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Angles in the SI—a practical dimensional metrologist viewpoint*

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CORRIGENDUM: Angles in the SI—a practical dimensional metrologist viewpoint* (2026 *Metrologia* **63** 013001)

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Abstract

In this corrigendum we correct some numerical and symbol errors in table 2 found in the original manuscript. These errors were independently detected by the authors and a reader during the final production of the paper, but too late for consideration as proof corrections. Table 2 is our version of a table from a paper by Lehman*et al* showing how our proposal for the radian would fit with their scheme. We took their table and extended it to include the radian, taking into account their later published corrigendum. However additional errors, which persisted after the corrigendum by Lehman *et al* were not noticed until the final stages of publication of our paper. This corrigendum fixes these errors and doing so has required a small additional change to the symbols in tables 1 and 2. Of our corrected lines in table 2 for the entries for m, kg, A and rad, the first three would also suffice as corrections to the Lehman paper. We thank the reader for informing us of the errors.

Original paper by Lehman *et al* [1]. Corrigendum by Lehman *et al* [2].

Table 1. The defining constants, number labels, numerical values, SI units and dimensionality of the defining constants for the base units of measurement, including the radian. Although π is irrational and therefore not exact, it can be defined or used to arbitrary precision to be consistent with the other numbers N_1 to N_4. We use 2π as the constant, referring to complete rotation, but could equally use just π and refer to half-rotation in the definition of plane angle. We do not propose to name, and have no preference for, any particular symbol to be used for the radian's defining constant, instead we simply use ↻ as the symbol representing the plane angle in a complete rotation.

				Dimensionality				
Defining constant	Number label	Exact defined numerical value	SI units	s	m	kg	A	rad
$\Delta\nu_{Cs}$	N_1	$9.192\,631\,770 \times 10^9$	s^{-1}	-1	0	0	0	0
c	N_2	$2.997\,924\,58 \times 10^8$	s^{-1} m	-1	1	0	0	0
h	N_3	$6.626\,070\,15 \times 10^{-34}$	s^{-1} m^2 kg	-1	2	1	0	0
e	N_4	$1.602\,176\,634 \times 10^{-19}$	s A	1	0	0	1	0
↻	N_5	2π	rad	0	0	0	0	1

* The views expressed are those of the authors and do not necessarily represent those of NPL Management Ltd.

Table 2. The defining formulae and multiplier values, of the base units of measurement, including the radian.

Base unit	Exact multiplier	Approximate multiplier	Full defining formula	Concise defining formula
s	N_1	$9.192\,631\,770 \times 10^9$	$\Delta\nu_{Cs}{}^{-1}c^0h^0e^0\circlearrowleft^0$	$\Delta\nu_{Cs}{}^{-1}$
m	$N_1N_2^{-1}$	$3.066\,331\,899 \times 10^1$	$\Delta\nu_{Cs}{}^{-1}c^1h^0e^0\circlearrowleft^0$	$\Delta\nu_{Cs}{}^{-1}c$
kg	$N_1^{-1}N_2^2N_3^{-1}$	$1.475\,521\,400 \times 10^{40}$	$\Delta\nu_{Cs}{}^{1}c^{-2}h^1e^0\circlearrowleft^0$	$\Delta\nu_{Cs}c^{-2}h$
A	$N_1^{-1}N_4^{-1}$	$6.789\,686\,817 \times 10^8$	$\Delta\nu_{Cs}{}^{1}c^0h^0e^1\circlearrowleft^0$	$\Delta\nu_{Cs}e$
rad	N_5^{-1}	$1.591\,549\,431 \times 10^{-1}$	$\Delta\nu_{Cs}{}^{0}c^0h^0e^0\circlearrowleft^1$	$\circlearrowleft$

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[1] Lehman J, Migdall A, Scherschligt J and Whitehead L 2024 *Metrologia* **61** 033001
[2] Lehman J, Migdall A, Scherschligt J and Whitehead L 2024 *Metrologia* **61** 049501

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Angles in the SI—a practical dimensional metrologist viewpoint*

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Abstract

The International System units of plane angle (radian) and solid angle (steradian) have been the subject of much international debate and discussion, stretching back over several decades. This letter presents the viewpoint from practising dimensional metrologists for whom the realisation of angular units is their responsibility.

1. Introduction

The status of plane and solid angles in systems of units, especially the International System of Units ('SI') has been in a state of flux over the preceding two centuries. Angular units were not included in the original 'metric system', nor in the following centimetre-gram-second, metre-kilogram-second (MKS) or the Giorgi Internatioal System of Measurement (also known as the MKSA or metre-kilogram-second-ampere system). When the SI was finally adopted in 1960, plane and solid angles were classed as dimensionless quantities, and their units radian and steradian were included into a separate class of supplementary units. In 1995 the 20th CGPM decided to interpret the radian and the steradian as dimensionless derived units and eliminate the class of supplementary units as a separate class in the SI.

Now the status of angle units is again the subject of international debate not only at the CIPM's Consultative Committee for Units, but also through a recent flurry of published journal papers. Such discussions tend to be clustered in time, with one paper re-starting the discussion after a few years, leading to several papers from different authors supporting new proposals or dismissing them due to generation of additional issues by what is proposed. After a short flurry of papers, with no firm consensus being reached, the discussion dies down and things are quiet again for several years.

An early set of discussions took place in publications [1–6] in the early to mid- twentieth century leading up to the first edition of the SI Brochure in 1970. It is interesting to note that in the 1st edition [7], the SI Brochure states:

Pour quelques unités du Système International, la Conférence Générale n'a pas ou n'a pas encore décidé s'il s'agit d'unités de base ou bien d'unités dérivées. Ces unités SI sont placées dans la troisième classe, dite des « unités supplémentaires» et l'on ets libre de les traiter soit comme des unités de base, soit comme des unités dérivées. (English translation: For some units of the SI, the General Conference has not or has not yet decided whether they are base units or derived units. These SI units are placed in the third class, called 'supplementary units' and one is free to treat them either as base units or as derived units). The General Conference was aware of the need for angular units but not certain how to classify them—the situation remains today.

Thus there was a set of proposals and discussions in the early 1980s [8–10] followed a few years later by a longer discussion [11–18]. All was quiet for a while until 1995 [19] and then nothing more was published until the early 2020s [20–25]. Around the time that the revision of the SI was being discussed, the subject of angles in the SI came to the fore again and seems to be a constant topic of discussion ever since [26–45].

2. A different viewpoint

So far discussions on angle in the SI have not included direct inputs from the dimensional metrology community—the experts at NMIs with responsibility for realising units of angle and for taking part in key comparisons concerning angle metrology (the

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K3 topic of the Consultative Committee for Length—CCL). At its 19th meeting in October 2024, the CCL noted the disparity between the way that plane angle is defined and how it is realised in practice and tasked the CCL Discussion Group 3 (on angles) to '... *discuss and prepare a consensus position on plane angles.*'[46].

The SI Brochure [47] frequently uses the term 'practical' when describing the SI, and although unit definitions do not need to be the same as their methods of realisation, the current definition of the unit of plane angle, the radian, could be improved (and several associated issues removed) by looking at how the unit is realised in practice. A note in the Brochure on how the radian is related to base units states:

'The radian is the coherent unit for plane angle. One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius'.

Within the dimensional metrology community, although small traceable angular displacements are often realised through a classical sine arm (a change in height at a known radius), the main method of realising large angles is the use of *intrinsic properties of the circle*, through the *calliper* principle [48]. From a purist point of view, realising the unit of plane angle by a method which actually requires a physical *rotation* through an angle confined to a plane seems intrinsically more 'real' than simply considering, for example, length ratios on a static diagram. Furthermore, since the radian is not a base unit, it has no *mise en pratique*. Surely, then, it is important that the only place where it is 'defined' in the SI (the SI Brochure) should define the unit in the way that it is practically realised by the dimensional metrology community or at least take note of the way it is realised?

2.1. Principle for circular division

Angular measurement and circular division have the significant advantage that no arbitrary standard of angle is required as all angles can be obtained from first principles by uniform division of the circle. The fundamental concept underlying the use of the principle is that a complete circular division of a circle (i.e. a complete circle or complete rotation) must be equal to 360° (or 2π rad, or whatever plane angle unit is desired).

As an example, consider the process to mark a circular disc (for example a bare clock dial) into 90° divisions (see figure 1). Firstly mark the edge of the disc in one location, (1), on its rim. Make a second mark (2) *approximately* 180° apart from (1) - one can be somewhat imprecise here as shown in figure 1.

Next set two indicators (e.g. microscopes) such that each one is aligned with one of the two marks. For example position each microscope such that the mark is centred in the field of view, ideally centred in the cross-wires of the reticle of the microscope.

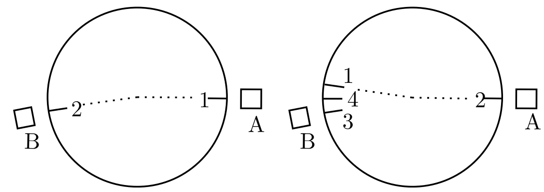

Figure 1. Dividing a circle using the calliper principle.

Microscope A points at mark (1) and microscope B points at mark (2).

Next, the disc is rotated until mark (2) is centred within microscope A. In general, unless the marks were initially exactly 180° apart, mark (1) will now not be centred in microscope B. Put a new mark (3) on the rim exactly where microscope B is centred.

The difference between lines (1) and (3) now represents twice the error from the intended 180° spacing, therefore place a new line (4) half way between lines (1) and (3), by simply estimating its location. Lines (2) and (4) now divide the circle into 180° divisions. (If the placing of line (4) was not accurate enough, re-position the microscopes to observe lines (1) and (4) and repeat the rotation process to derive a new line (5). These last two steps can be repeated until the required accuracy is obtained).

Additional lines are now placed at approximately the 90° and 270° positions and the microscopes are repositioned to be approximately 90° apart. A repeat of the above process allows the 90° and 270° line positions to be readjusted until the four final lines show no deviations from the microscope centre of view when rotated.

The disc has now been divided into 90° intervals, based only on the property that we defined a complete circle to *intrinsically* contain 360° and through the process of *rotation*. There was no need to derive a small angle through a sine arm (so no awkward length ratios which are reduced to metre/metre and thus appear dimensionless), nor a need to compare an arc length on the circumference with the radius (the current SI definition of the radian). In fact dimensional metrologists at NMIs rarely measure distances along arcs as our highest accuracy linescale standards are calibrated only when planar, *i.e.*, flat.

2.2. Principle for polygon calibration

Many NMIs have calibration services for optical polygons and these are mainly based on using the calliper (or closure) principle in reverse [49–51].

Consider a polygon with n faces, with the interface angles, denoted as $A_1, A_2, \ldots, A_n$ nominally $2\pi/n$ rad. Mount the polygon on a rotary table and position two autocollimators such that they face consecutive faces of the polygon and precisely align each autocollimator with its respective face. The angle between the autocollimators is denoted X. Denoting the deviations of the face angles from their nominal values by $a_1, a_2, \ldots, a_n$, and the deviation of the angle between the autocollimators from its nominal value by x, we have

$$A_i = 2\pi/n + a_i, i = 1, 2, \ldots, n, \tag{1}$$

$$X = 2\pi/n + x. \tag{2}$$

Then we have $A_i - X = a_i - x, i = 1, 2, \ldots, n$.

The constraint that all the angles in a circle add to 2π rad gives

$$2\pi = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} (2\pi/n + a_i) \tag{3}$$

$$= n(2\pi/n) + \sum_{1=1}^{n} a_i \tag{4}$$

$$\sum_{1=1}^{n} a_i = 0. \tag{5}$$

The system has $n+1$ unknowns—the n values of A_i and the value of X. However by rotating the polygon such that each face is successively aligned with one of the autocollimators and reading the value from the other autocollimator, we get n measurements of the deviation angles and adding to this the knowledge from equation 5, we have $n+1$ knowns so the system can be solved to obtain the values of all a_i and hence all values of A_i as well as a value for X. Of course, instead of using 2π rad we could have used 360° as the relevant constant, or any other defined value.

The critical feature here is the use of closure which is obtained by physical *rotation* of the polygon, i.e. physically exploring the angles between the faces and physically allowing and using the properties of circle closure. Again, no length ratios or comparison of radius with circumferential arcs is required.

3. Proposal

The current definition of plane angle in the SI is based around the relationship that the arc length along a circumference of a circle is equal to the radius of the same circle, when the arc subtends an angle of one radian. This immediately suggests a comparison of quantities of length as being the way to realise the unit of plane angle, the radian. Such a viewpoint causes issues as it leads to the consideration of length ratios and thus concludes that plane angles must be dimensionless or have the dimensions of unity simply because the mathematical relationships lead to the ratio of length/length which reduces to unity. In this reduction we have somehow lost the fact that angles are not merely length ratios but actually describe a physical phenomenon of *rotation*.

In any axiomatic system of units, we are forced to select *primitives* which cannot themselves be defined by reference to other properties. We therefore try to select properties of the Universe which are unique in relation to other primitives; we are unable to define length without recourse to the unit of length, and vice versa, but we believe that length is different to mass, which is different to electrical current and so forth.

Using a definition of plane angle which is implicitly or explicitly based on comparing quantities such as lengths inherently breaks the requirement that a base unit cannot be defined by a combination of other units. So one may conclude that perhaps angles are not base units. But we think this is wrong—it simply means that we have picked the wrong definition of the radian. We can define how to measure the magnitude of a rotation (as length ratios) but this misses out the physical act of something rotating. Rotation is a point-like phenomenon which only leads to measurable length ratios when the rotation is connected to a length such as through the radius of a spinning disc.

Rotations generate measurable phenomena which can be measured in units of SI base units when the experiment is specifically constructed to use such base units in the measuring process. This may seem to indicate that rotation is a physical phenomenon that is somehow different to the other SI base units but its effect can be measured in SI units. However there is an alternative viewpoint. A physical phenomenon which has a measurable property which is unique to the phenomenon is exactly what we consider to be one of the axiomatic primitives of a system of units, i.e. a base unit; mass has a measurable phenomenon which is completely different to, say, electrical current, so we conclude that mass and electrical current are base units. So why do we not allow plane angle to be a base unit? Perhaps it is because we can draw plane angles on a piece of paper and think that the space between them somehow *is* a plane angle whereas in fact it is only a *representation* of a plane angle in the same way that a pointer on a scale is a *representation* of a mass. If one doubles the mass, the pointer moves twice as far. We can measure ratios of masses as ratios of lengths on the scale, but it does not mean that we should use ratios of lengths in the definition of the kilogram as that would lead to problems. Likewise we should not be defining a unit of angle using length ratios.

Photons are fundamental particles that travel at speed c which has SI units of metre/second. Photons act as a scale factor between length and time in 4D spacetime and we use c as a defining constant in the SI for the unit of length, with it acting as a link to the unit of time. Rotations are also physical entities which act as a scale factor between SI base units, but in a way that the interaction is between two instances of the *same* base unit. In the same way that c acts as a scale factor between length and time, for rotations (where

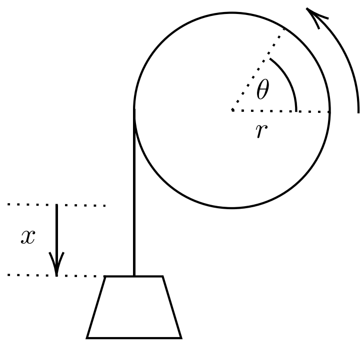

Figure 2. Physical example of simple pulley with weight attached by string around the pulley circumference.

the angle is measured in radians), the scale factor 2π links two instances of the same unit. In such a way, 2π can be considered to be the defining constant for rotations; it is a fundamental property of this Universe that circles or complete rotations contain 2π radians, in the same way that photons travel at speed c. If we are willing to allow length and time to both be SI base units, despite there being a coupling constant between them (c) we should allow plane angle to also be an SI base property with a relevant unit. In the same way that we choose values of the fundamental constants such that we end up with a consistent coherent set of base units, we should do the same for the unit of plane angle. We need to ensure that there is no additional numerical factor (other than the defining constant). Length couples coherently with time when we use the metre as the unit of length, the second as the unit of time, and c as the defining (coupling) constant. For plane angle, our equations become coherent when we use the radian as the unit of plane angle and use 2π as the defining constant.

As two orthogonal rotations can be used to generate a solid angle, there is no need to define a solid angle as a base unit in the way that the SI only defines a base unit of length and not a base unit of area.

Discussions about the dimensionality of angles and the status of angles in the SI have struggled because definitions of plane angle have been used which are ratios of entities with the same quantities (*e.g.* length/length) leading to dimensionless outcomes or dimension of unity. When we associate such arguments with measurements of properties of circles, a factor of 2π pops out of the mathematics (when we use radians as the angular units) and this causes consternation and confusion. We ought, instead, to embrace this numerical constant as a link between SI units in the same way that we embrace c as a fundamental property of this Universe linking length and time. When an object 'travels' in 4D spacetime, c governs certain properties; when an object rotates, 2π governs certain properties.

Figure 2 is a practical illustration of this concept. As the mass on the string descends under gravity, it rotates the pulley. There is a simple equation relating the vertical position, x, of the weight, the radius of the pulley, r, and the rotation angle, θ. One may think the equation is simply

$$x = r\theta \tag{6}$$

when θ is in radians, However the equation is then dimensionally unbalanced as x and r have dimensions of metres and θ has dimensions of radians, meaning the left hand side of the equation is in metres and the right hand side is in metre radians. However, if we use the definition of plane angle that there are 2π radians in a full rotation, then the equation correctly becomes

$$x = \frac{2\pi r\theta}{2\pi} \tag{7}$$

The 2π in the numerator is there because of the mathematics of circles—the circumference is 2π times the radius. The 2π in the denominator is there because we are using a fraction of a circle's circumference; the fraction is $\theta/2\pi$ when we measure the angle in radians, taking note that there are 2π radians in a full circle. As earlier, the dimensionalities of r and x are metres, but we now note that the dimensionality of θ is radians **as is the dimensionality of the 2π in the denominator**. Or, more explicitly, using square brackets to denote dimensions:

$$x\,[\text{metres}] = \frac{2\pi\, r\,[\text{metres}]\,\theta\,[\text{rad}]}{2\pi\,[\text{rad}]}. \tag{8}$$

The dimensions of both sides of the equation are now both metres. We simply have to be aware that sometimes 2π appears due to the mathematics of geometry, and sometimes due to working in radians when using angle units.

In summary, as a first step, we should change the definition of the unit of plane angle to follow the way that it is realised in practice, by simply stating that there are 2π radians or 360° in a full rotation. Next, there are arguments to suggest that, rather than defining plane angle as a derived unit, it should be defined as a base unit and the definition should include no extraneous detail such as other base units, but simply refer to a defining fundamental constant (2π if we use radian as the unit).

3.1. Links with recent proposals

In their 2024 paper [52] Lehman *et al* propose that, at the next revision of the SI (likely to be triggered by the redefinition of the second), the SI is redefined to have three categories of units: four base measurement units, physiologically relevant derived units, and the remaining units being derived from the base units. They limit the base units to the kilogram, metre, second, and ampere, and place the candela, mole and kelvin into the second class of physiologically relevant derived units.

Table 1. The defining constants, number labels, numerical values, SI units and dimensionality of the defining constants for the base units of measurement, including the radian. Although π is irrational and therefore not exact, it can be defined or used to arbitrary precision to be consistent with the other numbers N_1–N_4. We use 2π as the constant, referring to complete rotation, but could equally use just π and refer to half-rotation in the definition of plane angle.

Defining constant	Number label	Exact defined numerical value	SI units	Dimensionality				
				s	m	kg	A	rad
$\Delta\nu_{\mathrm{Cs}}$	N_1	$9.192\,631\,770 \times 10^9$	s^{-1}	-1	0	0	0	0
c	N_2	$2.997\,924\,58 \times 10^8$	$\mathrm{m\,s}^{-1}$	-1	1	0	0	0
h	N_3	$6.626\,070\,15 \times 10^{-34}$	$\mathrm{s}^{-1}\,\mathrm{m}^2\,\mathrm{kg}$	-1	2	1	0	0
e	N_4	$1.602\,176\,634 \times 10^{-19}$	s A	1	0	0	1	0
2π	N_5	2π	rad	0	0	0	0	1

Table 2. The defining formulae and multiplier values, of the base units of measurement, including the radian.

Base unit	Exact multiplier	Approximate multiplier	Full defining formula	Concise defining formula
s	N_1	$9.192\,631\,770 \times 10^9$	$\Delta\nu_{\mathrm{Cs}}{}^{-1}c^0h^0e^0\pi^0$	$\Delta\nu_{\mathrm{Cs}}{}^{-1}$
m	$N_1^{-1}N_2$	$3.066\,331\,899 \times 10^1$	$\Delta\nu_{\mathrm{Cs}}{}^{-1}c^1h^0e^0\pi^0$	$\Delta\nu_{\mathrm{Cs}}{}^{-1}c$
kg	$N_1N_2^{-2}N_3$	$1.475\,521\,340 \times 10^{40}$	$\Delta\nu_{\mathrm{Cs}}{}^{1}c^{-2}h^1e^0\pi^0$	$\Delta\nu_{\mathrm{Cs}}c^{-2}h$
A	$N_1^{-1}N_4^{-1}$	$6.790\,902\,147 \times 10^8$	$\Delta\nu_{\mathrm{Cs}}{}^{1}c^0h^0e^1\pi^0$	$\Delta\nu_{\mathrm{Cs}}e$
rad	N_5	1×10^0	$\Delta\nu_{\mathrm{Cs}}{}^{0}c^0h^0e^0\pi^1$	2π

The authors omit mention of the units of plane and solid angle. As they do not explicitly place these into either of the first two categories, the logical conclusion is that they would regard them as being in the third category of 'remaining units derived from the base units'. However, as discussed in this letter, the units of angle are not really derived from other units—they can be measured as ratios of other units but they are unique in terms of their physical properties and ratios of other units are not always angles.

Are angles 'physiologically relevant units'? They do not seem to match the criteria used for the kelvin or candela which are properties sensed by humans. Angles are also not the same as the mole, which is a unit used for counting elementary entities.

As stated above, angles are a physical property associated with rotation and none of the other units in the three categories defined by Lehman *et al* describe rotations. We should then conclude that they ought to be classed in the first category of base units with defining constants. With this in mind, revisions of tables **A.1 and A.2** in the Lehman paper are shown here as tables 1 and 2 in which the radian has been added as a base unit, and some errors in the original tables have been corrected as per their published corrigendum [53].

Inclusion of plane angles as a base quantity with units of radians fits well into the scheme of Lehman *et al* and causes no issues. The fundamental constant of plane angle rotation (2π) couples two instances of the same SI base unit (e.g. length to length) whereas the other fundamental defining constants of the SI couple different units. We should not see this as a contradiction or evidence that plane angle is not a base quantity, but as a prior limitation of choosing defining constants which include one or more SI unit(s). As Lehman *et al* show, the SI can be defined solely through four defining constants with the requirement that one constant ($\Delta\nu_{\mathrm{Cs}}$) contains just one SI unit and the three other constants contain $\Delta\nu_{\mathrm{Cs}}$ as well as other SI units. The four constants have dimensionality of 3, 2, 2, 1 in terms of SI units. It does not seem too odd to extend this to be 3, 2, 2, 1, 0 by including the radian as a base unit with defining constant 2π.

Additionally, classifying the radian as a base unit for plane angle, with a defining constant of 2π would avoid the need for a footnote in table 2 of the Letter to the Editor by Blattner and Brown [54].

3.2. Physical basis

According to general relativity and the standard model of particle physics, at a fundamental level, the Universe is made of particles, forces, and interactions, together with the fabric of spacetime itself. The SI must contain sufficient base units to allow measurement of all physical parameters of the Universe. In the proposal by Lehman *et al* [52] they state that their four base physical units of time, length, mass and electrical current (which are based on quantum mechanics and special relativity) are a sufficient and consistent set of base units for the SI—combinations of the units can be found that have the required dimensionality for all base and derived quantities. Our proposed definition for plane angle allows it to be used for point-like rotations, without reference to any other units, thereby allowing for rotation measurements of spacetime while maintaining compatibility with general relativity.

This consideration also shows why the principle can be used as described earlier without reference to ratios of lengths caused by rotation. A unit for plane angle rotation should be applicable to point-like rotation (with no linear extent). Intrinsic properties of rotation (such as angle closure) only occur when physical rotation is experienced. Definitions of angle related to static situations will lack these properties and this is why the current definition of angle in the SI leads to unsatisfactory consequences.

4. Conclusions

We summarise our conclusions as follows.

1. We have described how practising dimensional metrologists actually realise the unit of plane angle and have shown that this method does not use the definition of the unit as written in the SI Brochure.
2. The actual way that plane angles are realised in practice avoids many of the issues currently concerning discussions on plane angles by avoiding using length ratios and instead relying on physical rotation.
3. We have described rotation as being physically distinct from the other fundamental physical properties of the Universe, such as mass, length, time, electric charge. Plane angle can cause phenomena which *may* be measured in other SI units (usually as a ratio of two instances of the same unit), but the reverse mapping is not deterministic (one can measure an angle as a ratio of lengths, but not all length ratios are angles).
4. Plane angles relate SI units to themselves through a fundamental constant of 2π when the coherent unit for plane angle is the radian.
5. Following the above arguments, we propose a new definition of plane angle as follows:
   The SI base unit for plane angle is the radian. There are 2π radians in a full rotation.
6. The proposed new definition for the unit of plane angle not only follows current practice of realising the unit, it avoids issues of dimensionless ratios and removes reference to other base units from the definition.
7. Following the defining constant formulation, the definition of the SI as a whole can be updated by insertion of a final bullet point:
   The International System of Units, the SI, is the system of units in which
   - ...
   - the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, $K_c d$, is 683 lm W^{-1},
   - a complete planar rotation causes a change in plane angle of 2π radians,
   - ...
8. We have shown that there are arguments for including plane angle as an SI base quantity, being a fundamentally different property of the Universe to other quantities, and shown how it can fit in with proposals for reformulating the SI.
9. The proposed new definition: solves many of the issues associated with metrological dimensional analysis; it is conceptually easy to comprehend, requiring no fundamental changes to educational materials or software or re-dimensioning of existing quantities; it requires minimal adjustment of the existing SI; and it can both be used as a definition and as a suggestion of how to realise the unit in practice.

The debate concerning the *status* of plane angle may not be over, but we ask that at least the definition be updated to reflect the way that plane angle is realised in practice.

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Author contributions

AJL formulated the main idea of the work and prepared the first draft of the paper. TJC raised certain issues and offered alternative wording and viewpoints which were then used to update the paper.

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