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A FRAMEWORK FOR ASSESSING THE ECONOMIC SIGNIFICANCE OF NPL'S SPILLOVERS

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ABSTRACT

Spillovers are a significant part of the total impact the National Physical Laboratory generates. However, currently NPL does not have a systematic approach to understanding these spillovers. This document aims to address the issue and build a conceptual framework for the spillovers generated. By doing so, NPL can better understand the impact of its activities, beyond its direct userbase. In this document, two categories are used to classify knowledge spillovers: Indirect Benefits and Indirect Costs. The focus is on the indirect benefits which have two main streams: free to access information goods; and codified knowledge. Additionally, a conceptual framework is developed for each of the streams, to understand the channel through which the benefits disseminate to indirect users. The first stream concerns the market for calibration services and NPL’s role in the calibration chain. Using the concept of entropy in information theory, it finds that for £1 spent on improved measurement, the return on measurement is £1.30. The second stream explains NPL’s role in the creation and revision of standards and how that leads to product and process diffusion. Through the route of product diffusion, it finds that, on average, products have a lifetime of 13 years within an industry and 7 years within a firm. Moreover, it finds a 55-45 split between the direct and indirect benefits. The purpose of this document is to construct a top-level structure and a basic conceptual framework for the indirect benefits. This document makes a start at the mathematisation of the framework but developing a complete economic model for the spillovers requires further work.

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EXECUTIVE SUMMARY

Externalities are defined as the positive and negative impacts of an economic activity that affect other economic actors (society, business, government). Externalities cause market failure as the price equilibrium does not accurately reflect the true costs and benefits of a good/ service.

Spillovers are sometimes also called externalities but in the context of this document are defined as the indirect benefits and costs associated with acquiring technological knowledge created by others, commonly referred to as ‘knowledge’ spillovers. The reason why spillovers cause market failure is that the innovator does not take into account the value to the third party and therefore under invests or under produces the desired product. This outcome provides a rationale for government to intervene to correct the under-investment e.g., in the form of R&D tax incentives, to achieve social optimum. For this reason, it is important to understand the spillovers that NPL generates through its science programmes which will allow outcomes and effectiveness of the activities to be analysed.

The National Physical Laboratory (NPL) has systematic approaches to quantifying the direct benefit it generates through its widespread science activities.[1] However, we do not yet have an economic framework that would allow us to grasp the indirect benefits generated. Direct benefits of activities are confined to the direct users i.e., NPL’s customers, whereas indirect benefits extend beyond NPL’s customers to our customers’ userbase, and so on down the value chain. Indirect benefits are believed to be as large as the direct benefits, if not more. Therefore, there is a great need for developing a conceptual framework for the spillovers, which forms the basis for this document.

The document classifies knowledge spillovers into two categories: Indirect Benefits, and Indirect Costs. It focuses on the indirect benefits which are believed to have two main streams. A framework is developed for each of the streams; free to access information goods and codified knowledge.

The **first stream** uses the concept of information goods to explain the nature of calibration services. Information goods are classed as public goods (non-rivalrous and non-excludable to an extent). This makes calibration services subject to being sold by NPL’s users. This is because the primary calibration received by users is used as a baseline and sold to their userbase without any direct income coming to NPL. The absence of copyrights to such services, thus, leads to a market failure.

In addition, the Edgeworth model for a price-setting oligopoly is used to explain how a market for an information good, namely, calibration services, can still exist, even though the marginal cost of reselling NPL’s services is negligible compared to the amount it would have costed NPL to produce the service. This model concerns competition for market share between two firms with homogenous products and capacity constraints, such that these firms engage in continuous price wars, leading to a mixed strategy equilibrium.

Furthermore, this section also elaborates on the two approaches that would help quantify the indirect benefit of providing primary calibration services. The <u>first</u> one is a study that explains the value of information from providing calibration services, through a reduction in the cost of mistakes in conformance testing. This comes from the notion that using primary calibration

1 King, M. and Olakojo, S. (2023). NPL report IEA 15 NMS Business Case Model: An Explanatory Note.

services help eradicate systematic errors. Through NPL's role in the provision of better national measurement standards, the large number of organisations reliant on calibrated instruments can make better informed decisions, which decreases the likelihood of making a type I or type II mistake. The study finds that the GVA safeguarded through supplying high-quality, primary calibrations to manufacturing firms that work with the NMS labs is at least £197 million per annum. The second approach uses Claud Shannon's notion of entropy in information theory to estimate the value of the information gained through better measurement, which involves defining new information as "surprisal" and entropy as the "expected surprisal". Through an economic model for agents maximising the net-benefit generated from better measurement information, this report finds that for £1 spent on improved measurement, the return on measurement is £1.30. There is still much further work required to bring these two information-based approaches together in one model.

The **second stream** explains the role of NPL in the creation of codified knowledge. The primary method discussed is the creation and revision of standards which leads to product and process diffusion. Product diffusion is explained through a model based on firms' portfolio of products. The lifecycle of product creation, entrance to the portfolio and obsoletion/ displacement is shown through a conveyor belt. It uses data from the NMS survey to compute the lifetime of a product within a firm and within the industry. (Such data corresponds to similar data obtained in the UK Innovation Survey.) It finds that the products have a lifetime of 7 years within the firm that first created it, and a lifetime of 13 years within the firm's industry. In addition, using these numbers, the ratio between direct and indirect benefits is calculated. It finds a 55-45 spilt between direct and indirect benefits. The percentages are very close to the 50-50 split found in (Frontier Economics, 2023).[2] Process diffusion is explained through NPL's role in the revision of standards, and the creation and maintenance of best practice, which aids in diffusing knowledge surrounding standardised processes. This in turn contributes to increased efficiency and greater productivity growth.

Moreover, this document is of policy relevance as it builds and explains the classification system of spillovers generated by NPL. This document makes a start at the mathematisation of the framework but requires further work.

2 'Rate of return to investment in R&D' by Frontier Economics, and commissioned by the Department for Science, Innovation and Technology (DSIT).

1 INTRODUCTION

1.1 BACKGROUND TO NMS AND NPL

The UK National Measurement System (NMS) is the nation's technical infrastructure which exists to provide the UK with accurate and dependable measurements. The NMS has two central objectives:

1. To enable individuals and organisations to make measurements competently and accurately, and to demonstrate the validity of such measurement.
2. To coordinate the UK's measurement system with the measurement systems of other countries.

The NMS is delivered through the Department for Science, Innovation and Technology (DSIT). The science programmes are delivered by the UK's measurement institutes.

1. National Physical Laboratory (NPL)
2. National Measurement Laboratory (NML) at LGC
3. National Engineering Laboratory (TUV-NEL)
4. National Gear Metrology Laboratory (NGML)
5. National Institute for Airborne Acoustic Metrology
6. Medicines and Healthcare products Regulatory Agency (MHRA).

The bulk of the funding goes to three laboratories, namely, NPL, NML (hosted at LGC) and NEL. These labs provide world class measurement science that supports the public sector and businesses. They maintain the primary standards that underpin a distributed system for the certification of calibrations, and for ensuring comparability to corresponding standards around the world.

NPL conducts fundamental research and performs international measurement comparisons i.e., key comparison exercises and does research that generates articles in peer-reviewed scientific journals. This enables the development of cutting-edge measurement capabilities that support the creation of primary standards and state-of-the-art capabilities. The expertise is used to deliver calibration, testing, and training services to private businesses, hospitals, and universities. In addition, NPL works closely with Innovate-UK to offer grant-funded collaborative R&D projects which involve a mix of firms and research organisations. NPL carries out a multitude of activities across a wide range of areas – from quantum sensing and composite materials, to radiotherapy and emissions monitoring where impact generated is not just on the national level but international as well.

1.2 MEASUREMENT INFORMATION

On a general front, ‘information’ is a vital component of any decision-making process, which ultimately allows individuals and organisations to reduce the cost of making bad decisions. ‘Good Measurement’ is one of the main components that decision makers require to make better informed decisions. Some sectors (e.g., manufacturing and healthcare) of the system may rely more on measurement information than others (e.g., finance).

The evidence for the importance of measurement suggests that the total expenditure on measurement related R&D was approximately £2bn in 2017, 87% of which was performed by private businesses. Since 2009, expenditure on measurement related R&D in the UK has been steering upwards in both business and non-business sectors. Moreover, R&D in measurement related topics has accounted for between £3.2bn and £4.9bn to GDP (Fennelly and King, 2021).

Literature can agree that every measurement will have some degree of ‘error’ attached to it, in the form of systematic and random errors. There are also mistakes associated with the decision making, primarily based on the faulty measurement information attained. These come in the form of Type I (false positive) and Type II (false negative) errors. The effect of random errors can be largely eliminated by taking the average of multiple measurements. However, the same cannot be applied to systematic errors. These can be removed through calibrations and reference materials. Calibration is defined as a process of using a more accurate measuring device, or reference, to calibrate an instrument. Reference materials are defined as standards or controls that are used to calibrate instruments and validate methods. Additionally, good measurement practices, underpinned by reliable calibration services, can also contribute to decreasing the probability of making a Type I and Type II mistake (King and Nayak, 2023).

Measurement activity is possible without the work of NPL, but it wouldn’t be as reliable or effective (King and Nayak, 2023). NPL as the National Metrology Institute provides confidence in measurement through the role it plays in defining the globally agreed **International System of Units,** and by developing and maintaining the country's primary measurement standards. Building standardised measurements can contribute to reducing technical barriers to trade and therefore, transaction costs. NPL along with the other NMS laboratories play part of their role by providing ‘primary’ calibration services. These services create a knock-on benefit beyond the direct user and therefore, are classed as one of the streams of indirect benefit, details of which can be found in section 3.

1.3 LITERATURE ON SPILLOVERS

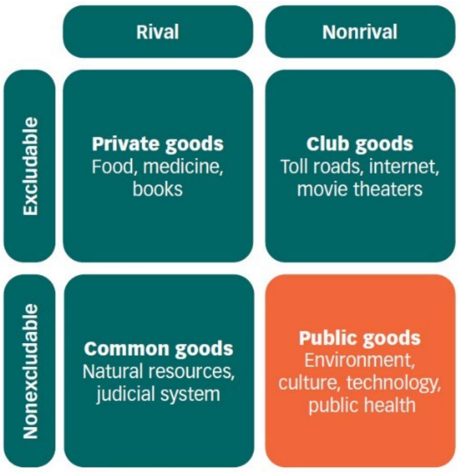
Externalities are defined as the positive and negative impacts of an economic activity that affect other economic actors (society, business, government). Traditionally, spillovers are also referred to as a type of externality. One definition is spillovers that arise when activities of one agent produces positive or negative effects on other agents in the market that are not wholly reflected in market prices (Conlon et al. 2012). Economists define “spillovers” as the idea of capturing benefits from other agents’ investment in information / knowledge without paying the full price (Bascavusoglu-Moreau and Cher Li , 2013). For instance, competing firms that replicate a successful innovation, and firms whose own research benefits from learning the successes and failures of others' research activities all gain such spillover benefits (Jaffe 1998). In fact, Grossman and Helpman (1992) use the following definition:

“By technological spillovers, we mean that firms can acquire information created by others without paying for that information in a market transaction, and the creators (or current owners) of the information have no effective recourse, under prevailing laws, if other firms utilize information so acquired”.

In the context of this document, the type of spillovers discussed are ‘knowledge’ spillovers defined as the indirect benefits and costs associated with acquiring technological knowledge created by others.

The concept of spillover effect was first explored by Adam Smith (1776) and then, by John Stuart Mill (1909). Smith believed that the most adequate outcome with respect to spillovers is attained with a competitive market with little to no intervention. On the other hand, Mill disagreed, and argued that government intervention in the market can be a useful tool when necessary to mitigate negative impacts. This concept was further developed by Arthur Pigou (1920). He argued that activities that produce a positive impact should be subsidized to further encourage the activity and negative impacts of an activity should incur an extra cost or tax to mitigate its effects. In line with this, a noble prize winner, James Meade (1952) argued that in the presence of positive externalities, a transaction failure or ‘market failure’ occurs between the interested parties where government intervention in the form of subsidies might be useful. The notion of ‘government intervention’ was questioned by another Nobel Prize winner, Ronald H. Coase (1960). He pointed out that market effected by externalities could be better understood in terms of the existence or absence of property rights. And that through property rights, interested parties could bargain and exchange rights in the markets This would solve the externality problem, without needing explicit government intervention.

The services that NPL offers, generate spillovers for instance, calibration services are one example. From the grid below, the category closest to explaining the nature of calibration services is ‘public’ good (information goods). The reason is that calibration services are non-rivalrous in nature and almost non excludable, as they can be copied and resold at a very low marginal cost (compared to the original cost of the service) by NPL’s users, to their own userbase and so on. They can be thought of as may be excludable in the first round where eager firms come to NPL and get the desired service for a premium. After that, the services can be copied and resold with not much



significant cost, giving rise to benefits that extend beyond the direct userbase, showcasing non-excludability. As an example, in the past, newly released films were readily available in the form of copied DVDs. The cost of making a film is very large as opposed to copying and selling the film on a DVD, which essentially has zero marginal cost in comparison.

As explained above, these types of goods lead to market failure i.e. product copying cannot be controlled, and the benefits are expected to disperse through the system. As information is a public good, spillovers imply that the benefit of the new information to society as a whole exceeds the loss of monopolistic rents the creator could have made if it wasn’t copied (Bascavusoglu-Moreau and Cher Li , 2013). In other words, the overflow of information is involuntary, and may cause the creator monetary losses but overall, a positive impact is generated on the economy and welfare of society (Zhu and Han, 2019). As argued by Ronald Coase (1960), presence of property rights provides a solution to this market failure. However, for such services, the detection of copying and its distribution may not be possible.

Tassey (1982) argued that measurement is like a public ‘infratechnology’ which means a technology that provides tools and techniques which can be applied across a range of sectors to encourage further innovation. In connection to this, recent literature (Estibals, 2012) suggests that both standards and measurement protocols form part of a national infratechnology. Therefore, codified knowledge in the form of standards can be another example of a public good. This is another key mechanism through which NPL generates indirect benefit. Swann (2000) conducted a first detailed survey on the existing literature on standards and standardisation, and found that;

- standards codify and diffuse knowledge, and best practice,
- standardisation helps build focus, cohesion and critical mass in the emerging stages of technologies and markets,
- standards for measurements and tests help innovative companies to show to the customer that their innovative products have the features they assert to have,
- open standardisation processes and standards enable competition between and within technologies and contribute innovation-led growth.

It is, therefore, not surprising that standards have a positive impact on emerging technologies. Standards allow common vocabularies and agreed definitions of terms to be established. This increases the confidence of investors and consumers which in turn, increases the speed at which companies can bring their products to market. (Allen & Sriram, 2000; van Merkerk & Robinson, 2006; Swann, 2010).

This also very strongly connects to Romer's knowledge production function (KPF). The KPF describes how knowledge is created and evolves; based on the idea that the rate of new knowledge production depends on the amount of R&D spent and the existing knowledge base. In other words, new ideas being non rival and partially excludable, are elementary for growth depicting increasing returns to knowledge. This can be explained with a classic quote “standing on the shoulder of giants” by Isaac Newton in his letter to Robert Hooke. It essentially means that an important component of new knowledge/ innovative idea is built upon the work of others (existing knowledge).

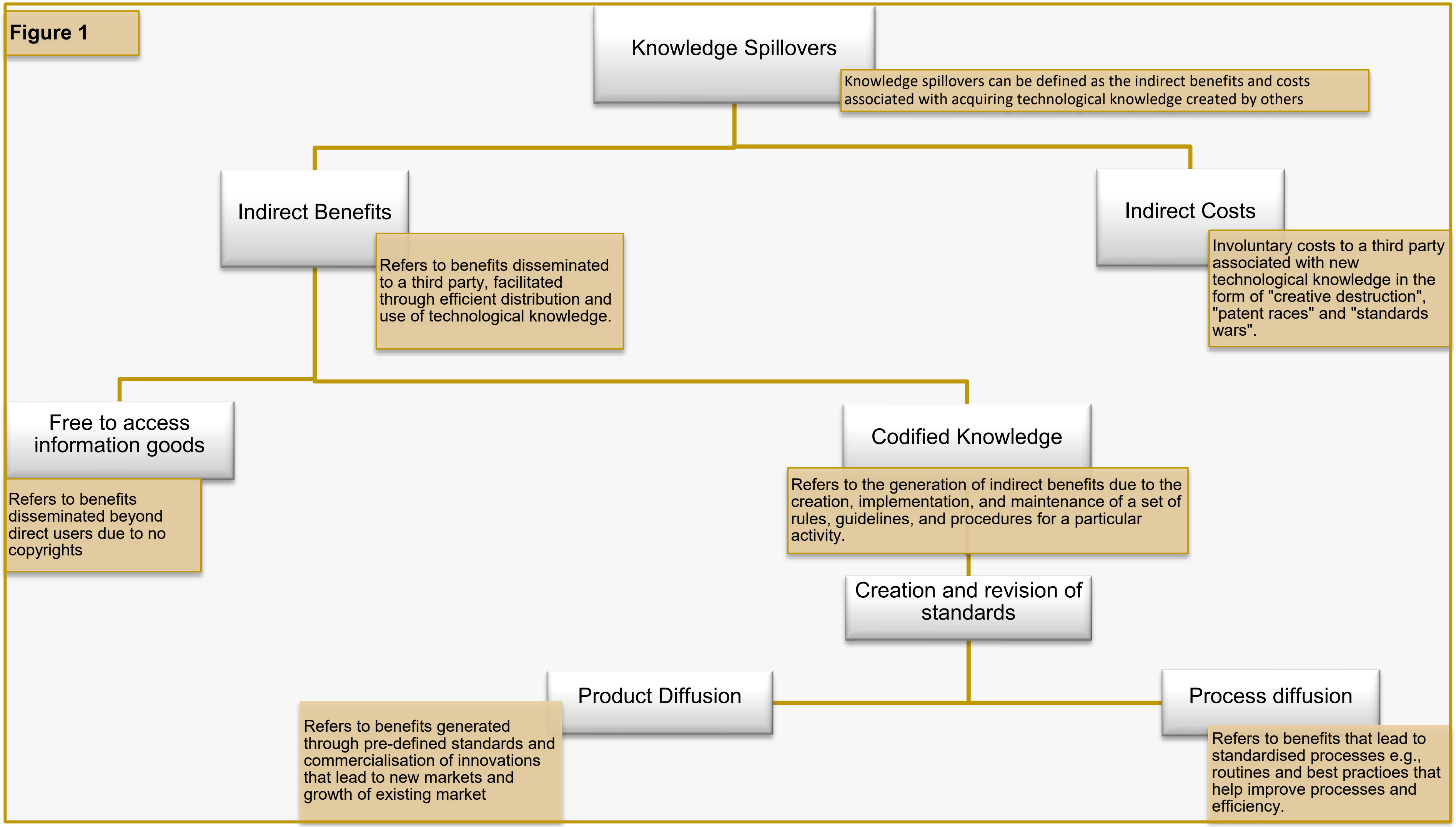
In the context of this document, these spillovers can be thought of as positive 'pecuniary externalities[3]', which can be defined as beneficial outcomes for participants that occur through the price system. The idea is that initially, the inventor of the innovation enjoys a high price and monopoly for its product which may be through patents. After some time, the technological knowledge diffuses, and other firms can produce better versions of the product which ultimately benefits the end user (customer) who enjoys a better product without any significant changes to the price it previously paid. This process happens quite quickly for the services that NPL provides. Take calibration services as an example; through the fanout, it gives indirect users access to these services at a price lower than what direct users would have paid to NPL.

The discussion above is important to set the background for the work done in this document. The aim of this document is to build an economic framework that quantifies the spillovers NPL generates by explaining the basic structure of the spillovers and developing a conceptual framework for each of the channels through which benefits disseminate beyond the direct userbase. Section 2 explains the basic structure using a figure. Sections 3 – 6 build the conceptual framework for the channels and explain how they link to generating indirect benefits. The final section summarises the discussion and draws out possible further work.

3 Worcester, D.A. (2016). Pecuniary and Technological Externality, Factor Rents, and Social Costs. *American Economic Review*, 59(5), pp.873–885.

2 BASIC STRUCTURE OF SPILLOVERS

The following figure sets out the classification system of the spillovers generated by NPL. Each of the terms are explained below in more detail.



Knowledge spillovers are defined as the indirect benefits and costs associated with acquiring technological knowledge created by others. Technological knowledge refers to the knowledge and understanding of technology and its application in various fields, which incurs indirect benefits (greater specialisation, increased product creation, greater productivity & growth), and indirect costs (the free rider problem, greater competition, R&D investment dissipation).

Indirect benefits refer to benefits disseminated to a third party, facilitated through efficient distribution and use of technological knowledge. For instance, competing firms that replicate a successful innovation, and firms whose own research benefits from learning the successes and failures of others' research activities, are examples of indirect benefits.

Indirect costs refer to the involuntary costs to a third party associated with new technological knowledge in the form of "Creative destruction"– economic concept introduced by Joseph Schumpeter that describes the process of new innovations replacing older ones, "Patent races" – competition between two or more companies to invent and patent a new idea first, and "Standards wars" – competition between incompatible technologies to gain market dominance. One example of an indirect cost is the cost to the immediate business' market share due to their innovation being copied or becoming obsolete due to a better version of the product ("market stealing").

Spillovers are classed into two categories: indirect benefits and indirect costs. The document focuses on the indirect benefits which primarily have two main streams:

1. Free to access Information Goods
   This refers to the benefits disseminated beyond direct users due to the nature of the services delivered e.g. calibration services. Information goods are best described as a public good (non-excludable to some extent and non-rivalrous in nature), subjecting them to copying and resale, by other participants of the system. Such goods cause market failure i.e., product copying cannot be controlled, and the benefits disperse beyond the direct userbase. Presence of property rights can deter the copying but, at times, the detection of copying and its distribution can be difficult.

2. Codified Knowledge
   This refers to wider benefits generated through the creation, implementation, and maintenance of a set of rules, guidelines, and procedures for a particular activity e.g. standards and patents. It also refers to benefits generated through best practice i.e., best practice guides, trainings, secondments, apprenticeships followed by knowledge NPL Alumni take to other organisations.
   Patents are an important channel of knowledge spillovers as they provide a paper trail of knowledge flows and enable other firms to build on existing technological knowledge e.g. patent citations. NPL has a role to play in supporting the development of patents for its users. In fact, NPL's evidence base suggests that the time to a new patent for firms that are regularly supported by NPL is 26% shorter for regularly supported companies compared to companies that may sometimes get support. The study also finds that the probability of developing a new patent is about 23% higher if the firm is regularly supported by NPL (Olakojo, Renedo and King, 2023). However, for the purposes of this document, the focus will be on standards as the main form of codified knowledge that generates spillovers.
   Standards create indirect benefit through various streams; efficiency (through improving processes), quality (through defining minimum requirements), innovation

(through sharing knowledge), productivity (through minimising trial and error), regulatory compliance, etc.

More broadly, standardisation contributes to

- product diffusion which refers to benefits generated through pre-defined standards and commercialisation of innovations that lead to new markets and growth of existing market. It is constituted as an indirect benefit because it allows the agents in the system to adopt the new innovations using these standards along with the experience of the portfolio of new products in the market / industry.
- process diffusion which refers to benefits that lead to standardised processes e.g., routines and best practices that help improve processes and efficiency.

The following sections model a framework for each of the ways by which the indirect benefits disseminate beyond NPL’s direct userbase.

3 FREE TO ACCESS INFORMATION GOODS

NPL offers a comprehensive range of high-quality calibration services, all of which are directly traceable to the internationally recognised, primary measurement standards, that it maintains on behalf of the UK. Most of these services are independently accredited by the United Kingdom Accreditation Services (UKAS) to ISO 17025. The others are at the forefront of metrology capability and are delivered under NPL's ISO 9001 compliant quality system. The direct benefit of delivering world class calibrations can be quantified in monetary terms, via invoicing data. However, the indirect benefits of providing traceable calibrations are more nuanced.

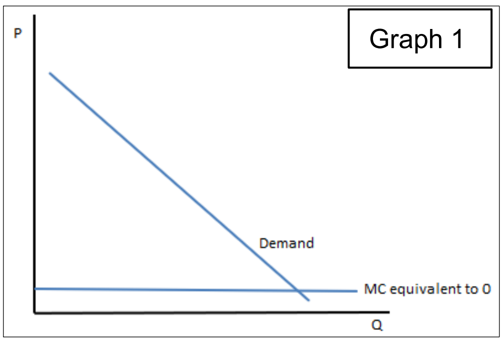
The next three subsections discuss the nature of calibration services, the market for calibration services and lastly approaches to quantifying the indirect benefit of calibration services.

3.1 CALIBRATION SERVICES AS INFORMATION GOODS

By the very nature of the system, calibration services can be regarded as information goods (type of public good – non-rivalrous nature and to an extent non-excludable); once a user receives a calibration service, the technological knowledge within spreads and can be resold to other users without income coming directly to the producer.

NPL as the National Measurement institute holds a unique position through providing ‘primary’ calibration services. It has regular users that obtain these calibrations. NPL’s data suggests, that on average, a set group of users come to NPL annually for these calibrations as it offers services that may not easily be available elsewhere. However, these services, once provided, are used as a baseline by NPL’s users, who then sell these services to their own userbase. This process does not generate any direct income to NPL. NPL’s calibration services ultimately lead to a “trickle-down effect”, where the knowledge from the services is spread and resold. This is referred by a term ‘fanout’. i.e., the accuracy in the firsthand calibrations provided by NPL is transferred through follow on calibration services provided to other firms by NPL’s users. In the grand scheme, fanout creates a much bigger societal benefit when compared to the monetary benefit, which NPL could have had if all the users came to NPL. Even if an organisation like NPL would have wanted to cash the monetary benefit, practically, it would result in “congestion” i.e., significant queuing for the services desired by the industry due to capacity constraint of NPL. In other words, NPL would not be able to cater the demand of the market.

The trickle-down effect of primary calibration services is a classic example of missing property rights. However, even with property rights, the detection of copying and distribution can be very difficult.



The process of provision of calibration services, its copying and reselling leads to market failure. This is because, the nature of calibration services allows consumption beyond direct users without any market transaction (with the original producer). In addition, there is a second level to the market failure which arises when the copied services are sold at different prices by NPL’s users. The law of one price (LOOP) states that equivalent goods should command the same price otherwise ‘haggling’ occurs i.e., where the buyer and seller negotiate on the price and buyer will try and pay the least amount possible. According to the law and the graph above, given the buyer knows that the cost to the seller is very low, it will try and negotiate to a point where price is equal to the marginal cost which in this case is equivalent to zero. So how can a market hold with price equivalent to zero? Therefore, this law cannot hold for the market of calibration services.

A model is needed that explains the existence of positive profits despite very low marginal costs. Two aspects that need to be considered are:

- There are fixed costs for setting up a calibration laboratory
- There are capacity constraints i.e., there is a limit to how much one firm can produce (cannot fulfil the market demand on its own).

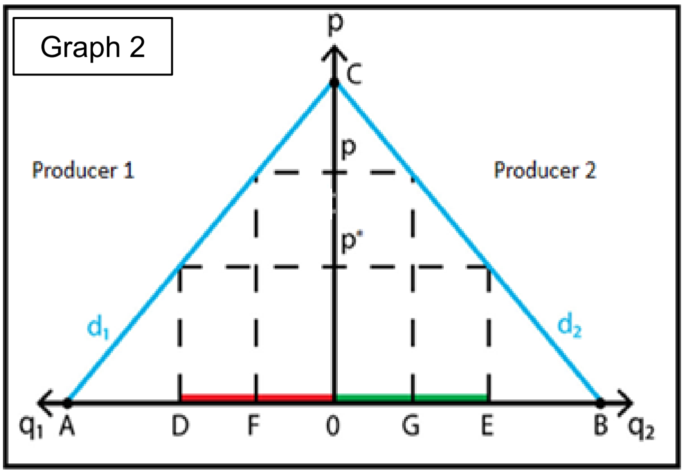
The Edgeworth model is the model that best explains this paradox. The next section explains the model and the link to calibration services in detail.

3.2 MARKET FOR CALIBRATION SERVICES

3.2.1 Introduction to Edgeworth Model

Edgeworth's Model is a price-setting duopoly which looks at what happens when there is a homogeneous product (i.e. consumers want to buy from the cheapest seller), and the output producers are willing and able to sell, at a given price, is limited. The limited output is considered a physical capacity constraint, which is the same at all price levels. It assumes that, in a certain time period, two prices can exist in the market, simultaneously. It also assumes that, under a certain price level, the output of a particular oligopoly cannot meet the market demand so, another producer can obtain the residual market demand. The model follows Bertrand's hypothesis, where each producer assumes that the price of its competitor, not its output, remains constant.

Suppose there are two firms, 1 and 2 in a market, facing the same demand curve in the market, denoted by d_1 and d_2 as shown in graph 2. If firms choose to collude, they will split and share the market and the production of the good. 1 will produce from O to F and 2 from O to G. The supply, therefore, is limited, and prices will be set at p. The revenues of each firm are denoted by the rectangle above FO and OG, and each firm enjoys an equal share. Note that d_1 and d_2 correspond to the total demand, each part being supplied by one of the firms.



Collusion is not always possible as firms have the incentive to break cooperation and earn greater profits. One of the firms will decide to lower its prices and increase production to gain market share from the other competitor. Consequentially, the other firm will do the same. Thus, the two firms will engage in a price war. This process will continue to the point where the maximum production of both firms is achieved. When this point is reached (OD for firm1 and OE for firm 2), price will not be reduced any further and will remain at p*. This is because each firm cannot supply anymore due to their capacity constraint. On the contrary, firms will have an incentive to increase their price to earn greater profit once again. And so, prices will begin to rise again, little by little, and the price war will begin again. Overtime, this process will repeat indefinitely, and prices will keep oscillating between p and p*.

Edgeworth's duopoly model suggests that since price and output are undetermined, the equilibrium is unstable and uncertain. It illustrates an important aspect of the Nash equilibrium concept: i.e., here the firms do not set a single price, but instead set different prices for each period based on a mixed strategy equilibrium (e.g. set a price of 20 with probability 0.2; a price of 25 with probability 0.18; etc.). This implies that the firms will set a price drawn from some probability distribution which is optimal against the other firm's probability distribution.

3.2.2 Calibration Services and Edgeworth Model

As discussed in beginning of this section, NPL provides primary calibration services to its core users. The nature of calibration services makes it subject to being copied and resold by users. In other words, users who classify themselves as commercial calibration laboratories, use the services that NPL provides as a baseline to then sell it to their own users at a very low marginal cost, without income coming to NPL directly.

In this model, NPL does not compete with commercial calibration laboratories. We can almost think of a vertically differentiated market where:

1. Some users are prepared to pay the premium for quality and don't want to wait for the new services to be adopted by other calibration laboratories.
2. Calibration laboratories can offer good but not perfect copies of NPL's services.
3. Incremental improvements first affect NPL's services (Users get the improved services without any time lag as NPL is always at the frontier).

For example, when a film is first released in cinemas. Some individuals are more eager than others to watch it in the best quality available rather than wait for a DVD.

As the National Metrology Institute, NPL is the high price firm that provides the most accurate measurement service. Due to a capacity constraint, it can only cater for a proportion of the market demand. Therefore, users that are willing and able to pay the price, come to NPL and use the knowledge from the services, as a baseline to then sell it to their own users. For the purposes of this model, let there be two firms, 1 and 2, who have used NPL's calibration services and can now re-sell it themselves. Firm 1 and 2 compete against each other for a greater market share of calibration services. Both firms have a capacity constraint so neither can fulfil the market demand on their own. The Edgeworth model illustrates the behaviour of the two firms. Both firms know the monopoly price (p) that they can charge and can earn a profit with. At this price they have an incentive to lower the price and capture their rival's market share. Once the price is lowered, the rival firm has an incentive to lower its price slightly below what was set by the other firm as they have the capacity to sell more. This will continue until the price reaches the level (p*) where they can sell their maximum output. At the competitive price, firm 1 will have an incentive to increase its price and earn a profit. Once the price is increased, firm 2 will have an incentive to increase their price above what firm 1 set. This war of increasing and decreasing prices between p and p* (referred in the graph above) will continue indefinitely. The market will eventually reach a point where there are mixed strategies such that each firm will set a price, drawn from some probability distribution, which is optimal against the other firm's probability mixture.

This theory of price fluctuations can also be supported by the fact that prices of such services are never advertised and constantly fluctuate. This means that, because there are no fixed prices, users don't expect a specific price, giving firms the liberty to change their prices. Additionally, in the real world, the services may not be 100% identical. Some services would be tailored according to the requirement of the user, contributing to the variability in prices.

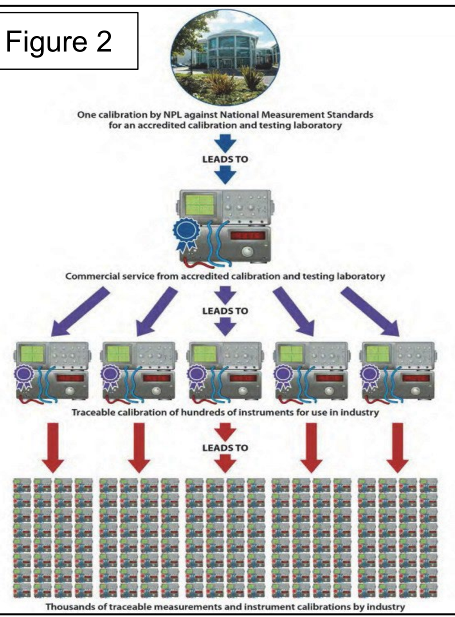
3.2.3 NPL and the Calibration Chain

NPL as the National Metrology Institute provides the most accurate measurement services, with its consumers recognising 'the value and quality of NPL's services'. This claim is supported by the NMS Survey in which users time and again refer to NPL's quality, stating "*NPL can be relied on to provide a high-quality service*". In the follow-up case studies from the NMS Survey 2022-23, the users implied that having NPL's name attached to the service helps them secure more sales - "*association with the NPL brand is also of benefit*" - and that the industry knows that services associated with NPL can be fully trusted, "*Being able to refer and trace measurements to a known and trusted national standards authority carries a lot of weight*". And so, they are willing to pay a premium for the quality of the first-hand calibration services they receive from NPL. This helps them in their brand image and allows them to justify a premium from their users.

NPL is the first vital link to the chain of calibrations through which the benefits then fanout across the economy. Evidence suggests that if some proportion of NPL's services ceased to exist, the whole chain would be weakened, i.e. the use of precise calibrations would decrease. This would imply that the users (UK based) of NPL's services can be expected to go to the next best alternative which based on distance and role would be the National Metrology Institute in Netherlands, VSL. By analysing a counterfactual, an estimate of how much the use of precise calibrations would decrease without the NMS labs, can be calculated. An econometric analysis (Renedo and King, 2020) finds that there is a strong negative relationship between distance and uptake of services. That is, the elasticity of invoices with respect to distance is -0.48 which means if the distance between the UK and another country was somehow to double, then demand for NPL's services would drop by 48%. The calculations in Annex A show that if the separation of "NPL" and its userbase is increased by 65%, this would cause the demand to decrease by 31%. This decrease may even be larger as there may be a drop in the capability of the services NPL offers.

Furthermore, the accuracy in the firsthand calibrations that NPL provides is, in-turn, transferred through follow on calibration services provided to other firms by NPL's users, who then behave like the firms in the Edgeworth model. This effect further spreads out across the economy. In fact, the NMS Survey 2022-23 found that the calibration services provided by the NMS labs have a fanout to ~75,500 organisations. This estimate only represents the 'first level' of fanout where services are provided to direct users by NPL and other NMS Labs, and they in turn provide services to their own customers. It does not include the services that are further provided by these recipients. It is within reason to assume that the 'second level' fanout would be significantly higher.

Figure 2



The next section explains the ways by which the indirect benefit attained from calibration services, could be quantified.

3.3 NPL'S ROLE IN UNDERPINNING MEASUREMENT INFORMATION

As discussed in the previous sections, measurement activity is possible without the work of NPL, but it wouldn't be as reliable or effective. Therefore, NPL plays a front and central role in providing primary calibration services that create a knock-on benefit, extending beyond the direct userbase.

Currently, we have two ways of approaching the quantification of this benefit; one is a scientific, more practical approach and the other is more mathematical. The two subsections below discuss each approach respectively.

3.3.1 Value attributable to high quality calibrations by reducing mistakes in measurement

As mentioned previously measurement comes with a degree of errors attached to it in the form of systematic and random errors. There are also mistakes associated with the decision making, primarily based on the faulty measurement information attained. These come in the form of Type I (false positive) and Type II (false negative) errors. Type I error is defined as the error of commission (i.e., wrongly including a 'false case') and type II error is defined as the error of omission (i.e., wrongly leaving out a 'true case'). Random errors can be eliminated by taking the average of multiple measurements. And systematic errors can be removed through calibrations and reference materials.

The study[4] published by NPL economists (Nayak and King, 2023) explains the value of information from providing calibration services, through the reduction in costs of mistakes. It covers how reducing systematic errors can decrease measurement uncertainty. Systematic errors can be minimised or may be fully eradicated through the primary calibration services that NPL offers. This essentially means, calibration helps to lower the standard deviation of a measurement process, which reduces the cost of measurement errors for a firm engaged in conformance testing of its products. In other words, with NPL providing more accurate measurements and better measurement techniques, agents are able to make better informed decisions which decreases the likelihood of making a type I and type II mistake. The table below explains the two types of mistakes.

Table 1: Type of mistakes		**Null hypothesis**	
		TRUE	**FALSE**
Decision about null hypothesis	**Not reject**	Correct inference (true negative)	Type II mistake (false negative
	Reject	Type I mistake (false positive)	Correct inference (true positive)

These can be explained further through the results of a covid test. If an individual does not have covid and has tested positive, this implies a false positive which is a type I mistake. And if an individual has covid but has tested negative, this means a false negative which is a type II mistake.

The study finds that the GVA safeguarded through supplying high-quality, primary calibrations to manufacturing firms that work with the NMS labs is approximately £197 million per annum.

4 An economic model for the value attributable to high- quality calibrations by reducing mistakes in conformance testing.

3.3.2 Entropy in information theory

The basis of the second approach comes from the fact that advances in metrology imply that SI units can be measured with ever greater accuracy. For instance, adding an additional decimal place to core units, gives greater accuracy. How can this greater accuracy – more information be quantified?

The answer is Entropy in information theory. Entropy was first recognised in the field of classical thermodynamics and is associated with a state of disorder, randomness, or uncertainty. Later, a famous physicist Ludwig Boltzmann introduced the concept of statistical disorder and probability distributions to entropy. According to this, entropy was defined as the measure of the number of ways a system can be arranged, often taken to be a measure of "disorder" (the higher the entropy, the higher the disorder).

In the 1940s, Claude Shannon was then the one who introduced the concept of information entropy. His definition of information theory implies that the entropy of a random variable quantifies the information associated with the variable's potential states or possible outcomes. Entropy here is derived from a set of axioms which define information and is primarily based on the concept, that an informational value of a communicated signal depends on the degree to which the content of the signal is surprising (certainty and impossibility are not surprising). For instance, if a highly unlikely event occurs, it is very informative as opposed to a highly likely event which occurred. The information content, also called the surprisal or self-information, of an event E is a function which increases as the probability $\mathrm{p(E)}$ of an event decreases. When $\mathrm{p(E)}$ is close to 1, the surprisal of the event is low, but if is close to 0, the surprisal of the event is high.

Shannon's Axioms for self-information

1. $\mathcal{I}(1) = 0$ i.e. an event with probability 100% is perfectly unsurprising and yields no information.
2. $\mathcal{I}(\mathrm{p})$ is monotonically decreasing in p. In other words, the less probable an event is, the more surprising it is and the more information it yields.
3. $\mathcal{I}(\mathrm{p_1 p_2}) = \mathcal{I}(\mathrm{p_1}) + \mathcal{I}(\mathrm{p_2})$ i.e. if two independent events are measured separately, the total amount of information is the sum of the self-informations of the individual events.

Logarithm is the only function that can satisfy all the axioms of self-information as explained in Annex B. Therefore, the information, or surprisal, of an event E is described as follows:

$$\mathcal{I}_E = \log_b\left(\frac{1}{p_E}\right) = -\log_b(p_E) \quad (1)$$

Entropy measures the expected (i.e., average) amount of information conveyed by identifying the outcome of a random trial. This implies that rolling a die has higher entropy than tossing a coin because each outcome of a die toss has smaller probability. Shannon defines entropy as the expected surprisal such that:

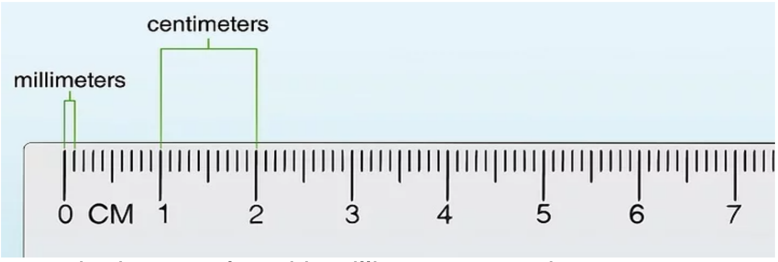
$$\mathrm{H} = \mathbb{E}[\mathcal{I}_E] = \mathbb{E}[-\log_2(p_E)] \quad (2)$$

Here $\mathbb{E}$ is the expected value operator, and $\mathcal{I}_E$ is the information content of an event. The choice of base for the logarithm varies for different applications. Base 2 gives the unit of 'bits' (or 'shannons'), while base e gives 'natural units (nat)', and base 10 gives units of 'dits', 'bans', or 'hartleys'. The base value of log is taken as 2 throughout, as done by Shannon.

Equation 2 can be rewritten as:

$$\mathrm{H} = -\sum p_E \log_2(p_E) = -(p_1\log_2(p_1) + p_2\log_2(p_2) + p_3\log_2(p_3) + \cdots) \tag{3}$$

Using this concept of information, entropy is further discussed in Kunzman et al 2005. The study argues that there is some prior knowledge with regards to a quantity (i.e. without any measurement). It can be assumed that the quantity lies within an estimated interval. Let that be denoted with ℓ_0(common sense interval). With good/accurate measurement, there is new information and to calculate the increase in this information, the interval is divided into 'm' sub-intervals. Let this be denoted with $\mathrm{U_m}$(interval as a result of good measurement). Measurements only give benefits if it gives information above the baseline level so a measurement process with a resolution worse than the common-sense interval gives no information ($-\log_2(1) = 0$). This can be further explained with the help of an example, of a ruler. A ruler with just centimetres can be thought of as ℓ_0 and a better ruler with millimetres contains more information i.e more divisions and therefore can be thought of as $\mathrm{U_m}$. The ratio of the two is simply the number of sub intervals in which the quantity will possibly lie.



$$m = \frac{\ell_0}{U_m} \tag{4}$$

With a uniform distribution the probability of the quantity lying in any of the sub intervals m is equally likely and denoted by:

$$\sum_{i=1}^{m} p_i = 1$$

$$p_i = p_j \; for \; all \; i, j \quad \Rightarrow p_i = constant \; for \; all \; i$$

$$\sum_{i=1}^{m} p_i = m\hat{p} \quad \Rightarrow \hat{p} = \frac{1}{m} \quad \Rightarrow p_i = \frac{1}{m} \; for \; all \; i$$

Using the above information and substituting it in equation 3, gives:

$$\mathrm{H} = -\sum_j^m p_j . \log_2(p_j)$$

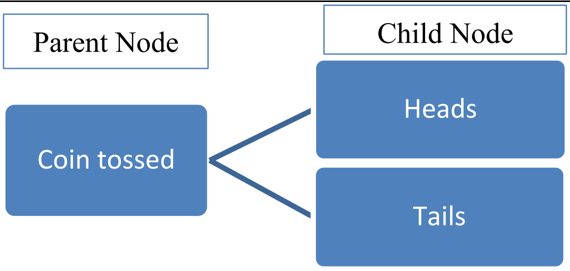
$$\mathrm{H} = -\sum_j^m \frac{1}{m} . \log_2\left(\frac{1}{m}\right)$$

$$\mathrm{H} = -m . \frac{1}{m} . \log_2\left(\frac{1}{m}\right)$$

$$\mathrm{H} = \log_2(m) \tag{5}$$

where H is the information that will be gained when the system settles into one of the m possible states. p_j is the probability that a random quantity lies in a sub interval j and m is the total number of sub-intervals.

An important aspect here is the fact that information and entropy have a negative relationship. The following expressions derive this relationship for further clarity.



When a coin is tossed, the probability that we land with either the head or the tail is 0.5. The coin possesses entropy when it is in the air because there is uncertainity whether it lands with a head or a tail. The entropy for the parent node would, therefore, be

$$H_B = -(0.5)\log_2(0.5) - (0.5)\log_2(0.5) = 0.5 + 0.5 = 1$$

Let's say the coin lands with a heads. This implies the following:

$$P(H) = 1$$

$$P(T) = 0$$

So the entropy of the child node will be:

$$H_A = -(1)\log_2(1) - (0)\log_2(0) = 0 - 0 = 0$$

Note that $\log_2(0)$ is undefined but using L'Hôpital's rule, multiplying zero with undefined gives a zero.

The change in entropy can be calculated as

$$Change\ in\ entropy\ =\ 0 - 1 = -1$$

The gain in information is calculated using the formula below.

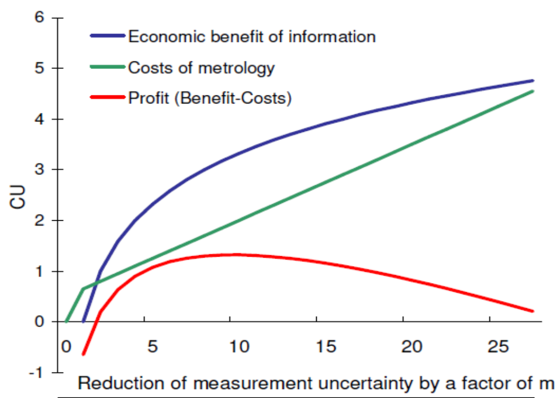
$$Gain\ =\ (Entropy\ of\ the\ parent\ node) - (average\ entropy\ of\ the\ child\ nodes)$$

$$Information\ gain\ =\ (1) - \left(\frac{1}{2}(0) + \frac{1}{2}(0)\right) = 1 - 0 = 1$$

Hence, $\Delta\mathcal{I} = -\Delta H$

Therefore, equation 5 can be used for the following interpretation: *If the uncertainty in a measurement can be reduced by m times, the gain in information will scale with the log of m with base value 2.*

Taking this a step further, Kunzman et al 2005 uses the graph 3 and assigns a monetary value to a shannon of information and shows the net benefits generated by measurements as a function of increasing measurement accuracy in terms of m. The graph implies two noteworthy cases:



Graph 3: Benefit, cost, and profit of information as a function of measurement uncertainty – expressed in currency units as a function of fractions m of the a priori interval.

- m=1: means the measurement uncertainty is the same as the uncertainty of the a-priori knowledge. Consequently, even with some investment on metrology, there is no increase of information. Those measurements cannot be qualified to be productive and should not be executed.

- m~10: corresponds to the traditional "Tool-Makers-Rule". This can be thought of as a rule of thumb as it has been considered reasonable historically.

Let the net benefits generated by measurements be denoted by the following expression

$$\pi = v_i \log_2 m - C(m) \quad (6)$$

Where π is the net benefit of measurement, v_i is the value of a piece of information measurement (like price in conventional profit function), m is the number of divisions in a measurement, $\log_2 m$ is like the quantity in conventional profit function and $C(m)$ is the cost of measurement.

Taking the derivative of the above equation w.r.t m:

$$\frac{d\pi}{dm} = \frac{v_i \log_2 e}{m} - C'(m) = 0$$

Rearranging the equation gives an expression for the value of measurement:

$$\frac{v_i \log_2 e}{m} = C'(m^*)$$

$$mC'(m^*) = v_i \log_2 e$$

$$\boxed{v_i = \frac{m.C'(m*)}{\log_2 e}} \quad (7)$$

Since, cost of measurement is $mC'(m^*)$ and the value of measurement is $\frac{mC'(m^*)\times\log_2 m^*}{\log_2 e}$, the benefit to cost ratio can be calculated using the formula below.

$$BCR = \frac{Total\ Benefit}{Total\ Cost} \quad (8)$$

Putting in the expressions for the benefit and cost gives:

$$\frac{Benefit}{Cost} = \frac{\frac{mC'(m^*)\times\log_2 m^*}{\log_2 e}}{mC'(m^*)} = \frac{\log_2 m^*}{\log_2 e}$$

Putting the value of m from the 'Tool-Makers-Rule' in the above equation.

$$\frac{Benefit}{Cost} = \frac{\log_2 m^*}{\log_2 e} = \frac{\log_2 10}{\log_2 e} = \frac{3.32}{1.44} = 2.31$$

Using the above to calculate the average return on measurement:

$$Average\ return\ on\ measurement = \frac{Benefit - Cost}{Cost} \quad (9)$$

$$Average\ return\ on\ measurement = \frac{3.32}{1.44} - 1 = 1.30$$

The value above infers that for investing in £1 of improved measurement, the return on measurement spent is £1.30.

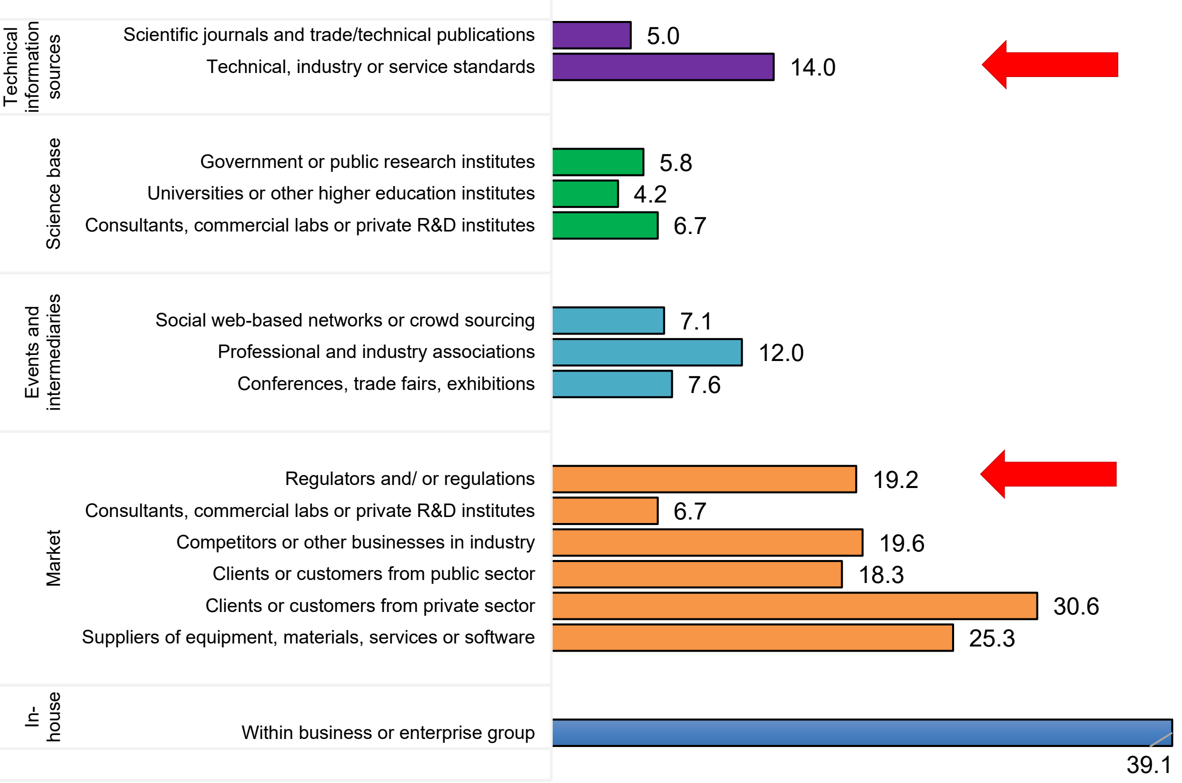
The idea here is to derive a method by which the value of information gained as a result of better measurement, can be estimated. The discussion above helps us quantify a baseline number which can then be used with the first approach to estimate the percentage increase in information that comes from reduced measurement uncertainty i.e. through better measurement. Further work is required to bridge the two approaches together.

4 STANDARDISATION

Standards are a conduit for the diffusion of knowledge; they form part of the infrastructure for innovation, helping to promote the adoption of best-practice, codify technological knowledge and simplify complex processes. Standards promote innovation, efficiency, and knowledge diffusion, which ultimately benefits broader segments of the economy. By facilitating innovation and reducing transaction costs, standards make it easier for companies to adopt cutting-edge practices and products, which can lead to cost savings and improved products. These improvements are then passed down to users through lower prices and enhanced products. As standards also enable firms to specialise and benefit from economies of scale, this further reduces costs and increases access to high-quality goods and services for more users. Over time, this fosters wide economic growth and innovation, leading to indirect benefits for society, as seen in the empirical findings of studies such as those by Peter Swann and Blind (2016).

The importance of standards can be shown through the rating that firms give to standards as found in the UK Innovation Survey (UKIS) 2023. Firms use standards and regulation as a source of information at a much higher rate compared to other sources of information, including scientific publications and universities.

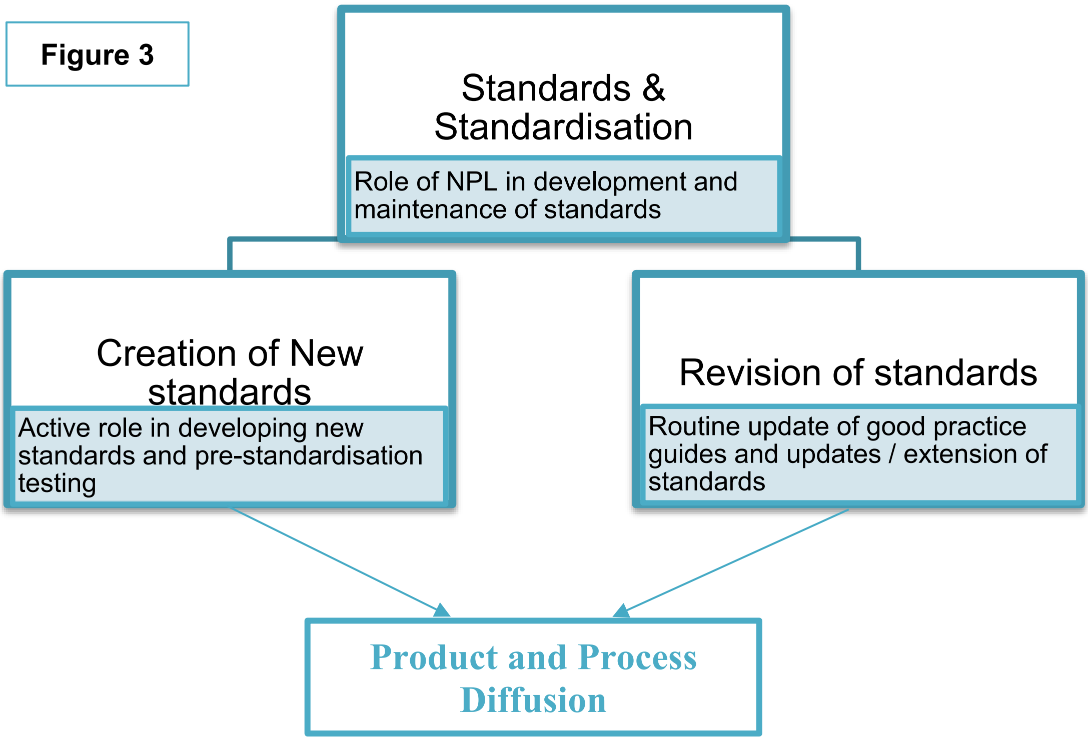
Graph 4: Sources of information (Percentage of broader innovators rating listed information sources as "highly important" to innovation activities), 2020 to 2022



Moreover, high quality is only achievable through robust standards. Robust standards provide the framework for ensuring that products, processes, and services consistently meet specific criteria, enabling them to maintain high quality. Without these standards, there is no reliable way to measure, assess, or guarantee the quality of outputs. Standards ensure uniformity, reliability, and safety across industries, which are essential for achieving and sustaining high quality. Therefore, high quality can only be attained when it is underpinned by strong, clearly defined standards, ensuring both consistency and continuous improvement. And that is where NPL plays a vital role.

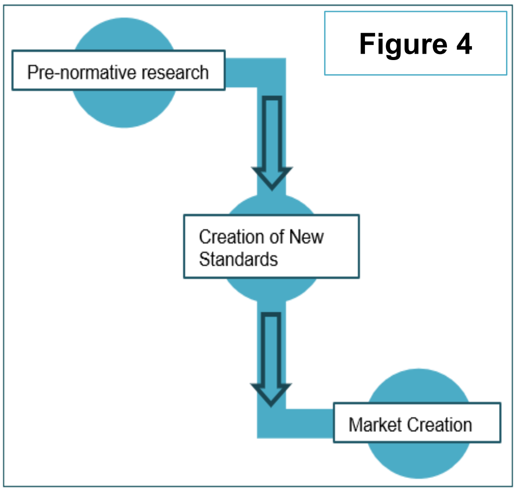
The chart below shows the two main streams by which NPL plays its role.

Figure 3



NPL plays a significant part in the creation of standards, which are critical to emerging technologies, through their involvement in pre-normative research. Pre-normative research is the foundational work conducted before a standard is officially established. This includes critical activities such as measurement, testing methods, and ensuring the reliability of conformance testing. The diagram explains that pre-normative research is key to creating a new standard, which is essential for creating a market, for the products associated with an emerging technology.

Figure 4



A good example of this is the major contribution of NPL in the development of the world's first quantum technology standard, ETSI GS QKD 011 on component characterisation for quantum key distribution (QKD). This, and other standards in progress, are key for the quantum industry as they:

- give agreed meanings for the terms used,
- establish trustworthy methods for characterisation and benchmarking,
- establish requirements for acceptable performance of products along with encouraging and facilitating trade by providing clarity in transactions between supplier and procurer.

NPL's work ensures that the standards are not only scientifically sound but also practical for industries to implement. Standards drive innovation, and NPL's research is crucial in bridging the gap between regulatory frameworks and market needs. By contributing to the development of reliable and robust standards, NPL supports both compliance with regulations and the drive for innovation. Furthermore, NPL acts as an arbiter, a mediator between regulation and industry, ensuring that standards remain relevant and adaptable in a rapidly evolving technological landscape. In essence, the role of NPL contributes to creating a market where buyers and sellers are confident about the new products associated with various emerging technologies. Additionally, NPL experts chair some CEN technical committees and many of the working groups developing new standards which further demonstrates NPL's front and central role for standards. This is evident from the case studies of the International Science Review 2020 which state that in the last 5 years, CEN and ISO have published 7 standards and technical specifications on emissions monitoring which were led by NPL staff and 15 more which have had significant inputs from NPL scientists. One example is by working in strong European collaborations with other NMIs, has led to the development of primary standards and associated electronics systems. These standards are in use both at NPL and around the world, delivering impact via provision of traceability for industrial customers. In fact, standards have contributed to about 13% of the growth in labour productivity in the UK over the period 1948-2002. This implies that standards are an important means by which drivers of technological change (accumulation of human capital) are realised[5].

The other end of the spectrum is NPL's function in the revision of standards which includes updates and extensions to existing standards. It also includes regular updates made to good practice guides. The good practice guides are practical and informative series of documents designed to meet the needs of industry. Based on NPL's expertise and experience, the guides enable users, their customers, and suppliers to agree on best practices. These are free to access on the NPL's website. In addition, they are a classic example of process diffusion as they provide knowledge to standardised processes in the various areas of measurement science. This contributes to increased efficiency and greater productivity growth i.e. through better and efficient processes. In fact, the NMS Survey 2022-23 finds that around 437 ~ 15% of private users utilise its downloads which includes good practice guides and software. This implies that NPL's experience and expertise in various areas of science which has been bundled up in these guides can be used by the industry to gain the knowledge they require to enter or expand into a field. The training courses that NPL delivers also play a great role in the transfer of general and technological knowledge. Below are some case studies of where NPL's Training has generated widespread benefits not just for the immediate users (including international users) but for their staff and potentially their suppliers.

"The National Measurement Institute of South Africa (NMISA) identified a need for cobalt-carbon and palladium-carbon reference temperature cells and the equipment to operate them. The National Physical Laboratory (NPL) not only manufactured and certified the temperature reference cells and the equipment required but also provided bespoke training to the NMISA team on how to use the equipment, resulting in international competitiveness for both the National Laboratory and local industries in South Africa."

5 DTI, 2005. The Empirical Economics of Standards: DTI Discussion Paper No. 12.

"Lockheed Martin opted to a fundamental level of training in dimensional measurement for their staff, and some requested for more advanced training. Through its NPL accreditation, staff at the firm are learning about the importance of good measurement practice and the right measurement behaviours, at times that best suit them. The company can even offer training to its suppliers, meaning the organisation can have greater confidence in its supply chain."

An example of NPL's role in standards is the work that has been done enhancing the capability of primary electrical standards in the realisation of the SI Ampere, by installing the Table-top Quantum Hall Resistance (TTQHR) system in the primary resistance scale. These primary electrical standards underpin the national traceability of electrical measurements under the framework of UK's National Measurement System (NMS). The impact is not limited to the national scale, but contributions to the international metrology community are leading to impacts at an international level. There is active collaboration with other NMIs on the design / validation / delivery of primary standards.

Furthermore, the case studies gathered for the International Science review 2016-2020 further confirm NPL's central role in the world of standards. One example: "*NPL has driven international standards for establishing validated air quality measurement methods, required by national regulations, including pollutants that are increasing despite policy initiatives (e.g. ammonia).*"

To summarise, NPL through the NMS, supports the UK's drive to influence national and international standards and offer an accurate and effective means of delivering traceability to them, which expedites regulation, innovation, and industrial competitiveness, as well as enabling local and international trade. This, in totality, feeds into the development of both national and international measurement infrastructure and enabling scientific innovation.

The next section elaborates the impact of product standards in diffusing the innovation knowledge and technology to indirect users and computes the lifetime of portfolio of products within a firm and industry.

5 PRODUCT DIFFUSION

Product standards are documented specifications that explain the characteristics of a product. They help to ensure that products are fit for their use and meet regulatory requirements. Such standards include details on their design, performance, testing and labelling. This allows the specification of a product to be defined which makes it easier for the market / industry to adopt the innovation, contributing to product diffusion. Product diffusion refers to the indirect benefits generated as a result of pre-defined standards and commercialisation of innovations.

Below is the simplest form of a model that aims to quantify the lifetime of a product within the industry and the firm. An add-on is to calculate the ratio of direct and indirect benefits delivered through product diffusion. Please refer to the Annex D for a detailed version of the model. The concept of a conveyor belt is used here to explain product diffusion by looking at the lifetime of a portfolio of products overtime in an industry. Before, moving to the conveyor belt model let's consider a set-up of an industry where there are only two firms.

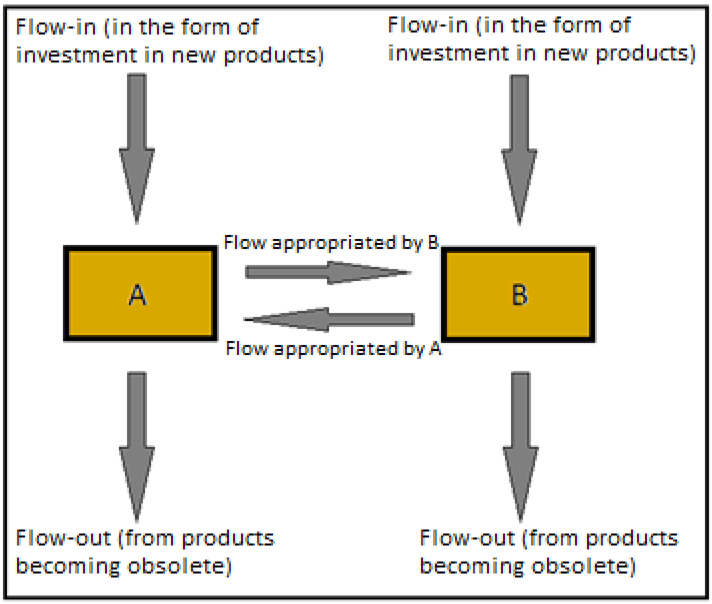
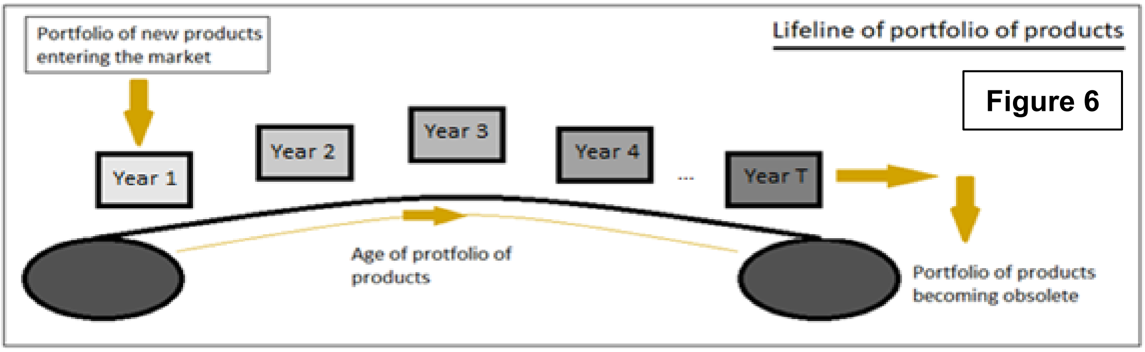


Figure 5

The assumption is that the two firms are symmetric. Both firms have two types of inflow denoted by X%. One which is the turnover that they receive from the sale of portfolio of products that are new to the market and the other is the percentage of turnover they make by adopting the other firm's innovation. They do so, by becoming more efficient at making the product compared to the original innovator. This concept of “market stealing” (indirect costs to the original innovator but indirect benefits for the end users through pecuniary externalities) essentially depicts process innovation. Similarly, there are two types of outflows denoted by Y%. One is in the form of the turnover lost due to the products becoming obsolete each year and the other is the percentage of turnover the firm loses due to the adoption of their innovation by the other firm. The concept of creative destruction (Schumpeter, 1942) is used here. It is defined as the "process of industrial mutation that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one." In line with this concept, the model assumes that each year the portfolio of products for a firm are updated where new products enter the market and each year some of them become obsolete. For the purposes of this model, the size of the industry is assumed to be stable when in equilibrium; it neither grows nor shrinks. And this assumption leads to the inflow equating to the outflow as explained in the annex. Moving to the conveyor belt model which looks at the industry on the whole. Each box represents the portfolio of products of all the firms at year t. For instance, box 1 is year 1 of the portfolio of products in the industry, box 2 is year 2 and so on.



As explained above, the inflow into the industry is denoted by X% which is the percentage of turnover that comes from the sales of the portfolio of products that are new to the market. Assuming that each year is equally productive (i.e. there is no yield loss), X% remains constant for each year. Each year some Y% of turnover is flowing out of the industry, due to some products becoming obsolete. At the steady state where the size of the industry remains the same, the turnover coming from new products each year becomes equal to the turnover lost due to some products becoming obsolete. The next step is to look at the totality of the lifetime of the portfolio of products which intuitively should be equal to 100%. We can take the example of a pie chart where the whole circle is equal to 100% (The conveyor belt can be thought of as cutting a pie chart in half and unfolding it).

Using this concept, the total lifetime of the portfolio of products within the industry and within the firm, is computed. This will allow us to calculate the proportion of benefit which is classed as direct. The leftover will lead to the quantification of the indirect benefit.
Let's consider the following equation:

$$\sum_1^T X_t\% = 100\% \quad (10)$$

This equation shows that a portfolio of products will last for T years. During this time, the portfolio will give you a certain X_t% of turnover each year. Across the lifetime, this should intuitively equate to 100%. Considering the assumption above that each year is equally productive, it can be asserted that:

$$\mathrm{X} = \mathrm{X}_1 = \mathrm{X}_2 = \mathrm{X}_3 \ldots\ldots\ldots\ldots\ldots = \mathrm{X}_\mathrm{T} \quad (11)$$

This in turn implies that:

$$\mathrm{TX}\% = 100\%$$

$$\mathrm{T} = \frac{100\%}{\mathrm{X}\%} \quad (12)$$

From the NMS survey, we know the value of X% which is the percentage of turnover that comes from the sale of goods and services new to the market. We also know the percentage of turnover that comes from the sale of goods and services new to the firm but not the

market[6]. Equation 13 only uses the percentage of turnover that comes from the sale of new goods and services, to compute the total lifetime of the portfolio of products in an industry. To compute the lifetime of the portfolio within the firm, both components; percentage of turnover that comes from the sale of products new to the market and those that were new to just the firm, have to be used.
The survey found that 23% of the turnover was for goods and services new to the market and 19% for goods and services that were new to just the firm. These results are for the three years combined (2020-2022). Therefore, to compute the annual value, it should be divided by 3. The total lifetime of the portfolio in the industry is calculated as:

$$\mathrm{T}_{total} = \frac{100\%}{\frac{23\%}{3}} \quad (13)$$

$$\mathrm{T}_{total} = 13.04$$

The total lifetime of a portfolio within the firm is calculated as:

$$\mathrm{T}_{firm} = \frac{100\%}{\frac{(23\%+19\%)}{3}} \quad (14)$$

$$\mathrm{T}_{firm} = 7.14$$

Finally, the lifetime within the firm, as a proportion of lifetime in the industry, becomes a measure for the direct benefit. The idea is that new products would have a certain lifetime within the firm (direct benefit generated) after which they will be adopted in some form within the industry. In other words, any leftover from the total product lifetime, must have entered the system, and by way, to other firms which essentially is the indirect effect. The table below summarises the results obtained.

Table 2	**Total product lifetime within the industry**	**Version lifetime within the firm**	**Direct (Original Version)**	**Indirect (Follow-on Versions)**
Total	13.04	7.14	55%	45%
Manufacturing	13.39	8.11	61%	39%
Non-Manufacturing	11.24	5.86	52%	48%

Table 1 results depict a 55-45 spilt between direct and indirect benefits, with percentages slightly varying when segmented by manufacturing and non-manufacturing sector. The percentages are very close to the split found in the study commissioned by the Department for Science, Innovation and Technology (DSIT).

To summarise, the total lifetime of a portfolio of products in the industry and the lifetime of a portfolio within the firm is computed, using the conveyor belt model, which essentially depicts product diffusion. This in turn provides a measure of the percentage split between the direct and indirect benefit of innovations that are supported by NPL. This is backed by the results obtained from the NMS Survey 2022-23 which show that NPL plays a vital role when it comes to the support it provides to various innovation activities undertaken by its userbase. It found that each year, around 920 of the UK-based businesses who have used the NMS labs,

6 In the NMS Survey 2022-23, two questions were asked regarding percentage of turnover from goods and services; one for goods and service new to the market and the other for goods and services that remain unchanged. The third category which is goods and services that were improved / new to the firm was calculated as a residual.

collectively attribute £500 million in sales revenue to innovations that wouldn't have succeeded without the support of NMS labs.

In addition, the conveyor belt model may also be used to explain that product and process diffusion are interlinked. This is backed by the fact that product diffusion occurs in a young market and process diffusion occurs when the market has matured. The concept in the conveyor belt model that a new product remains within the firm for a period of time after which it is copied and sold by others, known as "market stealing", is in fact, process innovation. The model can, therefore, be further developed to formalise the link.

6 CONCLUSION

The services NPL deliver have far reaching indirect impact and thus it is important to know the various ways in which the knowledge spillovers are spread beyond the direct userbase. NPL's knowledge spillovers are classified in to two categories: Indirect Benefit and Indirect Cost. The focus of this document is to build conceptual models that explain, and eventually quantify, the indirect benefits generated by NPL. Indirect benefit is explained through two main streams; free to access information goods and codified knowledge. Codified knowledge breaks down into creation and revision of standards, which leads to product and process diffusion.

NPL provides high class calibration services which are a classic example of an information good. A market failure with such a service is expected as the technological knowledge within the calibration services spreads and gets resold, without any direct income to NPL. However, transfer of accuracy in the firsthand calibrations ultimately benefits the industry. The only model that can explain how a market for an information good like calibration services can exist is the Edgeworth model i.e., the existence of a positive price and therefore, supernormal profits. Furthermore, it explains price randomisation. The accuracy in NPL's first hand calibration services is used as a baseline by its users to sell follow on calibrations. These users then compete against each other to gain market share. The Edgeworth model of price setting oligopoly explains this competition of market share between two firms with homogenous products and capacity constraints, and how firms engage in a price war leading to a mixed strategies equilibrium. In addition, it explains that NMS labs' firsthand calibrations are cascaded through the various tiers of the calibration chain and have a primary fanout to ~75,500 organisations.

Furthermore, it discusses two approaches to quantifying the indirect benefit generated through calibration services. The first one is a study that explains the value of information from providing calibration services, through the reduction in costs of mistakes. It finds that the GVA safeguarded through supplying high-quality, primary calibrations to manufacturing firms that work with the NMS labs is approximately £197 million per annum. The second approach uses Claud Shannon's entropy in information theory to estimate the value of information gained through better measurement. Through defining information as surprisal, entropy as the expected surprisal and the concept of net benefits generated from measurements, it finds that for £1 spent on improved measurement, the return on measurement is £1.30. However, further work is required to develop a general model with the help of the two approaches, discussed.

NPL's role in creating and maintaining codified and tacit knowledge is another stream through which indirect benefits are generated. NPL has an important role in the creation and revision of standards, through chairing CEN and ISO technical committees. These form basis for the development and revision of standards which are critical for emerging technologies. It also plays a vital role in the creation and maintenance of best practice through its free to download guides, trainings and apprenticeships.

In addition, the document links standards to product and process diffusion. The process of product diffusion is explained through the conveyor belt model which is about creative destruction and "market stealing". Using the percentage of turnover that comes from the sale of products new to the market, and those that were new to just the firm, it computes the lifetime of a product within a firm and that within the industry. The calculations are 7 years within the firm and 13 years within the industry. Using these numbers, it computes the ratio between direct and indirect benefits. It finds approximately a 55-45 spilt between direct and indirect benefits, with percentages slightly varying when segmented by manufacturing and non-manufacturing sector. The percentages are very close to the split found in the study commissioned by the Department for Science, Innovation and Technology (DSIT). Process diffusion is explained through NPL's role in revision of standards and how it helps diffuse knowledge surrounding standardised processes in various areas of measurement science.

The primary aim of this document has been to structure NPL's spillovers and build models which help in understanding the ways in which the indirect benefit is generated and channelled. The next step is to use this conceptual framework, and further develop them to fully mathematise the indirect benefits generated by NPL.

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8 ANNEX A – NPL AND PRIMARY CALIBRATION SERVICES

The organisations currently coming to NPL for calibrations represent the first vital link in the traceability chain; and it's through these organisations that the benefits then fanout across the economy. This is based on data from the latest NMS survey which shows that only 14% of domestic customers use a foreign NMI in conjunction with one of the NMS labs. This suggests that the chain of traceability is primarily anchored in the NMS labs. Thus, if an NMI ceased to provide some of its services, then the whole chain of traceability would be weakened.

The calculations below arrive at an estimate of how much the use of precise calibrations would decrease without the NMS labs, by analysing a counterfactual scenario. Given that VSL in the Netherlands is the closest major NMI to the UK, it would be the next best alternative. The calculations are based on a scenario where NPL shut down, and UK-based organisations had to take traceability from VSL.

The invoicing data suggests that approximately, 50% of NPL's revenue comes from overseas customers, particularly, those based in Europe. This suggests that NPL offers some services that aren't provided by the home NMI of overseas customers. An econometric analysis (King & Renedo (2020)) was performed using the dataset constructed from NPL's invoicing records to find the relationship between the volume of sales originating from customers in a particular country (number of invoices) and two factors of interest:

1. Price-level ratio between a country and the UK during a given year[7].
2. Strength of a country's own national measurement institute based on coverage of the Core Measurement Capabilities (CMCs) in the BIPM's database[8].

The headline results from the study, are:

- If the price-level ratio increases by 10%, then demand for NPL's services drops by 12.4%, which implies an elastic demand for services.
- If a foreign NMI increases its coverage of CMCs on BIPM's database by one percentage point, then demand for NPL's services drops by 2%.
- If the distance between the UK and another country was somehow to double, then demand for NPL's services would drop by 48%. In other words, the coefficient for logged distanced in the regression function is 0.48.

Using these headline figures, the effect on UK based user, of switching off NPL's services can be assessed by considering the increase in the distance that UK-based customers would need to ship the instruments that they wanted precisely calibrated. For this purpose, a conventional formula for home country's internal distance ($D = 0.67\sqrt{A/\pi}$, where A is the geographical area of a country) is used to calculate the average distance between NPL and its UK-based customer. The geographical area covered by the UK is 244,376km^2, giving an internal distance of 185.8km.

To model the shift from NPL to VSL for precise calibrations, we imagine that NPL was picked up and somehow transferred to Amsterdam, meaning that NPL's UK-based customers must now transport their instruments much further for calibrations. This implies that the current

7 This index is a country's purchasing power parity relative to the UK divided by the exchange rate expressed in local currency units per pound.
8 This index ranges from 0 to 100, where '100' signifies a full coverage of the CMCs in the BIPM's database and '0' signifies that there's nothing available.

distance of around 186km increases to 357km. Considering that logged distance features in the regression formula, it's appropriate to estimate the percentage change in the distance as follows:

$$\ln(357) - \ln(186) \approx 65\%$$

Hence, the consequent drop in demand from UK-based customers can be estimated as follows:

$$-0.48 \times [\ln(357) - \ln(186)] \approx -31\%$$

In other words, because the elasticity of invoices with respect to distance is -0.48, increasing the separation of "NPL" and its userbase by 65% would cause demand to drop by 31%. Furthermore, NPL currently covers a higher proportion of the CMCs than VSL, and so customers may encounter a drop in capability. Hence, a 31% decrease in the UK's use of precise calibrations may be a conservative estimate, with the actual drop probably being greater.

9 ANNEX B – INTRODUCTION TO ENTROPY

This section explains entropy in its basic form. Entropy is a scientific concept that is associated with a state of disorder, randomness, or uncertainty. It was first recognised in the field of classical thermodynamics where it was defined in terms of 'macroscopically' measurable physical properties, such as pressure and temperature. In the 19th century, a famous physicist Ludwig Boltzmann explained entropy as the measure of the number of possible 'microscopic' arrangements, or states of individual atoms and molecules of a system that yield this particular macroscopic condition of the system. He introduced the concept of statistical disorder into a new field of thermodynamics, called statistical mechanics. The interpretation of entropy in statistical mechanics is the measure of uncertainty, or disorder, which remains about a system after its observable macroscopic properties have been determined. *In other words, in statistical mechanics, entropy is a measure of the number of ways a system can be arranged, often taken to be a measure of "disorder" (the higher the entropy, the higher the disorder).* This definition describes the entropy as being proportional to the natural logarithm of the number of possible microscopic configurations of the individual atoms and molecules of the system (microstates) that could cause the observed macroscopic state.

In the mid-20th century, the mathematician Claude Shannon introduced the concept of information entropy. His definition of information theory implies that the entropy of a random variable quantifies the average level of uncertainty or information associated with the variable's potential states or possible outcomes. The entropy in information theory is directly analogous to the entropy in statistical mechanics. Hence, the formula for entropy under statistical mechanics is formally identical to Shannon's formula. The definition for entropy in information theory is derived from a set of axioms discussed below. The core idea of information theory is that the "informational value" of a communicated signal depends on the degree to which the content of the signal is surprising. If a highly likely event occurs, the message carries very little information. On the other hand, if a highly unlikely event occurs, the message is much more informative. For instance, the knowledge that some number will not be the winning number of a roulette wheel provides very little information, because any particular chosen number will almost certainly not win. However, knowledge that a particular number will win the bet has high informational value because it communicates the occurrence of a very low probability event. The information content, also called the surprisal or self-information, of an event E is a function which increases as the probability $\mathrm{p(E)}$ of an event decreases. When $\mathrm{p(E)}$ is close to 1, the surprisal of the event is low, but if is close to 0, the surprisal of the event is high. Below are the fundamental principles that Shannon uses to define the information.

Shannon's Axioms for self-information

4. $\mathcal{I}(1) = 0$ i.e. an event with probability 100% is perfectly unsurprising and yields no information.
5. $\mathcal{I}(\mathrm{p})$ is monotonically decreasing in p. In other words, the less probable an event is, the more surprising it is and the more information it yields.
6. $\mathcal{I}(\mathrm{p_1 p_2}) = \mathcal{I}(\mathrm{p_1}) + \mathcal{I}(\mathrm{p_2})$ i.e. if two independent events are measured separately, the total amount of information is the sum of the self-informations of the individual events.

Proposition A: Logarithm is the only function that can satisfy Shannnon's axioms of self-information. Therefore, the information (surprisal) of an event E can be described as follows:

$$\mathcal{I}_E = \log_b\left(\frac{1}{p_E}\right) = -\log_b(p_E) \quad (1)$$

where b is the base of the logarithm chosen on the basis of the application needed.

Proof: The following equations use the set of axioms defined above. Taking axiom 3:

$$\mathcal{I}(p_1 p_2) = \mathcal{I}(p_1) + \mathcal{I}(p_2) \quad (2)$$

Taking the derivative w.r.t to p_1

$$p_2\mathcal{I}'(p_1 p_2) = \mathcal{I}'(p_1) \quad (3)$$

Taking the second derivative w.r.t to p_2

$$\mathcal{I}'(p_1 p_2) + p_1 p_2 \mathcal{I}''(p_1 p_2) = 0 \quad (4)$$

Let $x \equiv p_1 p_2$ and substituting it in equation

$$\mathcal{I}'(x) + x.\mathcal{I}''(x) = 0 \quad (5)$$

Using the product rule. The above expression can be rewritten as

$$\frac{d}{dx}[x.\mathcal{I}'(x)] = 0 \quad (6)$$

If the derivative of the above expression is zero, then that means $x.\mathcal{I}'(x)$ itself must be a constant. Equating the constant term to k

$$x\frac{d\mathcal{I}}{dx} = k \quad (7)$$

Rearranging the expression and taking the integral

$$\frac{d\mathcal{I}}{dx} = \frac{k}{x}$$

$$\mathcal{I}(x) = k.\ln(x) + c \quad (8)$$

Now, combining equation 8 with axioms 1 and 2:

From axiom 1, $\mathcal{I}(1) = 0 \quad \Rightarrow \mathcal{I} = k.\ln(1) + c = 0 \quad \Rightarrow c = 0$

From Axiom 2, $\mathcal{I}(p) > 0 \; if \; p < 1$. This will hold only if $\mathrm{k} < 0$

Hence, we have proved the proposition. ■

Entropy measures the expected (i.e., average) amount of information conveyed by identifying the outcome of a random trial. This implies that rolling a die has higher entropy than tossing a coin because each outcome of a die toss has smaller probability. For a system that can be in a number of distinct states, Shannon defines entropy H as the expected surprisal such that:

$$\mathrm{H} = \mathbb{E}[\mathcal{I}_E] = \mathbb{E}[-\log_2(p_E)] \quad (9)$$

Here $\mathbb{E}$ is the expected value operator, and $\mathcal{I}_E$ is the information content of an event. The choice of base for the logarithm varies for different applications. Base 2 gives the unit of 'bits' (or 'shannons'), while base e gives 'natural units (nat)', and base 10 gives units of 'dits', 'bans', or 'hartleys'. The base value of log is taken as 2 throughout as done by Shannon.

Equation 9 can be rewritten as:

$$\mathrm{H} = -\sum p_E \log_2(p_E) = -(p_1\log_2(p_1) + p_2\log_2(p_2) + p_3\log_2(p_3) + \cdots) \quad (10)$$

10 ANNEX C – DERIVATION OF TOTAL LIFETIME OF A PRODUCT

Consider the following set-up of an industry. There are two firms that compete against each other. They sell different products and have a monopoly over their particular type of product. The products are considered to be substitutes so there is still competition.

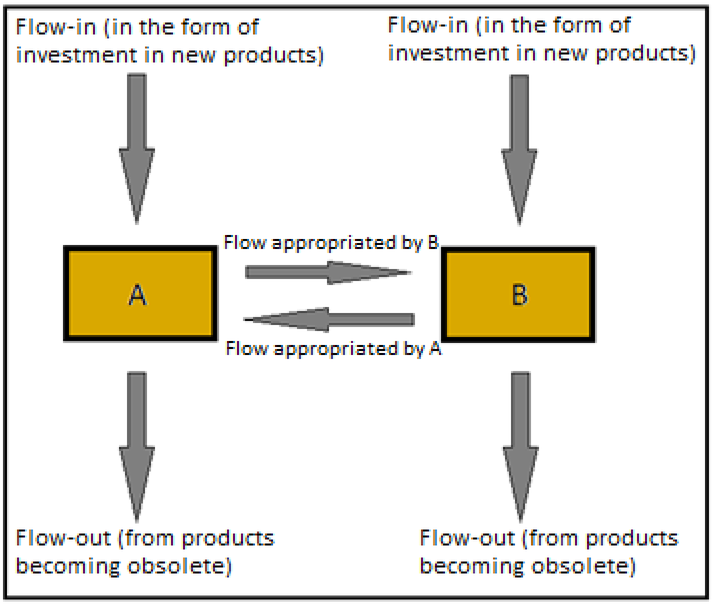

Figure 5

Consider the following first differential basic function as in the Solow model of Romer:

$$\dot{K} = s.Y - \delta K \tag{11}$$

where $\dot{K}$ is the change in capital stock, Y is the total production, s is share for investment and δ denotes the depreciation of the stock of capital.

The below equation adopts the Solow model to fit the industry set-up described above:

$$\dot{A} = a - \delta A - \theta A + \sigma B \tag{12}$$

$$\dot{B} = b - \delta B - \theta B + \sigma A$$

where A, B is the total turnover, $\dot{A}, \dot{B}$ is the change in total turnover, a, b is the turnover from new products, δ denotes the share of turnover lost due to some products dying and becoming obsolete, θ is the share of its turnover it loses to other firms due to product imitation and σ is the share of the turnover it gains as a result of imitating the other firm's products.

Eventually the system will settle into an equilibrium, where the market is neither growing nor shrinking that is steady state. This infers that at the steady state the overall change in the turnover will be zero as shown in equation 13.

$$\dot{A} + \dot{B} = 0 \tag{13}$$

Substituting the equations from 12 in equation 13:

$$\dot{A} + \dot{B} = a - \delta A - \theta A + \sigma B + b - \delta B - \theta B + \sigma A = 0$$

$$a + b - \delta(A + B) = 0$$

$$a + b = \delta(A + B) \tag{14}$$

$$\frac{a+b}{A+B} = \delta \tag{15}$$

where $A + B$ is the total turnover in pounds and $a + b$ is the turnover from the sales of new products in pounds. $\frac{a+b}{A+B}$ is simply X% which is the percentage of turnover that comes from the sale of new products. Equation 15 implies that the inflow rate is equal to the outflow rate. And because the inflow equals the outflow and the system is in equilibrium, the ages of the products must be uniformly distributed across the range of the portfolio of products. As mentioned above δ is simply the percentage of turnover lost due to some products becoming obsolete.

By definition, the total lifetime is:

$$\text{Total Lifetime} = \frac{A+B}{a+b} = \frac{\text{Total Stock}}{\text{Inflow}} \tag{16}$$

Taking the reciprocal of equation 15 and using equation 16 gives:

$$\text{Total Lifetime} = \frac{1}{\delta} \tag{17}$$

> **Numerical Example:** Let the total turnover be £100m and turnover from sales of new products be £15m each year. To compute the lifetime of a portfolio of products, it would be
> $T = \frac{£100m}{£15m}$
> Similarly, the other way is to use equation 19 to compute the lifetime of the portfolio. Since inflow is equal to the outflow in the steady state. The calculation will be:
> $T = \frac{1}{15\%} = \frac{100}{15}$
> which is essentially the same as above.

Consider the following equations for the firm level set-up.

$$\dot{A} = a - \delta A - \theta A + \sigma B \tag{18}$$

$$\dot{B} = b - \delta B - \theta B + \sigma A$$

where A, B is the total turnover, $\dot{A}, \dot{B}$ is the change in total turnover, a, b is the turnover from new products for each firm, δ denotes the share of turnover lost due to some products dying and becoming obsolete, θ is the share of its turnover it loses to other firms due to product imitation and σ is the share of the turnover it gains as a result of imitating the other firm's products.

Similar to above, in the steady state the market is neither growing nor shrinking which implies that the change in the turnover for each firm is zero.

$$\dot{A} = \dot{B} = 0$$

$$a - \delta A - \theta A + \sigma B = 0, \quad b - \delta B - \theta B + \sigma A = 0 \tag{19}$$

Both firms are identical so will have similar set of equations. For ease, the below equations are only for firm A.

$$a + \sigma B = \delta A + \theta A \quad \Rightarrow \quad \text{inflow to firm A = outflow from firm A}$$

$$a + \sigma A = (\delta + \theta)A \tag{20}$$

$$\frac{a+\sigma A}{A} = \delta + \theta \tag{21}$$

By definition, the total lifetime would be:

$$\text{Total Lifetime} = \frac{A}{\text{a}+\sigma\text{A}} = \frac{\text{Total Stock}}{\text{Inflow}} \quad (22)$$

Taking the reciprocal of equation 21 and using equation 22 gives:

$$\text{Total Lifetime} \ = \frac{1}{\delta+\theta} \quad (23)$$

Like in the industry set-up, at the steady state, the inflow rate becomes equal to the outflow rate, as shown in equation 21. The difference here is that the inflow for the firm level set-up is the percentage of turnover from sale of new products and the share it acquires year on year from imitating the products of firm B. And the outflow is percentage of turnover lost due to some products becoming obsolete as well as the percentage of turnover lost due its products being copied by the other firm.