ECM$\mu$ in the $j^{\text{th}}$ direction.




**The interface shear stresses** $\tau_j^{(\mu)}$ in Eqs. 1 are

$$\tau_j^{(\mu)} = K_j^{(\mu)}(\tilde{u}_j^{(\mu,1)} - \tilde{u}_j^{(\mu,2)}) \quad (2)$$

$$K_j^{(\mu)} = K_j = \frac{3\hat{G}_{j3}^{(1)}\hat{G}_{j3}^{(2)}}{h_2\hat{G}_{j3}^{(1)} + (1+(1-\eta)/2)\eta h_1 \hat{G}_{j3}^{(2)}}$$

$$\eta = t/h_1, \quad j = 1,2$$

$K_j$ are the improved shear lag parameters, $\hat{G}_{j3}^{(k)}$, $k=1,2$ are the out-of-plane shear moduli of the $k^{th}$ layer

Substitution of shear lag equations (Eqs 2) into equilibrium equations (Eqs 1) and subsequent differentiation yield

$$\frac{d^2}{dx_\mu^{(2)}}\tilde{\sigma}_{\mu\mu}^{(\mu,\mu)} + (-1)^\mu \frac{K_j}{h_\mu}(\tilde{\varepsilon}_{\mu\mu}^{(\mu,1)} - \tilde{\varepsilon}_{\mu\mu}^{(\mu,2)}) = 0,$$

$$\frac{d^2}{dx_\mu^{(2)}}\tilde{\sigma}_{12}^{(\mu,\mu)} + (-1)^\mu \frac{K_j}{h_\mu}(\tilde{\gamma}_{12}^{(\mu,1)} - \tilde{\gamma}_{12}^{(\mu,2)}) = 0$$

Strain differences $(\tilde{\varepsilon}_{\mu\mu}^{(\mu,1)} - \tilde{\varepsilon}_{\mu\mu}^{(\mu,2)})$ and $(\tilde{\gamma}_{12}^{(\mu,1)} - \tilde{\gamma}_{12}^{(\mu,2)})$ can be expressed in terms of stresses $\tilde{\sigma}_{\mu\mu}^{(\mu,\mu)}$ and $\tilde{\sigma}_{12}^{(\mu,\mu)}$ using

- equations of the global equilibrium of the laminate

$$\chi\tilde{\sigma}_{ij}^{(\mu,1)} + \tilde{\sigma}_{ij}^{(\mu,2)} = (1+\chi)\overline{\sigma}_{ij} \quad \chi = h_1/h_2$$

- stress-strain relationships for both plies

$$\{\tilde{\sigma}^{(\mu,\mu)}\} = [\hat{Q}^{(\mu)}]\{\tilde{\varepsilon}^{(\mu,\mu)}\}$$
$$\{\tilde{\sigma}^{(\mu,\kappa)}\} = [Q^{(\kappa)}]\{\tilde{\varepsilon}^{(\mu,\kappa)}\}$$

$[\hat{Q}^{(\mu)}]$ is the in-plane stiffness matrix of $\mu^{th}$ layer, and $[Q^{(\kappa)}]$ is the modified in-plane stiffness matrix of the equivalent constraint $\kappa^{th}$ layer

- generalised plane strain assumption

$$\tilde{\varepsilon}_\kappa^{(\mu,1)} = \tilde{\varepsilon}_\kappa^{(\mu,2)}, \quad \kappa \neq \mu$$

Finally, **the in-plane micro-stresses** in the laminated portion are found from equations

$$\frac{d^2\tilde{\sigma}_{\mu\mu}^{(\mu,\mu)}}{dx_\mu^2} - L_1^{(\mu)}\tilde{\sigma}_{\mu\mu}^{(\mu,\mu)} + \Omega_{11}^{(\mu)}\overline{\sigma}_{11} + \Omega_{22}^{(\mu)}\overline{\sigma}_{22} = 0$$

$$\frac{d^2\tilde{\sigma}_{12}^{(\mu,\mu)}}{dx_\mu^2} - L_2^{(\mu)}\tilde{\sigma}_{12}^{(\mu,\mu)} + \Omega_{12}^{(\mu)}\overline{\sigma}_{12} = 0$$

as

$$\tilde{\sigma}_{\mu\mu}^{(\mu,\mu)} = \frac{1}{L_1^{(\mu)}}\left(1 - \frac{\cosh[\sqrt{L_1^{(\mu)}}(x_\mu - s_\mu)]}{\cosh[\sqrt{L_1^{(\mu)}}(s_\mu - \ell_\mu)]}\right)(\Omega_{11}^{(\mu)}\overline{\sigma}_{11} + \Omega_{22}^{(\mu)}\overline{\sigma}_{22})$$

$$\tilde{\sigma}_{12}^{(\mu,\mu)} = \frac{1}{L_2^{(\mu)}}\left(1 - \frac{\cosh[\sqrt{L_2^{(\mu)}}(x_\mu - s_\mu)]}{\cosh[\sqrt{L_2^{(\mu)}}(s_\mu - \ell_\mu)]}\right)\Omega_{12}^{(\mu)}\overline{\sigma}_{12}$$

The in-plane macrostresses and macrostrains

$$\overline{\sigma}_{ij}^{(\mu,\mu)} = \frac{1}{2s_\mu}\int_{-s_\mu}^{s_\mu}\tilde{\sigma}_{ij}^{(\mu,\mu)}dx_\mu$$

$$\overline{\varepsilon}_{ij}^{(\mu,1)} = \overline{\varepsilon}_{ij}^{(\mu,2)} = \overline{\varepsilon}_{ij} = \frac{1}{2s_\mu}\int_{-s_\mu}^{s_\mu}\tilde{\varepsilon}_{ij}^{(\mu,\kappa)}dx_\mu$$

**The reduced stiffness properties of the $\mu^{\text{th}}$ layer**

can be determined by applying the laminate plate theory to the ECM$\mu$ laminate after replacing the explicitly damaged layer with an equivalent homogeneous one

$$\{\overline{\sigma}^{(\mu,\mu)}\} = [Q^{(\mu)}]\{\overline{\varepsilon}^{(\mu,\mu)}\}$$

$$[Q^{(\mu)}] = [\hat{Q}^{(\mu)}] - [\hat{R}^{(\mu)}][\Lambda^{(\mu)}]$$

Here

$$[\hat{R}^{(\mu)}] = \begin{bmatrix} \hat{R}_{11}^{(\mu)} & \hat{Q}_{12}^{(\mu)} & 0 \\ \hat{Q}_{12}^{(\mu)} & \hat{R}_{22}^{(\mu)} & 0 \\ 0 & 0 & \hat{Q}_{66}^{(\mu)} \end{bmatrix}$$

$$[\Lambda^{(\mu)}] = \begin{bmatrix} \Lambda_{22}^{(\mu)} & 0 & 0 \\ 0 & \Lambda_{22}^{(\mu)} & 0 \\ 0 & 0 & \Lambda_{66}^{(\mu)} \end{bmatrix}$$

$$\hat{R}_{\mu\mu}^{(\mu)} = \hat{Q}_{\mu\mu}^{(\mu)}, \quad \hat{R}_{22}^{(1)} = (\hat{Q}_{12}^{(1)})^2(\hat{Q}_{11}^{(1)})^{-1}, \quad \hat{R}_{11}^{(2)} = (\hat{Q}_{12}^{(2)})^2(\hat{Q}_{22}^{(2)})^{-1}$$

## In-situ Damage Effective Functions

$$\Lambda_{qq}^{(\mu)} = 1 - \frac{1 - \dfrac{D_\mu^{mc}}{\beta_q^{(\mu)}(1-D_\mu^{ld})} \tanh\left[\dfrac{\beta_q^{(\mu)}(1-D_\mu^{ld})}{D_\mu^{mc}}\right]}{\dfrac{1+\beta_q^{(\mu)} D_\mu^{ld}}{1-D_\mu^{ld}} + \gamma_q^{(\mu)} \dfrac{D_\mu^{mc}}{\beta_q^{(\mu)}(1-D_\mu^{ld})} \tanh\left[\dfrac{\beta_q^{(\mu)}(1-D_\mu^{ld})}{D_\mu^{mc}}\right]}$$

$$q = 2,6 \quad \mu = 1,2$$

Here

- $D_\mu^{mc} = h_s / s_\mu$, $D_\mu^{ld} = \ell_\mu / s_\mu$ are the relative crack density and relative delamination area for the $\mu^{\text{th}}$ layer
- $\{\beta_q^{(\mu)}, \gamma_q^{(\mu)}\} = \{\beta_q^{(\mu)}, \gamma_q^{(\mu)}\}(\hat{Q}_{ij}^{(\mu)}, Q_{ij}^{(\kappa)}, K_j, h_1 / h_2)$ are laminate constants.

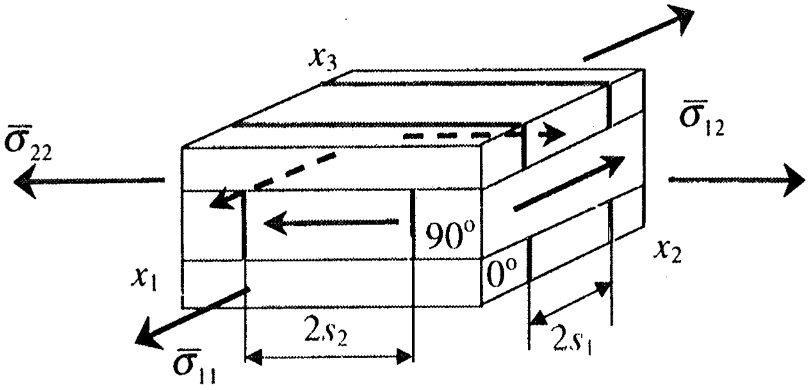
Through the reduced stiffness properties $Q_{ij}^{(\kappa)}$ of the equivalent constraint layer $\kappa^{\text{th}}$ layer, IDEFs $\Lambda_{22}^{(\mu)}$, $\Lambda_{66}^{(\mu)}$ implicitly depend on $D_\kappa^{mc} = h_\kappa / s_\kappa$ and $D_\kappa^{ld} = \ell_\kappa / s_\kappa$.

All four IDEFs form a system of simultaneous nonlinear algebraic equations

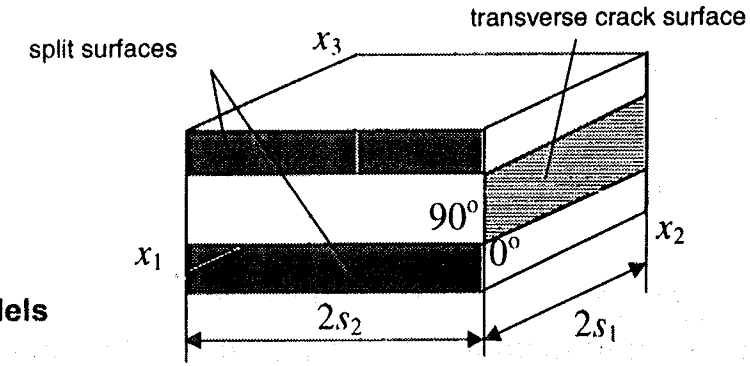
$$\begin{cases}
\Lambda_{22}^{(1)} = \Lambda_{22}^{(1)}(D_1^{mc}, D_1^{ld}, \hat{S}_{ij}^{(1)}, S_{ij}^{(2)}(D_2^{mc}, D_2^{ld}, \hat{S}_{ij}^{(2)}, \Lambda_{22}^{(2)}), \chi) \\
\Lambda_{66}^{(1)} = \Lambda_{66}^{(1)}(D_1^{mc}, D_1^{ld}, \hat{S}_{ij}^{(1)}, S_{ij}^{(2)}(D_2^{mc}, D_2^{ld}, \hat{S}_{ij}^{(2)}, \Lambda_{66}^{(2)}), \chi) \\
\Lambda_{22}^{(2)} = \Lambda_{22}^{(2)}(D_2^{mc}, D_2^{ld}, \hat{S}_{ij}^{(2)}, S_{ij}^{(1)}(D_1^{mc}, D_1^{ld}, \hat{S}_{ij}^{(1)}, \Lambda_{22}^{(1)}), \chi) \\
\Lambda_{66}^{(2)} = \Lambda_{66}^{(2)}(D_2^{mc}, D_2^{ld}, \hat{S}_{ij}^{(2)}, S_{ij}^{(1)}(D_1^{mc}, D_1^{ld}, \hat{S}_{ij}^{(1)}, \Lambda_{66}^{(1)}), \chi)
\end{cases}$$

which is solved computationally by a direct iterative procedure. All four IDEFs are determined as functions of **four** damage parameters $D_1^{mc}$, $D_2^{mc}$, $D_1^{ld}$, $D_2^{ld}$

# VERIFICATION OF ECM/2-D SHEAR LAG APPROACH (1)




previous models

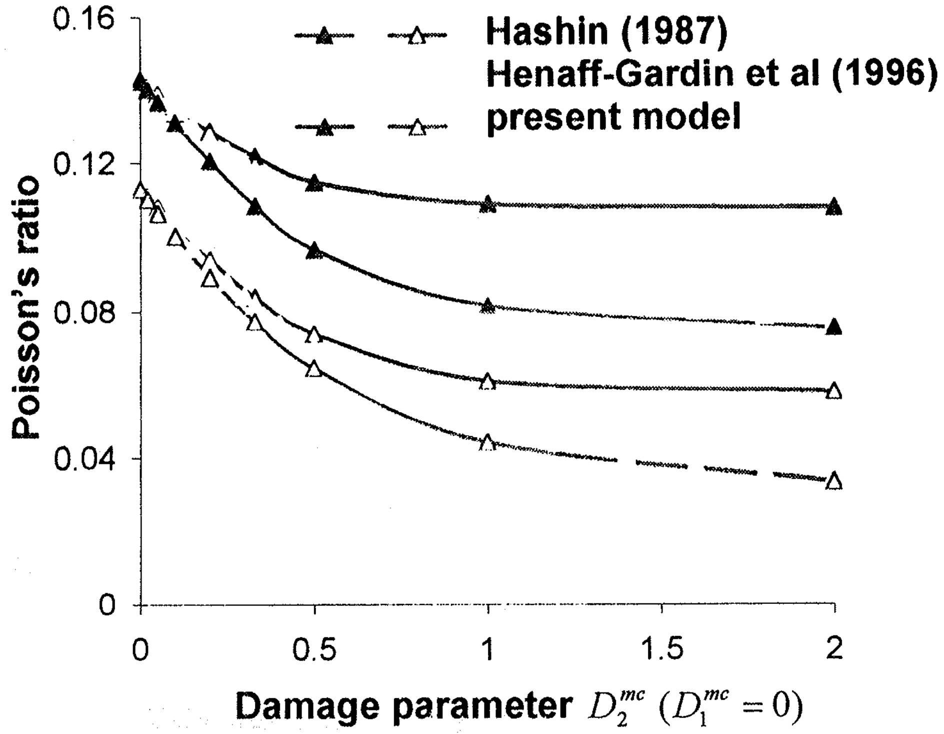


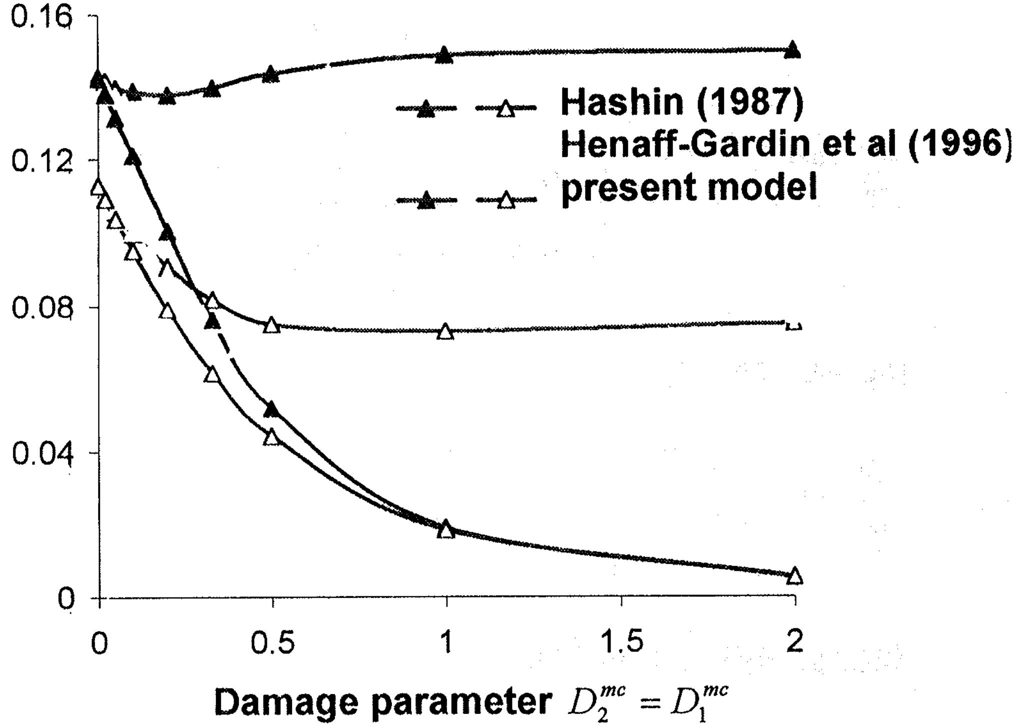

- **Hashin (1987)** — ***variational method***
- **Tsai & Daniel (1992)** — ***shear lag + finite differences methods***
- **Henaff-Gardin et al (1996)** — ***shear lag method***

| | Axial modulus reduction ratio | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Damage parameter** $D_2^{mc} = D_1^{mc}$ | **0.02** | **0.05** | **0.1** | **0.2** | **0.33** | **0.5** | **1.0** | **2.0** |
| Hashin (1987) | 0.990 | 0.975 | 0.951 | 0.907 | 0.853 | 0.804 | 0.762 | 0.757 |
| present model | 0.992 | 0.980 | 0.961 | 0.925 | 0.882 | 0.841 | 0.787 | 0.765 |
| Henaff-Gardin et al (1996) | 0.999 | 0.993 | 0.980 | 0.957 | 0.928 | 0.894 | 0.832 | 0.795 |

# VERIFICATION OF ECM/2-D SHEAR LAG APPROACH (2)

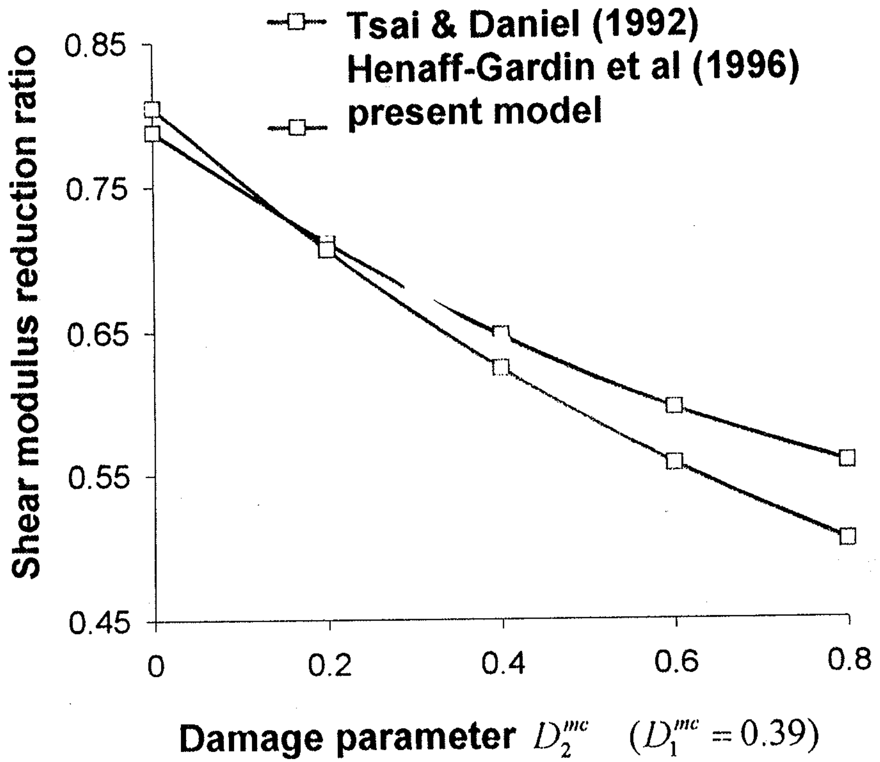
**GFRP** $[0/90]_s$ ▲ $[0/90_3]_s$ △



# VERIFICATION OF THE ECM/2-D SHEAR LAG APPROACH (3)

**AS4/3501-6** **$[0_3/90_3]_s$**




**Tsai and Daniel (1992)**

$$\rho_{T\&D} = \frac{1}{1 + \chi \dfrac{D_1^{mc}}{\lambda_2^{(1)}} \tanh \dfrac{\lambda_2^{(1)}}{D_1^{mc}} + \dfrac{1}{\chi} \dfrac{D_2^{mc}}{\lambda_2^{(2)}} \tanh \dfrac{\lambda_2^{(2)}}{D_2^{mc}}}$$

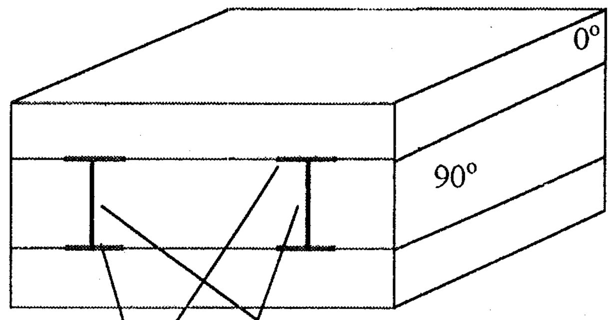
**Present model**

$$\rho^* = \frac{1 - \dfrac{D_1^{mc}}{\lambda_2^{(1)}} \dfrac{D_2^{mc}}{\lambda_2^{(2)}} \tanh \dfrac{\lambda_2^{(1)}}{D_1^{mc}} \tanh \dfrac{\lambda_2^{(2)}}{D_2^{mc}}}{1 + \chi \dfrac{D_1^{mc}}{\lambda_2^{(1)}} \tanh \dfrac{\lambda_2^{(1)}}{D_1^{mc}} + \dfrac{1}{\chi} \dfrac{D_2^{mc}}{\lambda_2^{(2)}} \tanh \dfrac{\lambda_2^{(2)}}{D_2^{mc}} + \dfrac{D_1^{mc}}{\lambda_2^{(1)}} \dfrac{D_2^{mc}}{\lambda_2^{(2)}} \tanh \dfrac{\lambda_2^{(1)}}{D_1^{mc}} \tanh \dfrac{\lambda_2^{(2)}}{D_2^{mc}}}$$

***No splitting***

$$\rho_{T\&D} = \rho^* = \frac{1}{1 + \dfrac{1}{\chi} \dfrac{D_2^{mc}}{\lambda_2^{(2)}} \tanh \dfrac{\lambda_2^{(2)}}{D_2^{mc}}}$$

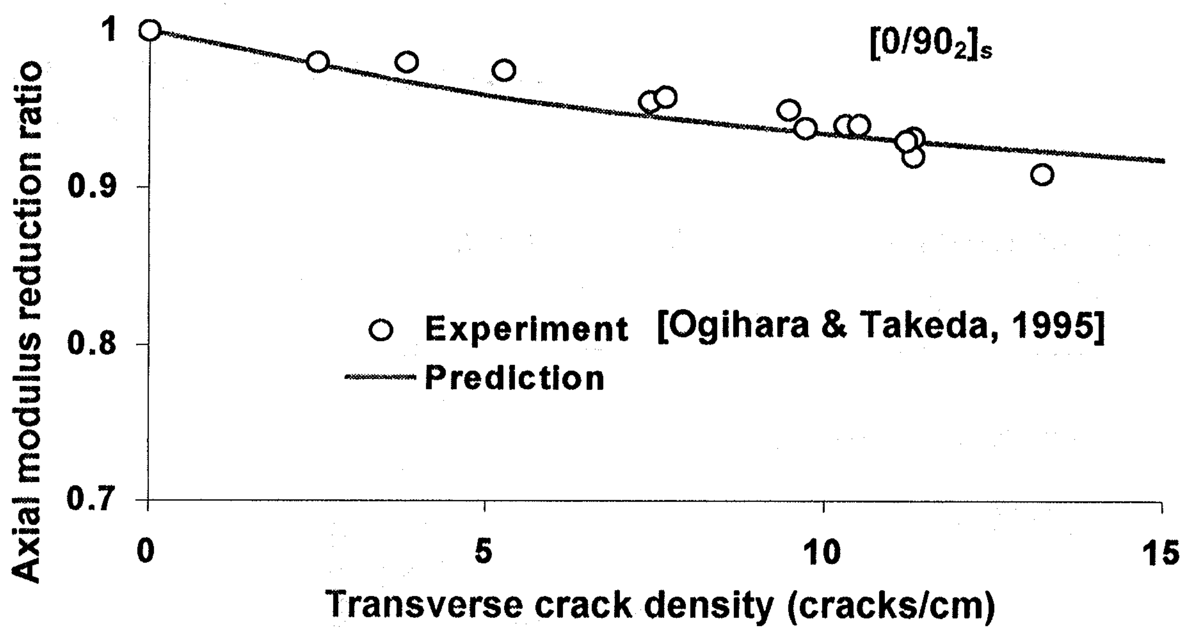
## *VERIFICATION OF THE MODEL:*
## *STIFFNESS DEGRADATION DUE TO TRANSVERSE CRACK TIP DELAMINATIONS*

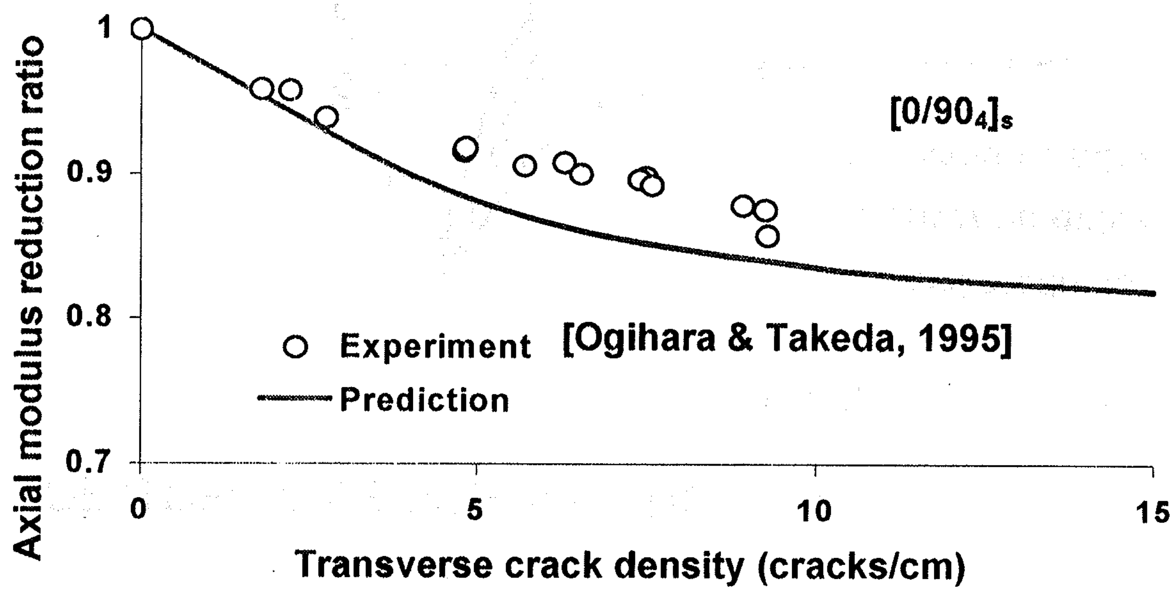



T800H/3631

$E_A$=152.2Gpa
$E_T$=9.57GPa
$G_A$=4.50GPa
$G_T$=3.21GPa
$v_A$=0.35
single ply thickness t=0.132mm

**relative delamination area 10%**

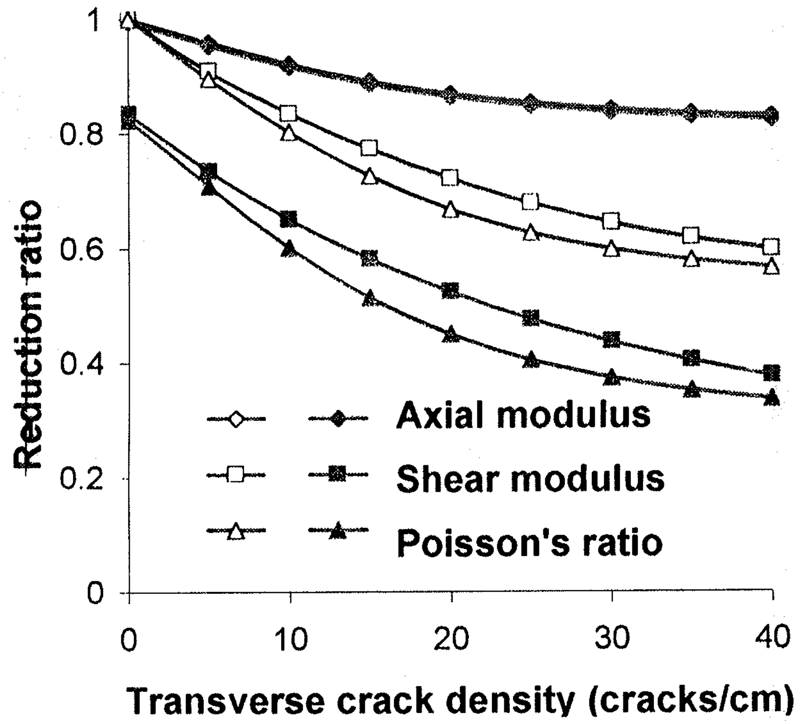


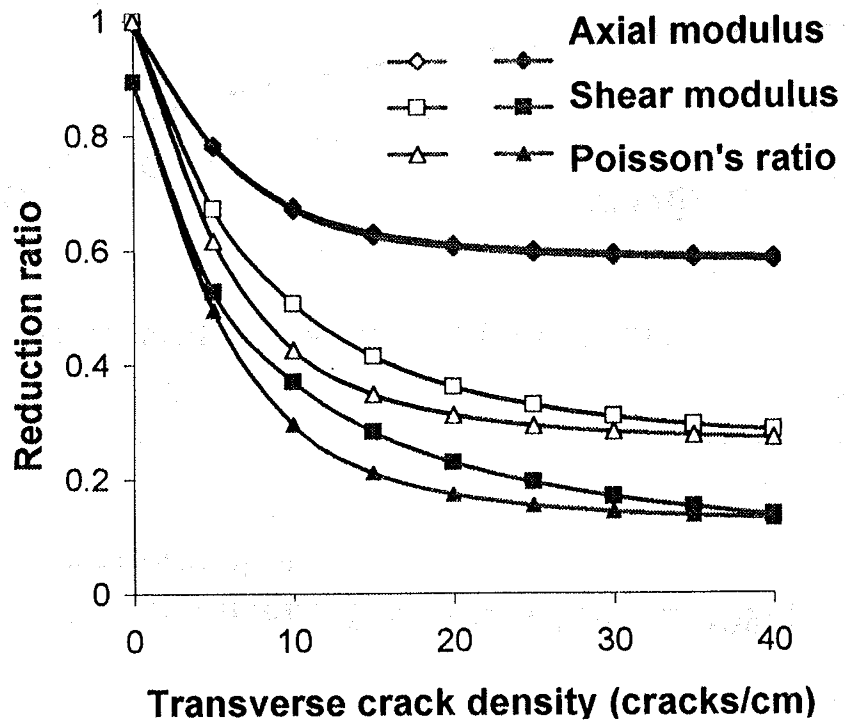
# PREDICTION OF STIFFNESS DEGRADATION (1)

E-glass/epoxy

$[0/90]_s$

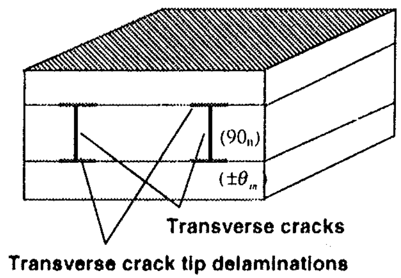



$[0/90_3]_s$

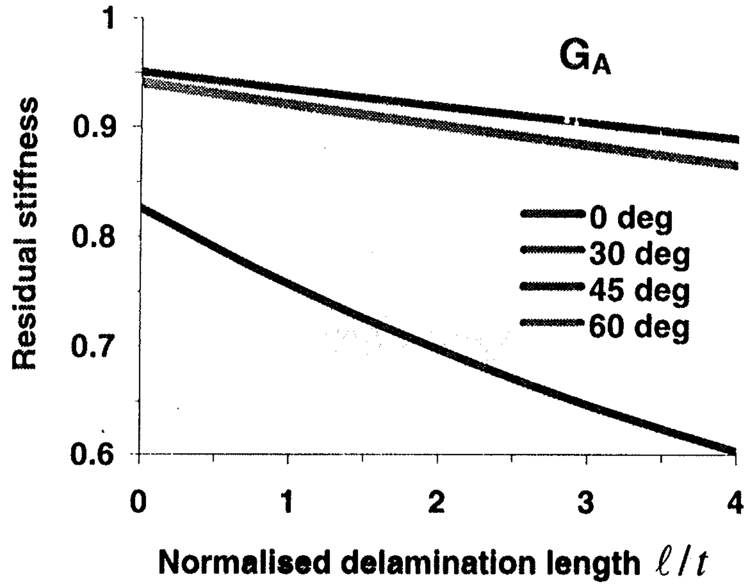


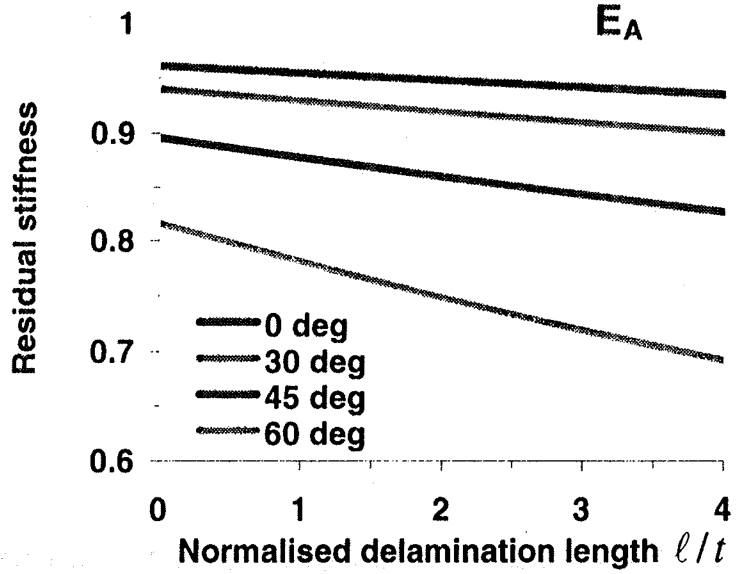

—— **Transverse cracking *without* splitting**
—— **Transverse cracking *and* splitting** (splitting density 10 splits/cm)

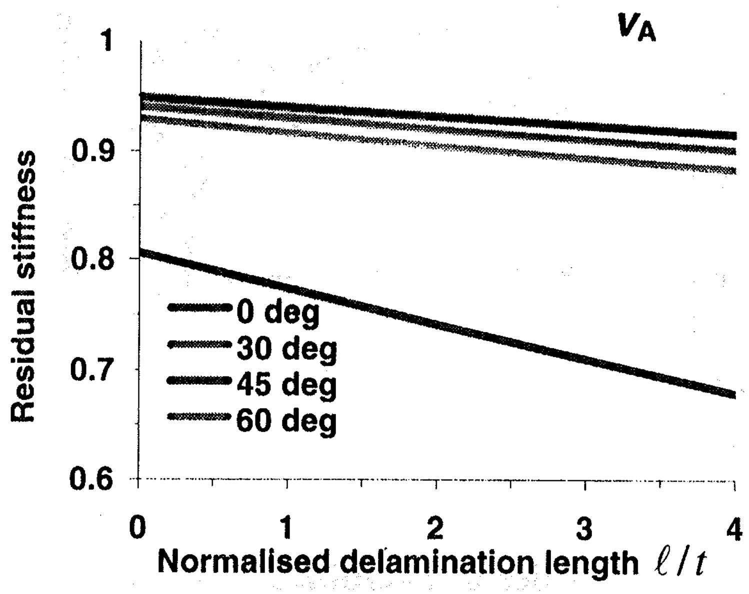
# Stiffness degradation due to transverse crack tip delaminations in $[+\theta/-\theta/90_4]_s$ laminates




**T300/934,** Crack spacing $s=16t$

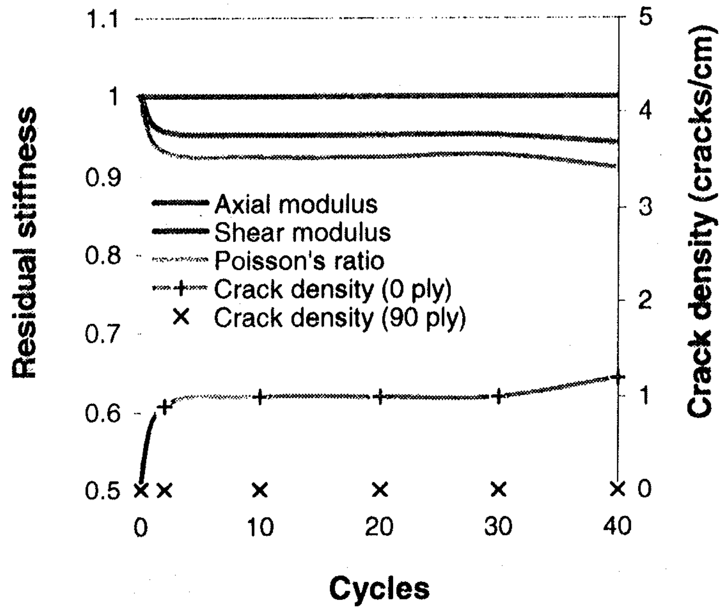


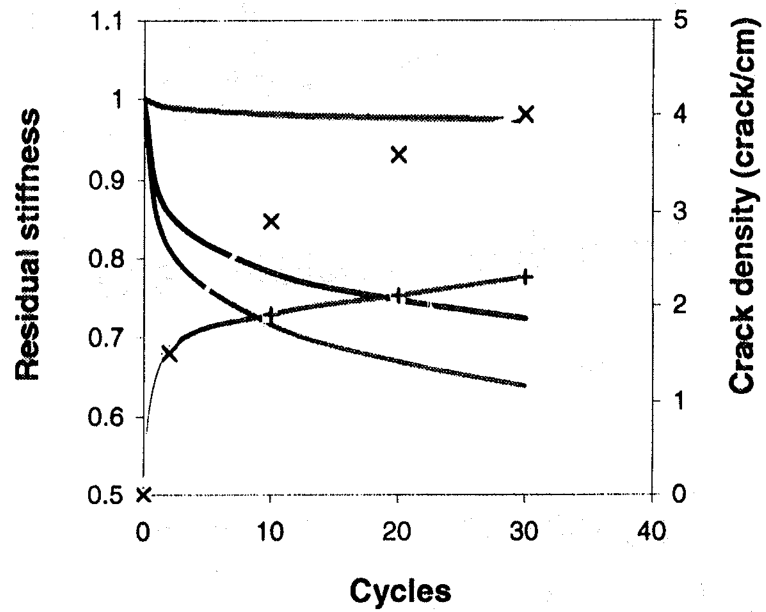



## Authentic crack patterns under thermal cycling

## T300/914 $[0_4/90_4]_s$



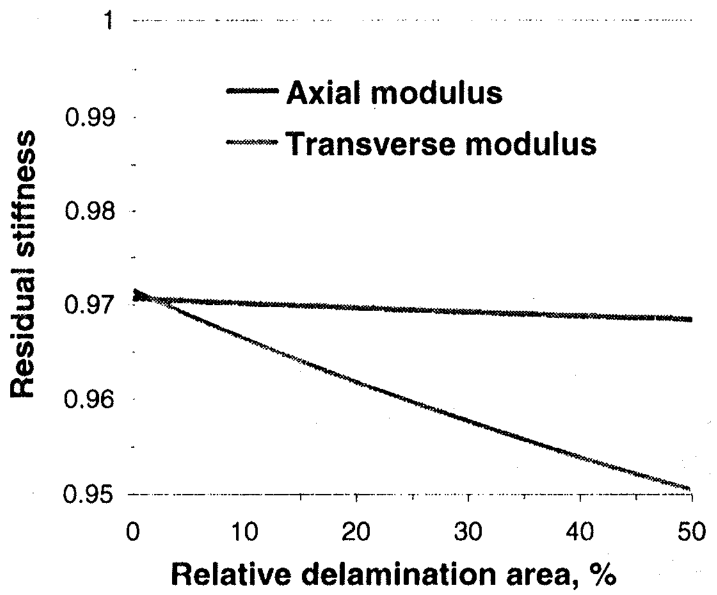

**-200°C/20°C**

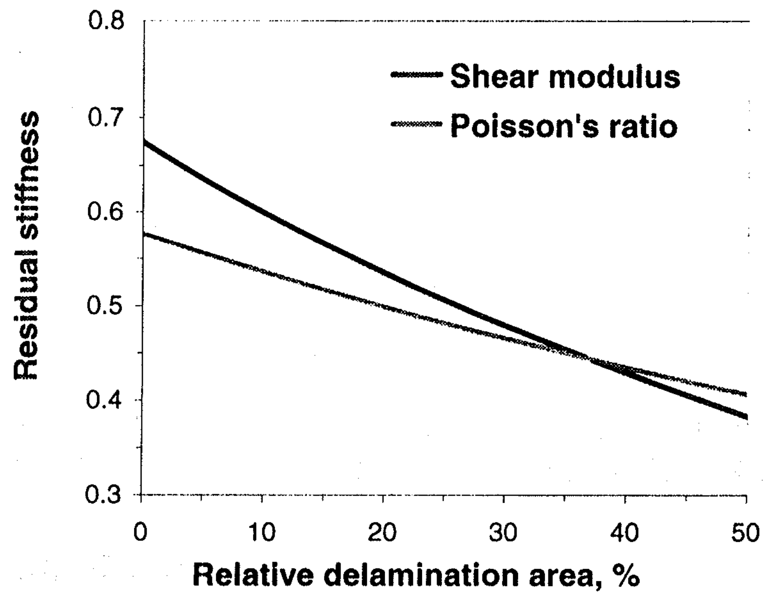



**-200°C/50°C**

## Effect of longitudinal delaminations

## T300/914 $[0_4/90_4]_s$




Crack density in the $0^{o}$ ply 3 cracks/cm
Crack density in the $90^{o}$ ply 4.5 cracks/cm

## CONCLUSIONS

- Equivalent Constraint Model (ECM) together with the improved 2-D shear lag method can be successfully used for analysis of composite laminates under fatigue loading
- The ECM/2-D shear lag approach shows acceptable agreement with earlier developed models as well as experimental data
- Longitudinal cracking (splitting) and delaminations in cross-ply laminates cause significant reduction of the shear modulus and Poisson's ratio
- The shear modulus and Poisson's ratio are much better non-destructive test parameters than the axial modulus and could be used to monitor damage development

# ACKNOWLEDGEMENTS

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*Prof. S. Ogihara* (University of Tokyo, Japan)

# Lifetime Prediction of FRP Components

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Imperial College of Science, Technology & Medicine

Mechanical Engineering Department

Centre for Composite Materials

# Fatigue life prediction of high performance composites

1

## Presentation Structure

- Introduction
- Previous Experimental Work
- Current Research
- Results
- Concluding Remarks

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## Previous Experimental Work

### Experiment Objective

- To study the mechanical performances of carbon-fibre and glass-fibre reinforced epoxy I-beams.
- To discuss the fatigue behaviour of the I-beams, under a four-point flexural configuration test.
- Produce fractographic observations of the failure of the I-beams.

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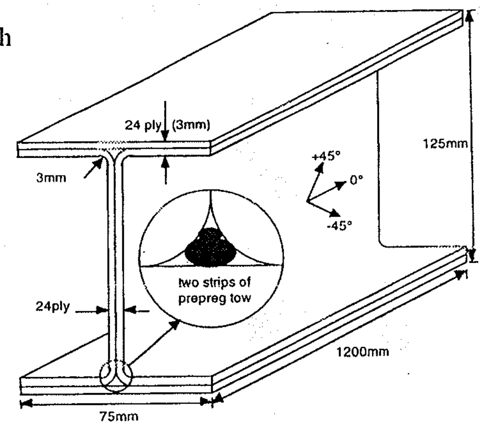
## Previous Experimental Work

- Two C- sections and two flange caps each has 12 plies of carbon/epoxy T300H/914
- Web and flanges have the same stacking sequence $(-45^\circ/0^\circ/+45^\circ)_{2s}$ $(+45^\circ/0^\circ/-45^\circ)_{2s}$
- Balanced antisymmetric combination of +45°, 0°, -45°
- 24 layer each of 0.125 mm, total thickness is 3 mm



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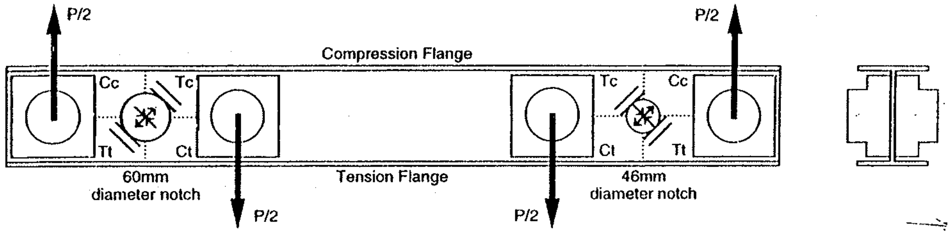
## Previous Experimental Work




Four-point flexure loading arrangement used for static and fatigue tests

$P_{min} = 5$ kN, $P_{max} = 50$ kN

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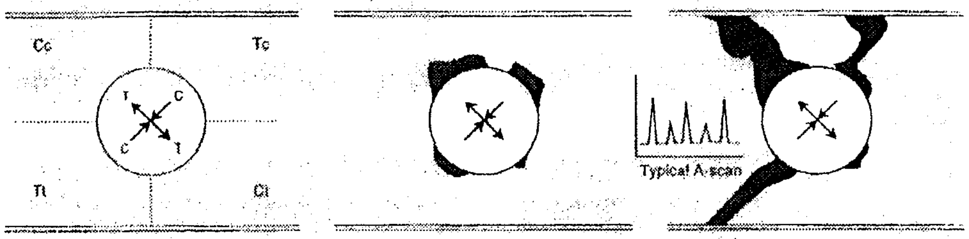
## Previous Experimental Work




After $0.74 \times 10^6$ cycles　　After $4.68 \times 10^6$ cycles　　After $4.78 \times 10^6$ cycles

Fatigue damage around the large notch (60 mm dia.) as detected using C-scan

## Previous Experimental Work

### Conclusions

- The unnotched I-beams did not exhibit any detectable damage between $1.2\times10^6$ and $8\times10^6$ fatigue cycles. The notched I-beam Fatigue ~~damage~~ failure occurred around the 60 mm diameter notch, after $4.78 \times 10^6$ cycles.
- Fatigue failure was due to matrix microcracking, delaminations and fibre fracture. Tensile stresses contributed largely to the damage around the notch.
- The principal mode of damage was matrix cracking, which led eventually to delaminations and fibre fracture.

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## Current Research

### Objectives

- Introduce a method for the calculation of the strain energy release rate by using the FEM.
- Establish a reliable theoretical procedure for life assessment of fibre reinforced composite components.
- Build an intelligent program which includes an automatic search technique for damage growth in composite laminates.
- Validate the technique by using it to predict the fatigue life of the I-beam.

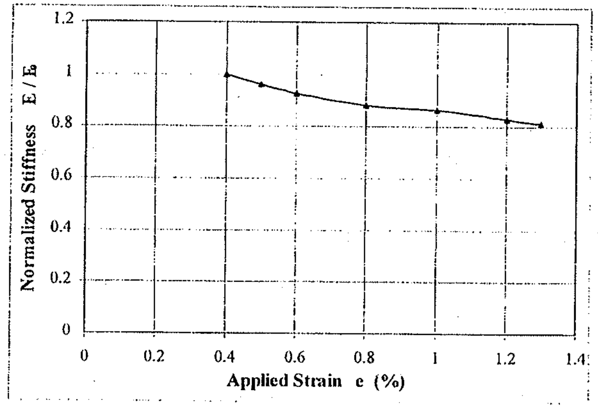
# Current Research

## Methodology

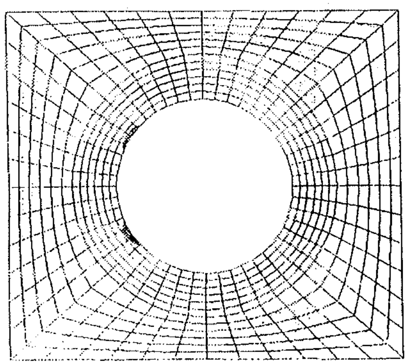
- The elements which have strain above the threshold matrix strain is detected and their combined areas is the damage area.
- The elastic strain energy and the work done by the external forces are to be recorded at the completion of each run.
- The mechanical properties of the damaged elements will be reduced according to "material" curve.



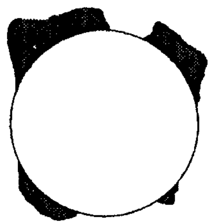
# Current Research




Material curve extracted from Tong *et al*, (1997)

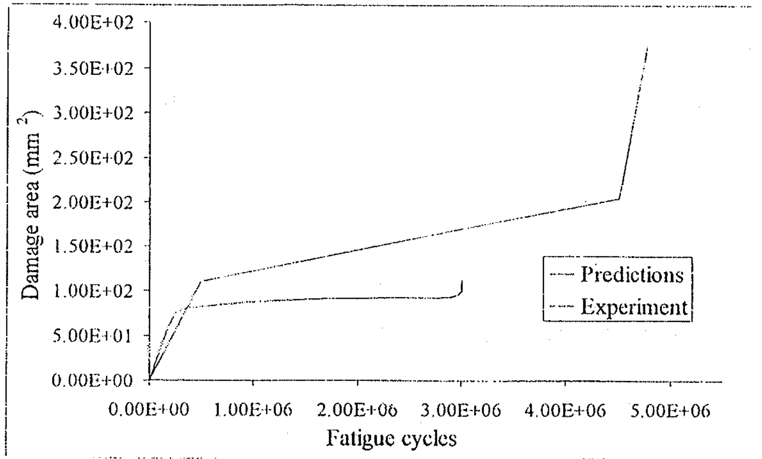


Matrix strain distribution produced by F.E.M. around the 60 mm hole in 0° layers.



Damage area around the 60 mm hole as detected by the fatigue test.

# Results



## Current Research

### Strain Energy Release Rate

*J*-Integral concept $$J = G = \frac{dU}{dA} = -\frac{d}{dA}(F - U_a)$$

$$G = \frac{dU}{dA} = \frac{U_2 - U_1}{A_2 - A_1}$$

Where $U$ is the total energy, $U_a$ is the change in the elastic strain energy, $F$ represents the work performed by external forces, $G$ is the energy release rate and $A$ is the damaged area.



## Current Research

### Role of Fracture Mechanics

Paris Law $$\frac{dA}{dN} = D\,G^n$$

Modified Paris Law $$\frac{dA}{dN} = D\,G_{max}^n \left\{ \frac{1 - \left(\frac{G_{th}}{G_{max}}\right)^{n1}}{1 - \left(\frac{G_{max}}{G_c}\right)^{n2}} \right\}$$

Where $D$, $n$, $n_1$, $n_2$, $G_{th}$ and $G_c$ are material constants to be found experimentally



## Current Research

### Role of Fracture Mechanics

$$N_f = \int_0^{A_f} \frac{1}{D\,G^n}\,dA$$

$$N_f = \frac{1}{2}\sum_{i=1}^{m}\left[A_{i+1} - A_i\right]\left[\left(\frac{1}{DG^n}\right)_i + \left(\frac{1}{DG^n}\right)_{i+1}\right]$$

Where $i = 1, 2, 3, \ldots\ldots\ldots m$ and $m$ is the number of ABAQUS runs/iterations



## Current Research

### Methodology

- A number of ABAQUS static analyis runs are needed to produce the stress and strain distributions of the composites.
- The matrix strain (transverse to the fibres) is checked within all layers at the end of each run.
- The fibre strain in all orientations is also checked for failure at the end of each run.

## Concluding Remarks

⇛ A reliable method has been developed for the prediction of the strain energy release rate.

⇛ The fatigue damaged area is the area which has a strain value above the threshold matrix strain.

⇛ An automatic search technique for the detection of the fatigue damage has been developed using an intelligent computer program.

## Concluding Remarks

⇛ The experimental value for the I-beam lifetime, was $4.78\times10^6$, the predicted values using the technique, the modified Paris Law and fracture mechanics values of $G_{th}$=50 and $G_c$=150, from Hojo *et al*, was $3.03\times10^6$.

⇛ The predicted lifetime for the coupon was around $5\times10^6$, while the fatigue test terminated around $7\times10^6$ due to the failure of the grips. This results indicates that the coupon would not make a suitable simulation of the beam, presumably because the stress field in the latter is much more complex.

## Concluding Remarks

⇛ The values of $n$ and $D$ were taken from best-fit plots to scattered results from DCB, and similar tests. Hence $n$ and $D$ can vary greatly depending on which experimental points are considered, and hence they have an effect on the fatigue lifetime predictions. Also, values of mechanical properties?

⇛ Subsequent investigation should be considered in the area of the material mechanical properties reduction, because this also has a strong effect on $G$, and hence on the predicted number of fatigue cycles to failure.

## Concluding Remarks

⇛ The prediction could be improved further if the 3-D finite element analysis is to be considered.

⇛ The programming search technique deals with composites which consists of any number of layers, but only up to three different fibres orientations, for example (−45°/0°/+45°). Therefore the technique could be developed further to include cases with more than three fibre orientations.

# A Constant Life Model for the Prediction of Fatigue Life in Reinforced Plastics

*Prof. Bryan Harris* (UNIVERSITY OF BATH)

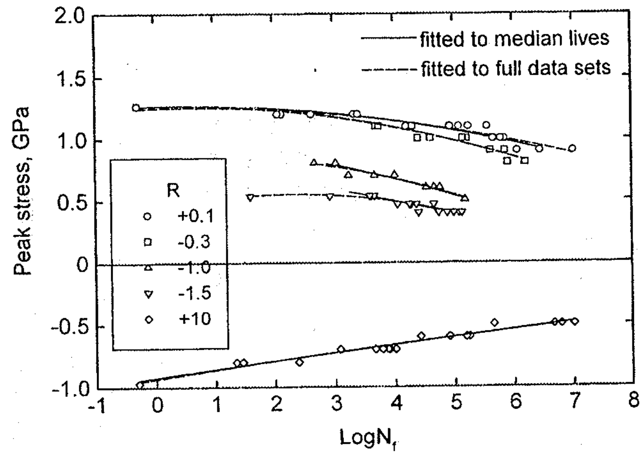
# FATIGUE LIFE PREDICTION FOR UNDAMAGED AND DAMAGED FIBRE COMPOSITES

**Bryan Harris**

Research funded by MoD/DERA

Collaborators:
T Adam, H Reiter, N Gathercole, DP Almond, MH Beheshty and JA Lee

University of BATH
Materials Science and Engineering




*S/logN data for HTA/913 laminate. The plotted points are the full fatigue data sets, and the curves are second-order polynomial fits to the data*

## Fatigue life prediction

constant-life model: $a = f(1 - m)^u (c + m)^v$

where: $a = \sigma_{alt} / \sigma_t$ $m = \sigma_{mean} / \sigma_t$ $c = \sigma_c / \sigma_t$

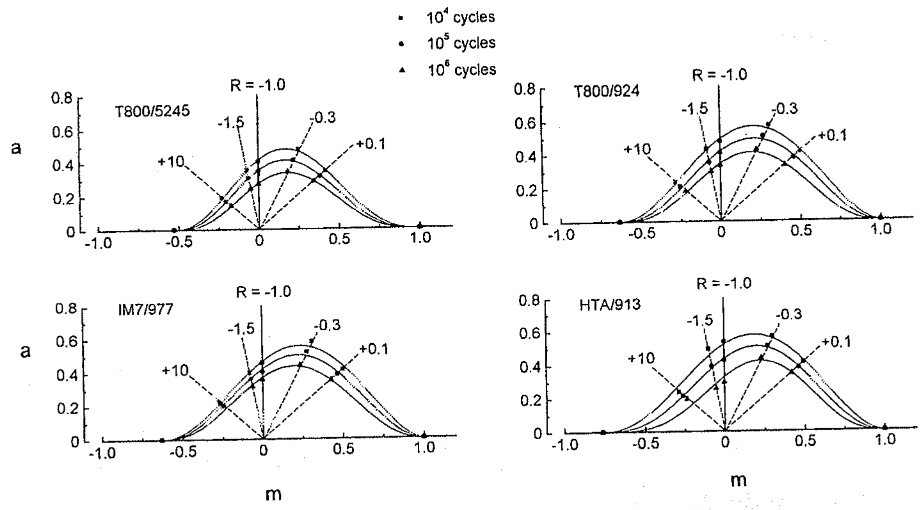
$f = \phi(c)$

u & v determine the shape of the curve

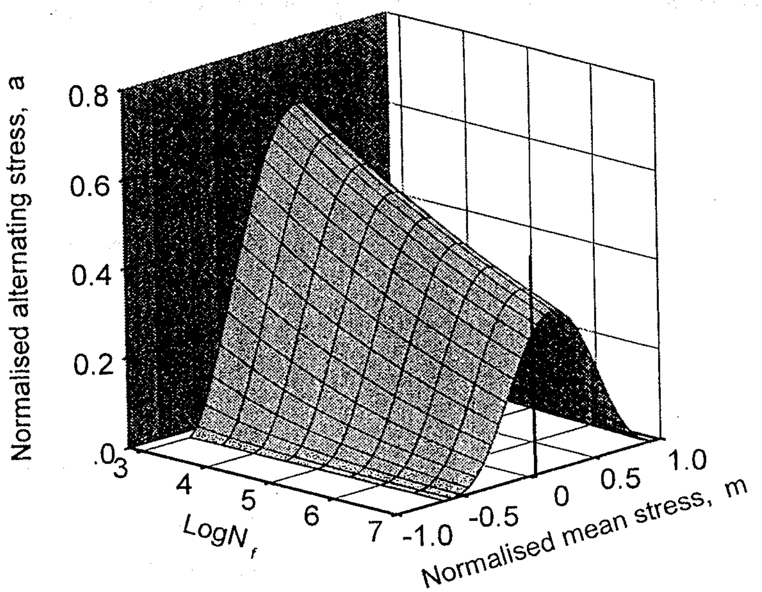
procedure:

- measure tension & compression strengths
- obtain some S/N data (different stresses & R ratios)
- determine values of f, v and u (and dependences on N )
- solve fatigue equations to obtain S/N curve for any R ratio
- do this for whatever failure probability is appropriate

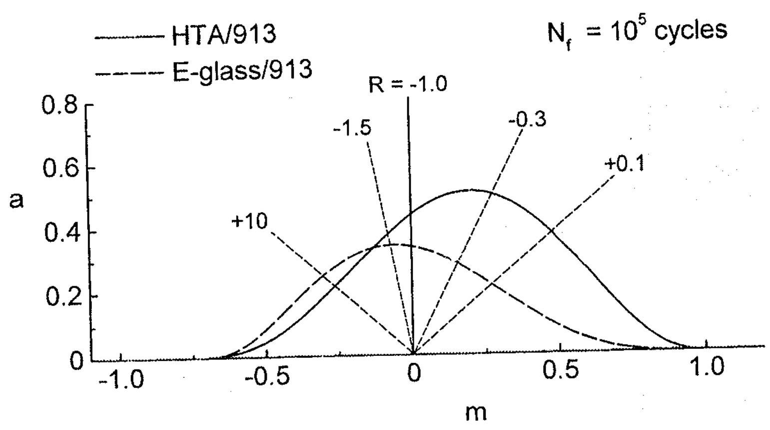
University of BATH
Materials Science and Engineering



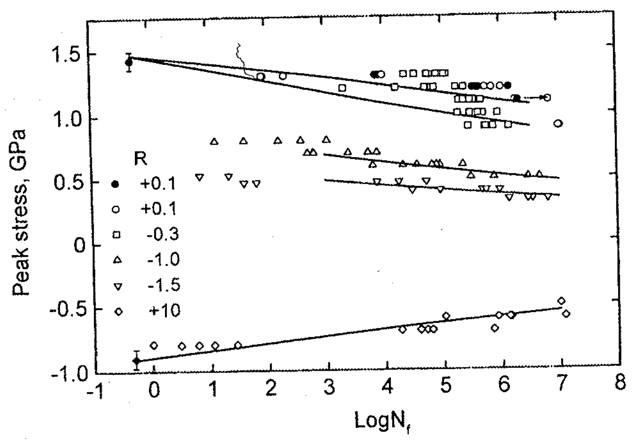

*Constant-life plots for four CFRP laminates of $[(\pm45,0_2)_2]_s$ construction.*



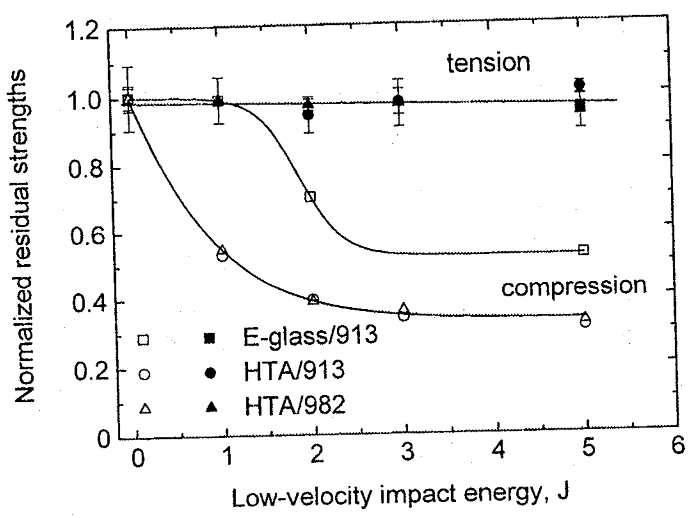

Predicted constant-life curves for a T800/924 CFRP laminate of $[(\pm45,0_2)_2]_S$ construction (Gathercole *et al*, 1984).



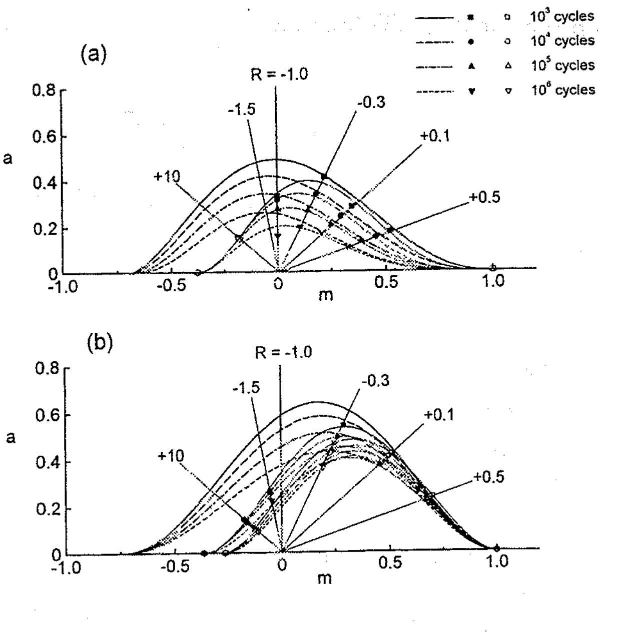

Constant-life plots for HTA/913 CFRP and E-glass/913 GRP laminates of $[(\pm45,0_2)_2]_s$construction for a life of $10^5$ cycles (Beheshty, 1997).



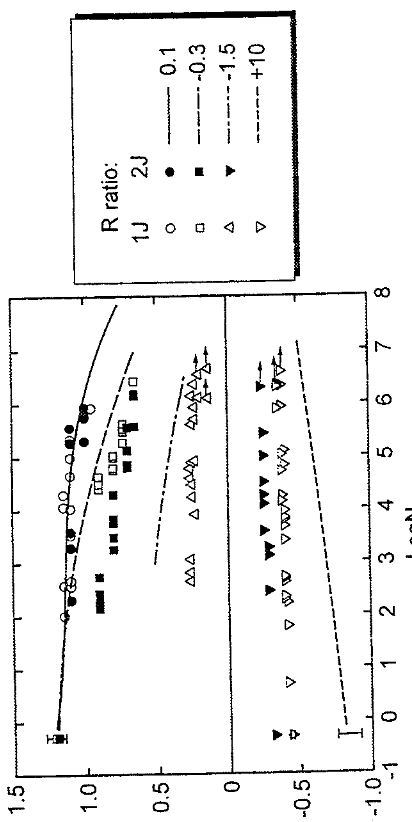

Stress/life test data (open symbols)for an IM7/977 CFRP laminate superposed on the preliminary predicted S/logN curves (lines) derived only from 5 test results (full symbols), the tension and compression strengths, and values of the fitting parameteres obtained previously for a different CFRP laminate (T800/924)



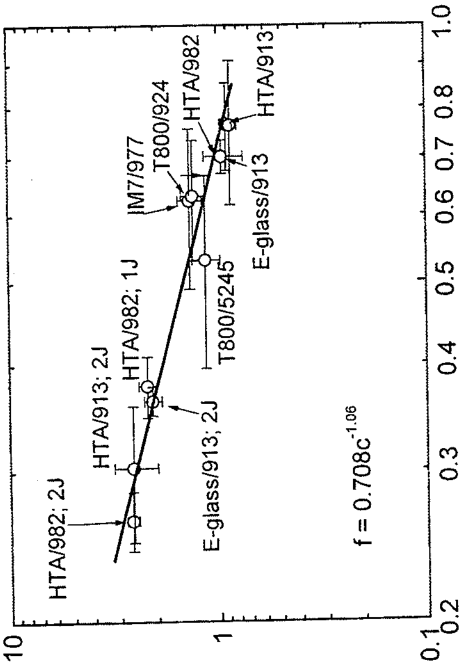

Effect of low-velocity impact on the residual compression and tensile strengths of HTA/913 and HTA/982 CFRP laminates and E-glass/913 (data are normalized with respect to the strength of the undamaged material). All laminates are of lay-up $[(\pm45,0_2)_2]_S$.




Constant-life plots for virgin and damaged $[(\pm45,0_2)_2]_s$ laminates.
(a) E-glass/913 glass/epoxy (2J impact damage);
(b) HTA/982 carbon/epoxy (1J (closed symbols) and 2J (open symbols) impact damage).
The data points are shown only for the damaged materials in order to avoid confusion.




$\sigma/\log N_f$ curves for HTA/982 CFRP laminate of $[(\pm45,0_2)_2]_s$ layup after damage by low-velocity impacts of 1 and 2J. The full lines are polynomial curves fitted to the data for the virgin laminate.




Relationship between f and c for all laminates, CFRP and GRP, virgin and damaged.
The f values are for a constant life of $10^5$ cycles (free fitting).
The best-fit line is $\log(f) = -0.15 - 1.06.\log(c)$

**Mathcad Worksheet for the Prediction of S/log$N_f$ curves from the constant-life function**

***Fatigue functions:*** $a = f(1 - m)^u(c + m)^v$  $a = \sigma_{alt}/\sigma_t$  $m = \sigma_m/\sigma_t$  $c = \sigma_c/\sigma_t$

$R = \sigma_{max}/\sigma_{min}$  $\sigma_{alt} = ½\sigma_{max}(1 - R)$

$\sigma_{alt} = \sigma_m(1-R)/(1+R)$  $\sigma_m = ½\sigma_{max}(1 + R)$

---

insert properties for laminate:

***Material properties:***
tensile strength: $\sigma_t := 1.267$
compression strength (+ve value): $\sigma_c := 1$  $c := \frac{\sigma_c}{\sigma_t}$  $c = 0.789$

f vs c relationship example for $N_f = 10^5$ cycles: $\log(f) = -0.15 - 1.058 \cdot \log(c)$

$f := 0.708 \cdot c^{1.058}$  $f = 0.551$

---

***Fatigue data.*** Enter stress/life data for R = 0.1 *(data file = data(stress, life)*

$\sigma_{max} := data^{<1>}$  $N_f := data^{<2>}$

$i := 1 .. \text{length}(N_f)$  $z_i := \log(N_{f_i})$

*From Constant-life plots:* $u := 2.13$  $v := 2.33$  *Mean u and v values for CFRPs: enter actual values if available*

*Dependence of 'f' on strength:* $f = \alpha c^{-\beta}$  *where α and β are both functions of logNf*

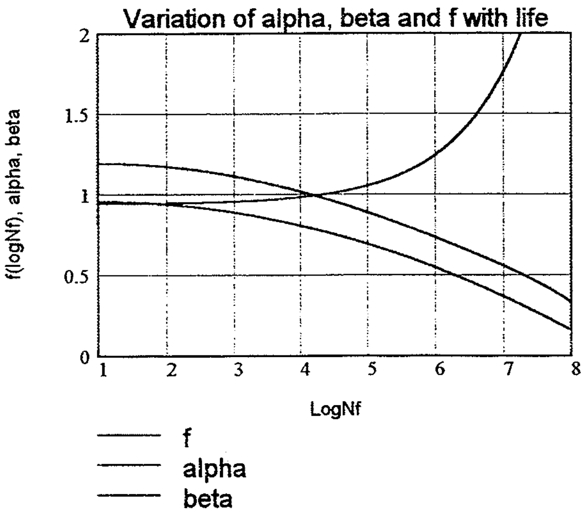
*For f as a function of log$N_f$,* if $n = \log N_f$

$\alpha(n) := 0.94 + 0.03 \cdot n - 0.016 \cdot n^2$

*(curve fits from Origin plots)*  $\beta(n) := 0.938 + (8.4 \cdot 10^{-4}) \cdot \exp\left(\frac{n}{1.016}\right)$  $\alpha(5) = 0.69$

$\beta(5) = 1.053$

$f(n) := \alpha(n) \cdot c^{-\beta(n)}$  $f(5) = 0.885$



## *Solution of two simultaneous equations:*

*(guesses)* $a := 0.3$

$m := 0.3$

Given

$$a = f(n)\cdot\left[(1-m)^{u}\cdot(c+m)^{v}\right]$$

$$a = m\cdot\frac{(1-R)}{(1+R)}$$

$am(n,f,R) := \mathrm{Find}(a,m)$

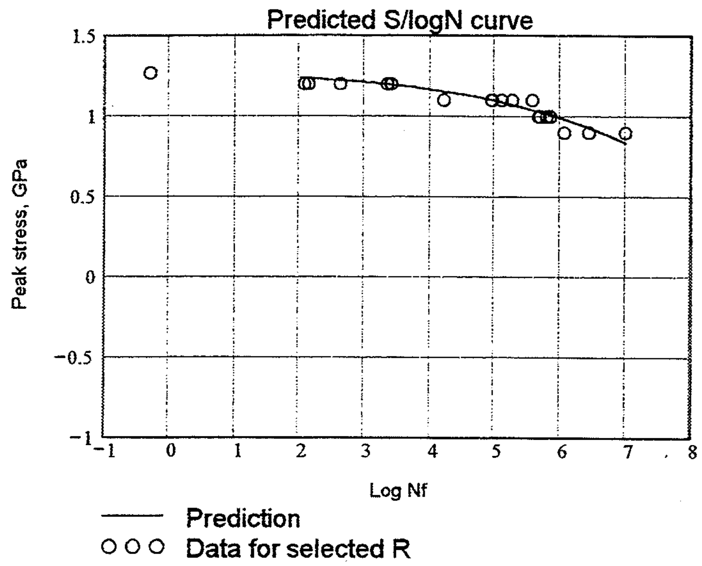
*select desired R ratio:*

For $R := 0.1$

$$C := \begin{cases} 20 & \text{if } R=10 \\ 2 & \text{otherwise} \end{cases}$$

and lives of: $n := 2, 2.2 .. 7$

$$smax(n) := \frac{C\cdot am(n,f,R)_{1}\cdot\sigma_{t}}{1-R}$$




*eg:* $smax(5) = 1.093$

$R = 0.1$

*For data output, select data table and right-click to use 'export' command*

For $R = 0.1$

| log life $n =$ | max stress $smax(n) =$ |
|---|---|
| 2 | 1.236 |
| 2.2 | 1.232 |
| 2.4 | 1.228 |
| 2.6 | 1.222 |
| 2.8 | 1.217 |
| 3 | 1.21 |
| 3.2 | 1.202 |
| 3.4 | 1.194 |
| 3.6 | 1.185 |
| 3.8 | 1.175 |
| 4 | 1.164 |
| 4.2 | 1.152 |
| 4.4 | 1.139 |
| 4.6 | 1.125 |
| 4.8 | 1.109 |
| 5 | 1.093 |