Uncertainties in the calibration of hydrophones at NPL by the three-transducer spherical-wave reciprocity method in the frequency range 10 kHz to 500 kHz

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CONTENTS

1 INTRODUCTION ................................................. 1
  1.1 BACKGROUND ........................................... 1

2 THE METHOD .................................................. 4
  2.1 DEFINITIONS .......................................... 4
  2.2 PRINCIPLE ............................................ 4
  2.3 PROCEDURE ........................................... 5

3 NPL IMPLEMENTATION ........................................ 8
  3.1 TANK FACILITY ....................................... 8
  3.2 INSTRUMENTATION ..................................... 9
  3.3 PROCEDURE ........................................... 10

4 SOURCES OF UNCERTAINTY ................................... 12
  4.1 PRINCIPLE SOURCES OF SYSTEMATIC UNCERTAINTY ........ 12
  4.2 ADDITIONAL SOURCES OF SYSTEMATIC UNCERTAINTY ...... 16
  4.3 RANDOM UNCERTAINTY .................................. 35

5 UNCERTAINTY BUDGET ......................................... 36
  5.1 ASSESSING THE UNCERTAINTIES ......................... 36
  5.2 COMBINING UNCERTAINTIES ............................. 36

6 COMPARISON WITH OPTICAL INTERFEROMETRY ............... 39
  6.1 THE NPL LASER INTERFEROMETER ....................... 39
  6.2 MEASUREMENTS ....................................... 39
  6.3 RESULTS ............................................. 40

7 DISCUSSION AND CONCLUSIONS ................................ 44

8 REFERENCES ................................................. 46

APPENDIX 1 MEASUREMENTS IN REVERBERANT TANKS USING PULSE
  TECHNIQUES ............................................. 49

APPENDIX 2 ELECTRICAL LOADING CORRECTIONS ............... 53
1 INTRODUCTION

This report gives an assessment of the uncertainties in the primary calibration of hydrophones by the method of three-transducer spherical-wave reciprocity in the frequency range 10 to 500 kHz. The work described here was undertaken at the National Physical Laboratory (NPL) using the NPL laboratory tank facility and some of the results shown relate specifically to that facility. However, much of the analysis is more general and will apply to similar implementations of the method.

A description of the method is given along with a brief description of the procedure used by NPL. This is followed by an assessment of uncertainties with particular emphasis on those which are important at high frequencies, ie the range 200-500 kHz. Results are then given of an intercomparison of the method at these frequencies with optical interferometry, which is the NPL primary standard calibration method for hydrophones at frequencies from 500 kHz to 20 MHz.

1.1 BACKGROUND

1.1.1 Requirements for primary standards

In underwater acoustical systems, electroacoustic transducers are used to generate and/or detect sound. To assess the performance of such transducers, there is a need to undertake absolute acoustic measurements which, if they are to be meaningful, must be traceable to common measurement standards. An essential element in the structure of measurement standards is the primary measurement standard which will provide the reference for acoustical measurement and testing. The primary standard can be embodied in a device or, as is the case here, it can be a primary standard calibration method.

The dissemination chain through which industrial users achieve traceability to the primary standard is by the use of intermediate secondary standard devices (transducers), which can be calibrated or characterised on an absolute basis by the primary standard method. The transducers most suitable for use as secondary standard devices are high quality reference hydrophones. Although sources of known acoustic output are often useful at the industrial level, it is more accurate to base absolute calibration methods on standard receivers rather than on standard sources because a standard receiver usually will compensate better for any lack of free-field conditions (ie, the output sound pressure level generated by an acoustic source for a given electrical stimulus is more dependent on the boundary conditions in the medium).

Also, standard receivers are broadband devices with a reasonably constant sensitivity at frequencies below resonance, whereas standard sources are less broadband and so several devices may be required to cover a desired frequency range.

The calibration accuracy required for secondary standard devices is typically $\pm 1$ dB or better to be sure of achieving the desired accuracy in field measurements. Taking into account the inevitable degradation which will occur at the dissemination stage, this implies that the accuracy of the primary standard should ideally be $\pm 0.5$ dB or better.
1.1.2 Realisation of the primary standard

The most common absolute methods used in the calibration of electroacoustic transducers depend on the principle of acoustic reciprocity. There are various ways of implementing reciprocity-based calibration methods\(^1\) including those prescribed by international standards\(^2\), but they all involve the electrical measurement of the current driving a transducer and the voltage generated by another transducer placed in the acoustic field produced by the first transducer. The full title of the calibration method adopted by NPL as the primary standard is *three-transducer spherical-wave reciprocity*. In this free-field method, three transducers are paired off in three measurement arrangements and the above electrical measurements are made. Given knowledge of the geometry of the acoustic field propagating between the two devices (in this case, a *spherical-wave* field) and assuming that at least one device is reciprocal, the transmitting or receiving sensitivity of any of the three transducers can be calculated from the purely electrical measurements. The type of acoustic field propagating (eg spherical-wave, plane-wave, etc) determines the reciprocity parameter, \(J\), which is the quotient of the receiving response and the transmitting response of the reciprocal transducer.

The low-frequency limit of this method is governed by the dimensions of the water tank used. For a given transducer size, the high-frequency limit also depends on the volume of water available if measurements are to be made in the far-field. In practice, at frequencies greater than 500 kHz, the method becomes more difficult to implement and other methods, for example optical interferometry, have the potential for greater accuracy.

1.1.3 The transducers

Hydrophones are electroacoustic transducers and in this report, the words hydrophone and transducer are used interchangeably. Where a particular function is implied, such as transmitting or receiving, this is stated in the text. Although the calibration method described applies to electroacoustic transducers in general, this report is concerned with the calibration of reference hydrophones suitable for use as secondary standard devices. These most commonly take the form of piezoelectric transducers designed to be small compared to the acoustic wavelength in order to appear like point receivers, with an ideally omnidirectional directivity and a flat frequency response. As the frequency is increased, eventually the wavelength will be comparable in size to the dimensions of the transducer. At this point, diffraction around the transducer and resonances within the transducer will cause variation in both the frequency response and the directional response. The design of the transducers often makes use of symmetrical sensing elements to improve the directional response at frequencies close to resonance\(^3\).

The useful operating range of a hydrophone is usually considered to end just above resonance where the sensitivity begins to fall off rapidly with frequency (in general, with a slope inversely proportional to the square of the frequency). Ideally therefore, a hydrophone used as a reference device should have a resonance frequency higher than the highest frequency of interest. However, for the frequency range 300-500 kHz this would require a transducer with a resonance frequency in excess of 500 kHz. Small, omnidirectional reference hydrophones
are not generally available with such high resonance frequencies though some new devices have been reported which may serve this function in the future. Therefore, either larger directional devices (eg plane-piston transducers) are used and the desired omnidirectional response is sacrificed or, alternatively, hydrophones such as the B&K 8103, the Reson TC4034 and the ITC 1089 are used at frequencies above resonance. Care is needed when using transducers above resonance because their performance is likely to be less well-behaved.
2 THE METHOD

The principle of reciprocity as applied to electro-acoustics was first introduced by Schottkey in 1926. MacLean and Cook first used it for calibration purposes in 1940 and 1941.

2.1 DEFINITIONS

Before the method can be described, there are some important definitions which need to be stated. The free-field sensitivity, $M$, of a hydrophone at a chosen frequency and in a stated direction, is given by

$$M = \frac{e}{p}$$  \hspace{1cm} (1)

where $e$ is the open-circuit voltage of the hydrophone and $p$ is the acoustic pressure which would exist at the position of the hydrophone in the acoustic field if that hydrophone were removed.

The transmitting response, $S$, of a projector at a given frequency and in a given direction is defined as the acoustic pressure at a reference distance from the acoustic centre of the projector divided by the current flowing through its terminals. From this definition, the far-field acoustic pressure at a distance $d$ from a projector being driven by a current $i$ can be expressed as

$$p = Si \frac{d_0}{d}$$  \hspace{1cm} (2)

assuming a spherical-wave field, where the acoustic pressure falls off inversely with distance. The reference distance, $d_0$, is usually chosen to be 1 metre and is then omitted from the equation. For a pair of transducers, the transfer impedance, $Z$, is defined as the ratio of the open circuit voltage, $e_H$, across the terminals of the receiving device to the current, $i_p$, through the transmitting device, and is therefore given by

$$Z = \frac{e_H}{i_p}$$  \hspace{1cm} (3)

2.2 PRINCIPLE

All reciprocity methods depend on one of the transducers being reciprocal; that is, the ratio of its receiving sensitivity $M$ to its transmitting response $S$ is equal to a constant factor, $J$, known as the reciprocity parameter. This parameter varies depending on the type of acoustic field being produced.

For a spherical-wave field, the reciprocity parameter is given by
\[ J = \frac{2 d_0}{\rho f} \]  

(4)

where \( \rho \) is the density of water and \( f \) is the frequency of the acoustic wave. The distance \( d_0 \) is the same reference distance that is used in the definition of the transmitting response. It can be shown that the reciprocity parameter is the ratio of the volume velocity emanating from the reciprocal transducer to the resulting sound pressure used in the definition of the transmitting response (i.e., the pressure at a point 1 m away)\(^9\). The parameter is thus a transfer acoustical admittance of the medium and medium boundaries. This means that the expression for the reciprocity parameter depends upon both the geometry of the transducer and the experimental conditions. For example, the reciprocity parameter used in free-field reciprocity will differ from that used in coupler or plane-wave reciprocity.

2.3 PROCEDURE

Three transducers are paired off in three measurement arrangements as illustrated by Figure 1. Of the three devices, one is designated as a projector (P), one is the hydrophone (H) to be calibrated and one is the reciprocal transducer (T).

In the first measurement, the hydrophone is placed in the acoustic field of the projector at a distance \( d_1 \). From Equations (1) and (2), it can be seen that if the projector is driven with a current \( i_{PH} \), the hydrophone will develop an open-circuit voltage, \( e_{PH} \), given by

\[ e_{PH} = \frac{M_H S_P i_{PH}}{d_1} \]  

(5)

From Equation (3), the transfer impedance is then given by

\[ Z_{PH} = \frac{S_P M_H}{d_1} \]  

(6)

For the second measurement, the hydrophone is replaced by the reciprocal transducer which develops an open-circuit voltage \( e_{PT} \) across its terminals when the projector is driven by a current \( i_{PT} \) and is positioned a distance \( d_2 \) away. By similar reasoning to the above, the transfer impedance for this pair is

\[ Z_{PT} = \frac{S_P M_T}{d_2} \]  

(7)

Similarly, in the third arrangement the reciprocal transducer is driven by a current \( i_{TH} \), causing an open-circuit voltage \( e_{TH} \) at the hydrophone a distance \( d_3 \) away. In this case, the transfer impedance is
Equations (6) to (8) form a set of three simultaneous equations in four unknowns, namely $M_H$, $S_P$, $M_T$ and $S_T$. However, for the reciprocal transducer, the reciprocity theorem states that the receiving and transmitting response are related such that $M_T/S_T=J$ where $J$ is defined by Equation (4). This relationship enables Equations (6) to (8) to be solved giving the sensitivity of the hydrophone as

$$M_H = \left( J \frac{d_1 d_3 Z_{PH} Z_{TH}}{d_2 Z_{PT}} \right)^{ij} \tag{9}$$

In some implementations of the reciprocity method, the same separation between source and receiver is used at each measurement stage. This is equivalent to setting $d_1=d_2=d_3$ which simplifies Equation (9) slightly. However, since this is not necessarily the case, Equation (9) is a more general form of the equation for the sensitivity. It should be noted that in Equation (9) the magnitudes of the transfer impedances are used. Phase information can be included\cite{10,11} (for example by use of complex notation) but has been omitted for clarity. Phase calibration
is not within the scope of this report.

In the method, neither the transducer used as a projector (P) nor the hydrophone to be calibrated (H) need be reciprocal devices. However, if P and H are reciprocal, and small piezoelectric hydrophones usually are, then this enables further measurements to be made. For example, the fourth measurement shown schematically in Figure 1 can be performed if P is a reciprocal device. The four measurements which now make up the complete calibration include an element of redundancy which allows the sensitivity of the hydrophone to be calculated in two ways: either using Equation (9) as stated, or by using the same equation but with $Z_{TP}$ substituted for $Z_{PR}$. This provides an extremely useful check on the accuracy of the method.

The fourth measurement described above is often referred to as a reciprocity check and is used to validate the assumption that $Z_{TP}=Z_{RT}$. This is important since the method depends upon the transducer(s) being reciprocal. Note that if the hydrophone is a reciprocal device so that $Z_{PH}=Z_{HP}$, it is even possible to calculate the theoretical transmitting response of the hydrophone even though it was never used as such in the measurements.
3 NPL IMPLEMENTATION

3.1 TANK FACILITY

Most of the work reported in this document was performed in the NPL laboratory tank. The tank is 2 metres in length, 1.5 metres wide and 1.5 metres deep and is pictured in Figure 2. It is constructed from welded polypropylene sheet strengthened by an external steel frame and is free standing on a concrete plinth. The tank size is relatively small for such a facility, the main constraint being the space available to house it.

![The NPL laboratory tank and positioning system.](image)

A high-quality positioning system has been installed on the tank to allow for accurate alignment and positioning of transducers, an important factor at higher frequencies where transducers typically become more directional. The system comprises two independent positioning carriages providing translation of a source and receiver in the water tank. The complete system is mounted on a profile support structure which is bolted to a concrete floor on specially levelled metal plates. The system makes use of cruciform section guidage rails with precision racks machined into one edge. These provide linear translation along three orthogonal axes (x, y and z) for both carriages. Rotation is carried out by rotation stages mounted on the carriages. In addition, one of the carriages provides a tilt facility of ±4°. All stages are controlled via encoded stepper motors with the control units interfaced to a computer. The smallest programmable step size is 0.01 mm for the linear axes or 0.01° for the rotational axes. Accuracy for these is 0.2 mm and 0.03° respectively giving an overall positioning accuracy of ±0.5 mm for the carriages alone. Transducers under test can be fixed
to a variety of mounts which are connected to the z-axis carriage rails by hollow aluminium poles which locate in collet chucks bolted on to the rails.

To perform free-field measurements in a laboratory tank such as this requires either the elimination or the avoidance of reflections from the water boundaries. This could be achieved by lining the inner walls of the tank with an acoustic absorber, thereby rendering the tank anechoic. However, to render the NPL tank anechoic over a broad range of frequencies is a difficult task and could drastically reduce the usable volume of the tank.

A common way to overcome these limitations and to avoid the problems of reflections is to use pulse techniques\textsuperscript{12} and such techniques have been adopted at NPL. In this method, a tone-burst containing a few cycles of the desired frequency is transmitted from the projecting device. The signal detected by the hydrophone consists of the direct-path signal followed by reflections from the tank boundaries. Gating techniques are then used to isolate the direct-path signal from the unwanted reflections. Measurements are then made on the steady-state portion of the direct-path signal.

When using pulse techniques in a laboratory tank, the size of the tank poses constraints on the lowest usable frequency, the pulse duration, the distance between transducers and the repetition rate between pulses. For these reasons, it is difficult to perform measurements in the NPL tank at frequencies below 10 kHz.

3.2 INSTRUMENTATION

A schematic diagram of the equipment used is shown in Figure 3. The output of a sinusoidal oscillator is gated to produce a tone-burst before being fed into a power amplifier. This amplifier drives the transducer being used as the projector via a current probe. The signal generated by the hydrophone is then amplified by a preamplifier, filtered and measured on the digitising oscilloscope. The output of the current probe, which nominally has unity sensitivity into a 50 Ω load, is fed into a calibrated programmable attenuator which is connected to the same preamplifier as used to amplify the hydrophone signal. The attenuator is adjusted so that the magnitude of the current signal displayed on the oscilloscope is approximately the same as that of the measured hydrophone voltage. The current and voltage signals are nominally of the same magnitude and pass through the same signal processing instrumentation (preamplifier, filter and digitiser), so the linearity of the equipment is not a significant source of uncertainty in the calibration, the only equipment needing absolute calibration being the current probe and attenuator. A computer controller is used to control the experiment and process the data. All data is transferred between the instruments and computer via an IEEE 488 interface and the results can be printed or plotted at the end of the calibration.
3.3 PROCEDURE

The measurement procedure adopted conforms to IEC 565: *The calibration of hydrophones*\(^2\). Measurements are made at a series of discrete frequencies, for example the standard third-octave frequencies recommended by the International Organization for Standardization (ISO)\(^1\). However, measurements are often performed at other more closely-spaced frequencies, for example when operating close to the resonance frequency of a transducer. For each measurement of the transfer impedance, a measurement is made of the signal frequency, water temperature, separation distance, current driving the projector, voltage generated by the hydrophone and the setting of the attenuator. The computer guides the user through the measurement procedure with on-screen instructions and a system of menus.

Each set of measurements consists of the measurement of four transfer impedances, \(Z_{ph}\), \(Z_{th}\), \(Z_{pt}\) and \(Z_{tp}\). The measurement of both \(Z_{pt}\) and \(Z_{tp}\) forms the reciprocity check mentioned in Section 2.3. This provides some redundancy in the measurements allowing the hydrophone sensitivity to be calculated in two ways. It also enables a check to be made on whether the transducers are behaving reciprocally, a crucial requirement of the method.

Before measurements are made, the transducers are wetted with a weak solution of detergent and soaked for a predetermined time (usually overnight) in the water tank to ensure that
thermal equilibrium is reached. For a calibration, four independent sets of measurements are performed allowing four independent calculations of sensitivity to be made. To ensure each set of measurements is truly independent, between sets all three transducers are removed from the water and from their mounts, remounted, rewetted and resoaked in the water for a short time.

Each set of measurements is performed at a different separation distance. The use of four separation distances allows checks to be made to highlight any lack of free-field conditions caused for example by interference from reflections. In addition, it allows distance-loss checks to be performed to verify the existence of a spherically-spreading acoustic field. These checks might fail at one frequency and only two distances because two errors of different kinds happen to cancel out. The probability of error cancellation at several distances and over a frequency range is negligible\(^1\).

In every calibration of an "unknown" hydrophone (ie a device which has not been calibrated previously), two reference transducers are chosen as companion devices in the calibration. For example, when determining the receiving sensitivity of a hydrophone, the hydrophone is used as transducer H in the measurement procedure (see Section 2.3) with reference transducers used as P and T. These reference devices have a calibration history (they have been calibrated a number of times previously) and so considerable knowledge has been built up regarding their performance. This knowledge allows checks to made of the measurements in the calibration since the performance of the reference devices can be predicted if the devices are stable. Although both P and T are used as a source in the calibration, the hydrophone, H, is not (unless a transmitting sensitivity calibration is required). Of course, it may not be possible to use the hydrophone as a source anyway, for example if it has an integrated preamplifier.

The checks described above allow an assessment to be made of the validity of the calibration and give an indication of the uncertainties in the calibration.
4 SOURCES OF UNCERTAINTY

In any calibration method, it is important to make an assessment of the uncertainties in the method, and it is usual to discriminate between the uncertainties according to how they are determined\textsuperscript{14,15}. The systematic uncertainty is estimated from an assessment of the likely contributions within the method, for example, knowledge of the calibration accuracy of the measurement instrumentation or the accuracy of important assumptions inherent in the method. In contrast, the random uncertainty can be measured by making repeated measurements of a quantity and observing the variation in the result.

By careful experimental design, it is possible to reduce or even eliminate some sources of uncertainty, for example by use of a calibrated attenuator to equalise two voltages, thereby avoiding potential inaccuracy due to the limitations in the linearity of the measurement equipment. This approach has been adopted at NPL where possible. In addition, where it is difficult to make an accurate assessment of certain systematic uncertainties, an attempt is made to randomise them (i.e. cause them to contribute to the random uncertainties). It must be stressed that in the calibration of electroacoustic transducers, many of the contributions to the uncertainties are device dependent. Consequently, it is difficult to make generalised statements regarding the uncertainties without a knowledge of the characteristics of the transducers under test.

Some potential sources of systematic uncertainty are apparent from a \textit{prima facie} examination of the method and the equation used to calculate the sensitivity and these are described in Section 4.1. Other sources are less straightforward in origin and more difficult to assess. These are covered in Section 4.2.

4.1 PRINCIPLE SOURCES OF SYSTEMATIC UNCERTAINTY

From Equation (9), the following equation for determining the receiving sensitivity, $M_H$, of a hydrophone by the reciprocity method can be derived:

$$M_H^2 = J \frac{d_1 d_3}{d_2} \frac{Z_{HH}}{Z_{RT}}$$

(10)

This equation is divisible into three parts. The first part derives from the reciprocity parameter, the second relates to the distances between the devices, and the third is the ratio of the transfer impedances for each pair of devices used in the calibration. Each of these can be treated separately.

4.1.1 Reciprocity parameter

The first term in Equation (10) is the reciprocity parameter, $J$, which for a spherical-wave field was given by Equation (4) as equal to $2/\rho f$ (neglecting the reference distance $d_0$ which is assumed to be 1 metre). This requires values for the density of water, $\rho$, and the frequency of the acoustic wave, $f$. The density of the water depends on temperature and can be looked
up in tables or calculated to high accuracy. The error in taking the density as 1000 kg m$^{-3}$ is only 0.2% at 20°C.

The frequency of the signal can be measured to a high accuracy using a counter or frequency meter. When measured at the output of the oscillator using a nanosecond universal counter, the frequency can be determined to ±0.1% or better. In fact, the programmable oscillator used in the NPL system is of high accuracy and resolution: the frequency display of the oscillator agrees with that of the frequency meter to better than 0.1%. It is also important to know that the frequency of the signal detected by the hydrophone is of the correct frequency once steady state has been reached. Measuring the frequency after digitisation using a computer algorithm based on measuring the periods between zero crossings also shows good agreement with the frequency counter (typically within ±0.2%).

If the frequency is in error, it will not only affect the sensitivity calculated from Equation (10), but also introduces a shift along the frequency response curve of the transducer. This is not such a problem when the response is flat, but when it is sharply rising or falling, as when close to a resonance, the error could become significant and the uncertainty may have to be increased. It is also important to use a signal source of low distortion when driving transducers. At frequencies well below resonance, the transmitting voltage response has a slope of +12 dB per octave ($S \propto f^3$) so any harmonics present in the drive signal will be amplified by the transducer response and produce distorted signals. The programmable signal source used at NPL has a total harmonic distortion of less than 0.1% (equivalent to -60 dB) over the frequency range 50 Hz to 1 MHz.

It should be remembered that the expression for the reciprocity parameter given in Equation (4) is valid only for a spherically-spread field and so any deviation from spherical-wave propagation will lead to errors in the calibration. These errors are discussed in more detail in Section 4.2.5.

4.1.2 Measurement of distance

The calibration method requires the determination of the separation distance between the pairs of transducers. This distance must be measured between the acoustic centres of the devices. The acoustic centre is defined as the point from which the spherically divergent sound waves emitted by that transducer appear to diverge when viewed in the far-field. Fortunately, most transducers exhibit some symmetry which enables the acoustic centre to be estimated with reasonable certainty. If there is doubt about the exact position of the acoustic centre of a transducer, a plot of the reciprocal value of the hydrophone voltage versus separation distance should be made which should be a straight line through the origin if the acoustic centre has been estimated accurately. It should be remembered that it is possible for the acoustic centre of some devices (for example, some piston devices) to change with frequency.

The most straightforward method of measuring the distance is to use a metre rule or tape measure. At NPL, the positioning system gives a read-out which allows the separation of the carriages to be worked out automatically. Each transducer is mounted so that its axis is
coincident with the axis of the carriage of the positioning system. The separation is usually taken as the distance between the centres of devices (if spherical or cylindrical) or from the front face (if planar or piston). If there is any offset in the mounting arrangement, this must be included in the calculation of separation distance.

Although the positioning system accuracy is \( \pm 0.5 \) mm for the carriages alone, the accuracy with which the transducers can be positioned depends upon the straightness of the mounting poles and the reliability with which the transducers can be attached to these poles. To check the accuracy of the positioning system, two fine drill bits were inserted into the chucks on the ends of the z-axis carriage rails. A metal bar was then placed under these and indent made in the bar by rotating the two drill bits. The distance between the indentations was then measured using a finely graduated metal rule. Two hydrophones were then mounted in their associated poles and placed in the holders as for a standard calibration. The distance between the centres of the hydrophones was then measured and the devices were rotated to check for any eccentricity. The results of this exercise showed that the worst case difference between the separation of the acoustic centres as compared to the positioning system reading was 4 mm. An uncertainty value of 0.5\% has been assigned to the separation distance measurement (5 mm at 1 metre).

An alternative method of measuring the separation distance is to use the "time-of-flight" method, i.e. to measure the propagation delay before the arrival of an acoustic pulse using an oscilloscope. The distance can then be calculated if the speed of sound is known. It must be remembered that the sound speed in water varies with temperature and the error involved in taking the speed of sound to be 1480 ms\(^{-1}\) is as high as 1\% at 15°C. Therefore the value for the speed of sound at the appropriate water temperature must be obtained from tables or formulae in the literature\(^{16}\). An oscilloscope with a calibrated timebase must also be used.

In practice, the acoustic method is prone to error and should be undertaken with care. The start of the received pulse is often embedded in noise, making it hard to determine its exact position. An alternative is to measure the delay between, say, the first positive peaks of the two signals. However, when operating near the transducer resonance, the phase response of the devices can cause errors in the measurements. Also, an acoustic time-of-flight measurement may not provide a measurement of the desired distance between the acoustic centres of the devices. This is because the transducers are not point sources or receivers but have a finite size. Taking the example of a spherical hydrophone with the acoustic centre at the centre of the spherical element, the hydrophone element will start to respond to the incident acoustic wave when the wave first impinges on the outer edge of the transducer. Thus, an acoustic measurement of the separation distance between two spherical transducers (or by the same reasoning, cylindrical ones) will be an underestimate of the distance required.

4.1.3 Measurement of electrical transfer impedance

In Equation (9), the hydrophone sensitivity is expressed in terms of the transfer impedances \( Z_{ph} \), \( Z_{pr} \) and \( Z_{pp} \), each transfer impedance being the ratio of a voltage generated in a receiving transducer to a current driving a transmitting transducer. There is more than one way of
measuring the current driving the transmitting device. A common method is to measure the voltage drop across a standard precision resistor. However, a disadvantage of this method is that electrical problems can be experienced during measurements due to there being several different electrical paths to ground. The method adopted by NPL is to use a calibrated current probe which has the advantage that it does not make electrical contact with the wire conducting the current to the transmitting device and so is less prone to the above problems. Both the voltage generated by the current probe and the output voltage of the hydrophone pass through the same amplifier and filter before being sampled by the 12 bit digitising oscilloscope. The magnitudes of the two signals are made nearly equal (to within 5%) by attenuating the current probe voltage using an attenuator. The transfer impedance is then given by:

\[ Z = \frac{V_h}{V_i} G \left( \frac{A}{20} \right) \]  

(11)

where \( V_h \) is the measured voltage from the receiving transducer, \( V_i \) is the measured voltage corresponding to the attenuated drive current \((V_i = V_0)\), \( G \) is the current probe gain (or sensitivity) and \( A \) is the attenuator setting in dB. Since this equation contains the ratio of two voltages measured with the same preamplifier, filter and digitiser, absolute calibration of this equipment is not necessary. Since the two voltages are made nearly equal by the attenuator, any uncertainty contribution due to non-linearity of this equipment is also negligible. From the above equation, it is evident that the uncertainty in the electrical measurements for each transfer impedance results from the combined uncertainties of the attenuator and current probe, both of which must be calibrated.

(a) Current probe calibration

The current probe works by forming the secondary winding of a 1:25 transformer. A current of one twenty-fifth of that in the drive circuit is induced in the coil of the probe. An internal 50 ohm load in parallel with a 50 ohm external load causes the induced current to flow through 25 ohms, giving a unity gain. Of course, the gain is not exactly unity, and the probe must be calibrated over the frequency range and amplitude range of interest. One of the difficulties of calibrating such a current probe is that the sensitivity depends on the probe geometry, i.e. the exact angle and position of the wire as it passes through the coil. For this reason, the NPL probes have been fixed in position in sealed metal boxes with external BNC connectors so that any movement or vibration cannot change the sensitivity. NPL has two current probes each of which is calibrated as a sealed unit along with their own 50 ohm load. The calibrations are performed annually by a NAMAS accredited laboratory with an accuracy of ±0.6% in the frequency range 1 to 100 kHz, ±1.0% in the range 100-200 kHz and ±2.0% in the range 200-500 kHz. The calibration values for the current probe are entered into the calibration software and corrections are made after every measurement of current.

(b) Attenuator calibration

The programmable attenuator and its associated switch are controlled by computer, allowing
considerable automation of the measurement procedure. The attenuator is calibrated annually by a NAMAS accredited laboratory. The uncertainty in the calibration varies depending on the attenuator setting, typically being 0.005 dB per decade setting. For the attenuator settings used during hydrophone calibrations, the uncertainty due to the attenuator calibration is typically 0.01 dB. As with the current probe, the calibration values for the attenuator are stored in the calibration software and corrections are made after every voltage measurement.

c) Measurement of the voltages

When measuring the voltage signals, selective windowing must be used so that the measurement is made only on the steady-state portion of the tone-burst. The measurement window is carefully set to measure the same cycles of the tone-burst for both the current and voltage signals. The measurement can be performed on the digitiser using its internal algorithms, or by transferring the waveform to the computer and then using software algorithms to perform similar functions. To evaluate any discrepancies between the two methods a comparison was carried out. Measurements were first performed on a captured waveform using the digitising oscilloscope. The same waveform was then transferred to the computer and measurements made in software. Measurements made on the digitiser included using the peak function, RMS function and onboard FFT. The software algorithms used calculated values for the peak, RMS and discrete Fourier transform. The maximum difference observed between all of these measurements was 0.25% and the mean values obtained for each method agreed to within ±0.1%.

The method used at NPL to measure the voltage and current signals is to perform a periodic RMS measurement using the internal measurement functions of the digitiser. Errors may occur due to the fact that an integer number of cycles may not fit exactly into an integer number of digitisation sample points. This error is very small when the window is large and is randomised to a degree in the measurements. Since the same method is used for the measurement of both $V_h$ and $V_v$, there will be some cancellation of errors derived from the digitisation and measurement of the signals.

Hydrophones are relatively high impedance devices and so it is generally necessary to correct for the loading of the hydrophone by the amplifier. In the NPL system, this correction is made every time the hydrophone voltage is measured using the equation in Appendix 2.

4.2 ADDITIONAL SOURCES OF SYSTEMATIC UNCERTAINTY

4.2.1 Directionality of hydrophones

All transducers exhibit some directionality in their response, that is the receiving sensitivity (and by the reciprocity theorem, the transmitting sensitivity) varies with the angle of incidence of the acoustic wave. The directionality is more pronounced at high frequencies when operating close to and above the resonance frequency of the device. It is clear that errors can occur during calibration if the sensitivity depends on direction, and for this reason, accurate plots of the transducer's directional response are made before a calibration is attempted.
Figure 4  Directional response plots for a B&K 8100 at 200 kHz in the a) x-y and b) x-z planes. The scaling on the plots is linear.

Figure 5  Directional response plots for a Reson System TC4034 at 400 kHz in the a) x-y plane and b) x-z plane. The scaling on the plots in linear.

When specifying the directional response of transducers, it is important to define the coordinate axes of the devices. NPL has adopted the convention of using three orthogonal axes, x, y, and z, with the origin coincident with the acoustic centre of the transducer. The z direction is chosen to coincide with the transducer axis which, for an omnidirectional hydrophone, will coincide with the direction defined by the hydrophone body and cable. The x and y directions define a plane orthogonal to the transducer axis containing the acoustic
centre. The x direction is usually taken as the direction of the reference mark on the hydrophone body. It is then possible to perform directional response measurements in three planes, two "vertical" planes (x-z and y-z) and one "horizontal" (x-y).

![Diagram of directional response plots for a B&K 8103 at 500 kHz in the a) x-y and b) x-z planes. The scaling on the plots is linear.](image)

**Figure 6** Directional response plots for a B&K 8103 at 500 kHz in the a) x-y and b) x-z planes. The scaling on the plots is linear.

During the 1988 UK national intercomparison of calibration methods\textsuperscript{17}, the directionality of the hydrophone response was identified as one of the main sources of uncertainty at high frequencies. Although this is undoubtedly the case, the errors so caused are not always easy to quantify. If we make the assumption that the response of a cylindrical hydrophone (eg the Brüel & Kjær 8104) can be approximated by that of a continuous line source, the directional response function $D(\theta)$, as a function of angle $\theta$, is given by\textsuperscript{1,18}

$$D(\theta) = \frac{\sin\left(\left(\frac{\pi l}{\lambda}\right)\sin\theta\right)}{\left(\frac{\pi l}{\lambda}\right)\sin\theta}$$

(12)

where $\lambda$ is the acoustic wavelength and $l$ is the length of the piezoelectric cylindrical element (equal to about 20 mm for the B&K 8104). Equation (12) dictates that the vertical orientation must be within $\pm 3^\circ$ to give a response not more than 0.3 dB down from the maximum at 200 kHz. For the Brüel & Kjær 8103, which has an 8 mm long element, the orientation must be within $\pm 8^\circ$ to give the same drop in sensitivity. As can be seen from Equation (12), the directional response depends on the ratio $l/\lambda$. Therefore, the smaller the element size measured in wavelengths, the less sensitive it is to errors in alignment. Figures 4, 5 and 6 show measured directional responses in both the x-y plane (horizontal) and in the x-z plane (vertical) for a B&K 8100 at 200 kHz, a Reson System TC 4034 at 400 kHz and a B&K 8103 at 500 kHz respectively.
A directional transducer (e.g., a piston) can be aligned by looking for the maximum acoustic signal which should occur on axis. It is also possible to align symmetrical transducers in this way using the positioning system on the NPL tank. However, to do this is time consuming, may not be feasible for industrial users not possessing the means to accurately manipulate the transducer, and anyway is only valid for one frequency since at other frequencies the maximum signal may be obtained at some other angle. It is more common to align a symmetrical transducer using a mark on the transducer body. At NPL, alignment marks on the mounting poles allow the hydrophones to be orientated by sight to better than ±3° in the x-y plane. In the x-z plane, the positioning system and mounting poles fix the orientation to within ±2°. To determine the uncertainty this causes in the calibration, reference must be made to the directional response of the particular transducer in use at that frequency. The uncertainty is therefore device dependent, but typical values can be estimated for a given hydrophone type from a knowledge of the typical directional responses and from the kind of analysis shown in Equation (12). For example, for a B&K 8104 hydrophone, the uncertainty in the measured voltage will be typically ±0.2% at 50 kHz rising to ±2.5% at 200 kHz. For a B&K 8103, the uncertainty will be typically ±0.1% at 100 kHz and ±2.5% at 500 kHz. For smaller spherical hydrophones such as the Reson System TC4034, the uncertainties at high frequencies are reduced due to the more omnidirectional response. The uncertainty arising from misalignment will be randomised somewhat with the NPL procedure since the transducers are remounted and realigned between repeats.

4.2.2 Reflections

One of the most important potential sources of error at low frequencies is the problem of acoustic reflections from the boundaries of the water tank.

A practical way of identifying multipath reflections is to examine the stability of the tone-burst to changes in frequency. Reflections cause rapid changes in peak amplitude (due to constructive and destructive interference) which are absent from the direct path signal. A good general check for reflections is to repeat the measurements at different separations or make them at closely spaced frequencies where interference is manifested as a periodic ripple superimposed on the steady trend of measurements. This type of check enables any residual error due to reflections to be quantified. To randomise any residual effects of reflections in calibrations at NPL, each repeated measurement is performed at a different separation distance.

4.2.3 Mounting

The manner in which a transducer is mounted can affect the measured sensitivity. This can arise for a variety of reasons such as acoustic reverberation and reflection from structures close to the sensing element, and even vibration transmitted into the body of the transducer through the mount. These effects can be minimised by making the mounting poles out of a material which has a close match in acoustic impedance to that of water, and by cushioning the transducer against the mount with absorber or acoustic release material. However, there is a trade-off between good acoustic impedance matching and the rigidity required for accurate
positioning. At NPL, specially straightened free-flooding aluminium poles are used to connect the transducers to the positioning system. The transducers can be connected to the poles with a number of different types of mounts made of aluminium or nylon-66. Symmetrical transducers are connected such that their axes coincide with the z-axis of the carriage of the positioning system.

Rather than attempt to estimate the uncertainty caused in the calibration by the use of a particular mount, it is preferable that the transducer is calibrated in the mount in which it will be used in the field to make measurements. This is equivalent to saying that the calibration is characteristic of the mount as well as the transducer. Adopting this policy, a contribution to the uncertainties for transducer mounting is not required.

![Hydrophone voltage vs time](image)

**Figure 7** A typical unfiltered hydrophone waveform for a measurement made at 10 kHz showing the effect of transients and reflections at low frequencies.

4.2.4 Transients

The first few cycles of the received pulse can be very irregular both in amplitude and frequency. The irregularity occurs because the projector-receiver combination consists essentially of series resonant circuits being driven off resonance. As stated in Appendix 1, it takes approximately $Q$ cycles of the resonance frequency for the circuit to settle to its steady-state response where measurements can be taken. For this reason, transducers with a high Q-factor are difficult to measure in such a small tank as the one at NPL because a steady-state may not be reached before reflections interfere with the signal.

The problem caused by transients is worse at lower frequencies since there are fewer cycles in a tone-burst of a given time duration. When the first few cycles are distorted it is risky to rely on the remaining few for the measurement. Figure 7 shows a typical waveform obtained from the receiving transducer for a measurement made at 10 kHz. The transmitting transducer
has initially attempted to vibrate at its resonance frequency before settling to the driving frequency. The effect of interference by reflected signals is evident after only a few cycles of the signal. Under these conditions, the location and measurement of the steady-state part of the signal become less reliable. Also, small hydrophones can be very inefficient projectors at low frequencies. The consequent poor signal-to-noise ratio can be overcome by using larger projectors which have higher transmitting sensitivities at lower frequencies. However, since the resonance frequency is also lower, the transient response at the beginning of the tone-burst lasts for a longer time which can make the use of more powerful projectors disadvantageous. Figure 8 is a hydrophone waveform for the same transducers but at 50 kHz, showing the effect of reflections and transients.

![Hydrophone waveform](image)

**Figure 8** A typical unfiltered hydrophone waveform for a measurement made at 50 kHz showing the effect of transients and reflections.

An uncertainty must be introduced to account for this potential inaccuracy, but the uncertainty may be difficult to confirm. One way of assessing the effect transients may have on the measurement is to scan a narrow measurement window across the entire waveform. If this window is only one wavelength wide then it should be possible to find a position where the voltage does not alter with window position, ie a flat portion. Figure 9 shows the measured voltage as a function of the start position of the window. The frequency is 10 kHz and the width of the window is 0.1 ms (one period). It can be seen that the flattest portion is at around 800 samples (0.3 ms). Although far from completely adequate, this technique does give some idea of the variation in voltage that can occur when the window is not set correctly and hence gives an estimate of the uncertainties involved. At 50 kHz (see Figure 10) the flat portion is much more pronounced.
Figure 9  Variation of measured voltage with window position on a 10 kHz waveform.

Figure 10  Variation of measured voltage with window position on a 50 kHz waveform.

The uncertainty resulting from the transients on tone-bursts is inevitably device dependent and
generally, the higher the Q-factor, the larger the uncertainty. Typically for a B&K 8104 hydrophone calibrated in the NPL tank, the uncertainty in the voltage measurement is ±3% at 10 kHz reducing to ±1% at 50 kHz. To perform a definitive validation of the calibration, comparisons must be made with measurements made in a much larger volume of water using longer tone-bursts where steady-state can be reached.

![Figure 11](image.png)  
Figure 11 Percentage deviation from average transfer impedance versus frequency.

4.2.5 Non-reciprocal behaviour

The calibration method depends upon at least one of the electroacoustic transducers being a reciprocal device. To be reciprocal, a transducer must be linear, passive and reversible. Unfortunately, not all linear, passive and reversible transducers are reciprocal, for example a transducer that contains both piezoelectric and magnetostrictive elements\(^1\). In fact, there is no definitive method for determining whether a transducer is reciprocal. However, it is possible to ascertain that a pair of transducers is extremely likely to be reciprocal by performing the reciprocity check described in Section 2. This involves comparing the magnitudes of the two transfer impedances when the rôles of projector and receiver are interchanged. If this check shows that the pair are not reciprocal then each transducer must be paired up with another device and the check repeated. By a method of elimination it will be possible to find a pair of devices that are reciprocal. It is important for the reciprocity check that the two reciprocal transducers are of different types. If the two devices are of identical construction, the transfer impedances could coincidentally be equal, as would be the case if both transducers had identical nonlinear characteristics.

IEC 565 permits a tolerance of ±10% in the reciprocity check\(^2\), but for accurate measurements
a higher tolerance of ±1% should be aimed for. This is generally achievable at normal signal levels, but the transfer impedances \( Z_{TP} \) and \( Z_{PT} \) can differ by more than this, especially at frequencies close to and above resonance, and for high drive voltages. Figure 11 shows the deviation from the mean value of transfer impedance for a pair of transducers over a range of frequencies. As can be seen, the variation is less than ±1% in the range 10-250 kHz.

In the procedure adopted at NPL, the uncertainty contribution for non-reciprocal behaviour is assessed automatically by software at each frequency for every calibration by examining the agreement obtained with the reciprocity check, typically ±1% for transducers used below resonance. When a transducer is used at or above its resonance frequency, it is more difficult to obtain good reciprocal behaviour (i.e., for \( Z_{TP} \) and \( Z_{PT} \) to agree to within ±1%), and the uncertainty contribution must be increased accordingly.

![Graph](image_url)

**Figure 12** Deviation from mean of product of voltage and distance (+) and transfer impedance and distance (×) versus distance at a frequency of 50 kHz.

4.2.6 Non-spherical acoustic field

In the description of the method given in Section 2, it was assumed that the acoustic field generated by the transmitting transducer propagates as a spherically diverging wave (i.e., has a \( 1/r \) acoustic pressure dependence with distance, \( r \)). This assumption is in fact explicit within the definition of the transmitting response, \( S \). Similarly, the expression for the reciprocity parameter given in Section 2.2 is appropriate only for a spherical-wave field. That is not to say that reciprocity-based calibration methods are not possible for other field-geometries (e.g., cylindrical-wave, plane wave, etc), merely that the expression for the reciprocity parameter
is then different.

![Graph showing deviation from mean](image)

**Figure 13** As for Figure 12 but with the projector rotated by 90°.

Since the calibration method depends upon the assumption of spherical-wave propagation, it is important to assess the accuracy of this assumption. The assumption is not unreasonable for small hydrophones since at a large enough distance they should appear like simple sources, especially when used below resonance where the transducer dimensions are less than the acoustic wavelength. The analysis in Appendix 1 shows the minimum separation distance required between source and receiver for the assumption of a spherically spreading field to be valid to within ±2%. As a general rule, the distance, $d_{\text{min}}$, is given by $(L_1^2 + L_2^2)/\lambda$ where $L_1$ and $L_2$ are the maximum dimensions of the transducers and $\lambda$ is the acoustic wavelength².

To assess whether the acoustic field is truly spherically spreading for distances greater than defined above, measurements were made of the acoustic signal detected by a hydrophone as the separation distance from the transmitting transducer was increased. A plot was then made of the product of the voltage (or transfer impedance magnitude) and the separation distance versus distance. The product should theoretically be a constant giving a horizontal straight line. Figure 12 shows data plotted in this format at a frequency of 50 kHz for a transducer pair consisting of a B&K 8104 (source) and a B&K 8105 (receiver). Clearly the plot is not flat, showing variation of up to ±2%. The transfer impedance magnitude is plotted along with voltage, showing good agreement. Since for a transfer impedance measurement the voltage and current are measured after equalization using the calibrated attenuator, the effects of any nonlinearities in the measuring equipment are eliminated.
There are several potential causes of trends in the results shown in Figure 12 which need to be eliminated. Apparent non \( 1/r \) dependence can be caused by the two transducers being at different depths and by peculiarities of the mounting configuration used. Also, if the separation distance is incorrectly measured (perhaps due to an incorrect assumption about the acoustic centre of a transducer), this can lead to a steady trend in the results. This can be simulated by making measurements with a deliberate offset introduced into the mounting pole of the transmitting device. All of the above possibilities were eliminated by independent testing of the positioning system accuracy and by using several different mounting arrangements. Figure 13 shows the same type of plot as Figure 12 but this time the measurements were made with the transmitting transducer rotated through 90° keeping all other conditions the same. The trend of the results has changed considerably from that shown in Figure 12, evidence that the effect is acoustic in nature due to the finite size of source and receiver.

![Graph showing deviation from mean of the product of voltage and distance versus distance.](image)

**Figure 14** Deviation from mean of the product of voltage and distance versus distance. Measurements performed in large tank at 200 kHz.

Similar trends have been observed with other transducers, particularly when used at or above resonance, although the exact form of the deviation is dependent on the particular transducers used. In general, the results show a greater agreement with spherical spreading as the distance increases, as would be expected theoretically. This is further illustrated by Figure 14 which shows measurements using the same hydrophones but performed at 200 kHz and in a larger tank. The above evidence leads to the conclusion that the larger the separation distance, the better the agreement with an ideal spherical field and therefore the more accurate the calibration. However, when performing measurements in small tanks using pulse techniques, there are limitations on the maximum separation distance that can be used if other sources of
error are not to become large (see Appendix 1). Consequently, a compromise distance must be used. From the results of the voltage-distance measurements made using a number of transducer combinations, it is possible to say that for separation distances greater than 0.75 m in the NPL tank, the typical error due to this source is not greater than ±1% for transducers used below resonance. This contribution must be increased to ±2% or more if shorter separation distances are used or if hydrophones are used at high frequencies where the device dimensions are equivalent to several wavelengths and the behaviour is more complex. The contribution will affect each measurement of transfer impedance since the transfer impedance depends on the definition of the transmitting response given in Equation (2).

For calibrations in the NPL tank, four different separation distances are generally chosen in the range 0.7-1.2 m. A check on the transfer impedances can then be made during each calibration to determine the deviation from ideal spherical spreading and an estimate made of the uncertainty contribution. To determine how the measured sensitivity varies with separation distance, calibrations were performed at a number of distances between 0.4 and 1.2 m without remounting, realignment or rewetting. Figures 15 to 17 show the results for a B&K 8103, an ITC 1089C and a Reson TC 4034 at 250, 400 and 500 kHz respectively. The results show that the sensitivities typically vary by only ±0.1 dB at distances of 0.7 m or greater.

Figure 15  Variation of measured sensitivity with distance at 250 kHz for a B&K 8103.
Figure 16  Variation of measured sensitivity with distance for an ITC1089C at 400 kHz.

Figure 17  Variation of measured sensitivity with distance for a Reson TC 4034 at 500 kHz.
4.2.7 Hydrophone non-linearity

The transducers used in underwater acoustics are not linear under all conditions.\textsuperscript{21,22} This is important since any non-linearity breaks one of the conditions for the reciprocity theorem to hold. Hydrophones can exhibit a non-linear receiving response when exposed to high acoustic pressures and a non-linear transmitting response when driven by large currents. Fortunately, the acoustic pressures generated during calibration are relatively low (typically 100 Pa at the hydrophone) and should not cause non-linear response within the receiving device. Similarly, the drive currents used are typically a few hundred milliamps, which should be well within the linear range for most transducers.

![Graph showing deviation from mean vs. drive current](image)

**Figure 18** Variation of transfer impedance with projector drive level at 50 kHz.

To test for any hydrophone non-linearity, measurements of transfer impedance were made for transducer pairs using different projector drive currents. The results shown in Figure 18 (typical for a pair of reference hydrophones) demonstrate that any effect on the measured transfer impedance is generally less than $\pm 0.5\%$. The slightly increased variation seen at very low drive currents is due to poor signal-to-noise ratio.

4.2.8 Wetting

Inaccuracies can be caused during measurements by inadequate wetting of the hydrophone surfaces\textsuperscript{33}. A thin layer of air, sometimes observable as a silvery sheen, can cling to the hydrophone surface and cause errors in the measurements, frequently causing the measured hydrophone voltages to drift with time.
The problem of air bubbles attaching to the surface of hydrophones is increased if there is a temperature difference between the hydrophones and the water. The problem is solved by cleaning the hydrophones thoroughly before use using a soft brush and a weak solution of detergent and then soaking the hydrophones in the water tank for a period of time. If the temperature difference is large then it may be necessary to soak the devices for several hours.

![Graph showing the effect of soaking time on hydrophone voltage](image)

**Figure 19** Effect of soaking time on hydrophone voltage at 10 (×), 50 (+), 100 (○) and 200 (Δ) kHz when both hydrophone and projector are initially dry.

Figure 19 shows the variation in hydrophone voltage with time when two dry devices have been placed in the water tank. The two devices had been previously cleaned but then allowed to dry in air for over 24 hours. The variation shown indicates that the devices in this instance needed to soak for nearly two hours before a steady state was reached. It should be noted that only the hydrophone voltage was measured and so there may have been some slight drift due to equipment heating up. There appears to be very little frequency dependence although it is interesting to note that the 100 kHz curve drifts downwards whereas the others drift in the opposite direction.

In general, it is very difficult to assign an uncertainty for this effect since some transducers will require far longer than others to become fully wetted. It is better to wet the hydrophone during calibrations in the same way as is used during measurements in the field. The wetting time should then be stated on the calibration documentation. At NPL, the general policy adopted is to wet the hydrophones with a weak detergent solution when cleaning them and leave them overnight to soak in the water tank. Before each repeated measurement, the hydrophones are remounted, rewetted and resoaked for 30 minutes.
4.2.9 Cavitation

Generally, the negative acoustic pressures encountered in the underwater acoustical calibrations reported here are not large enough to cause dissolved gas to be brought out of solution and form bubbles by the action of cavitation. However, bubbles tend to form naturally in the saturated water, frequently clinging to a surface rather than drifting freely. A bubble driven at a frequency close to its resonance oscillates vigorously, re-radiating uniformly in all directions. If the bubble is assumed to be spherical and of radius $a$, it has an angular frequency of resonance, $\omega_0$, such that

$$\omega_0 = \frac{1}{a} \left( \frac{3 \gamma P_0}{\rho_0} \right)^{1/2}$$ (13)

where $\gamma$ is the ratio of specific heats for an ideal gas, $P_0$ is the hydrostatic pressure and $\rho_0$ is the density of water. It is clear from Equation (13) that the resonance frequency is proportional to the square root of the static pressure and inversely proportional to the static radius. Therefore, a bubble near the surface (i.e. at one atmosphere of pressure) and of 1 mm radius will oscillate at 6.7 kHz. The presence of bubbles (or the onset of cavitation) is indicated by certain characteristics of the received signal such as a ragged appearance and the presence of the second harmonic of the excitation frequency.

Although potentially a significant source of error, the presence of resonant gas bubbles is not a major problem for calibrations of reference hydrophones in a small tank if care is taken. The bubbles can largely be eliminated by a combination of soaking and use of wetting agents. However, visual checks for bubbles are still necessary, any bubbles present being easily removed with a soft brush. The maintenance of a stable water temperature also helps to reduce the number of bubbles formed in the water.

4.2.10 Temperature

The sensitivity of piezoelectric hydrophones is dependent on the ambient temperature. The dependency arises because the properties of the sensor and encapsulant materials used in hydrophone elements vary with temperature. Consequently, the variation for any hydrophone will depend on the type of material used in construction. The variation is not necessarily linear over the range -15 to +60°C, although the variation is fairly smooth and can be approximated by a linear variation for a small temperature change. Typically, at about 20°C the sensitivity might change by 0.04 dB for every 1°C change in temperature for a Brüel and Kjær hydrophone.

There are two approaches to addressing this problem. The first approach is to correct the sensitivity after calibration to a standard temperature, e.g. 20°C. This allows a direct comparison between calibrations performed at different temperatures. However, the problem with correcting the measurements is that the coefficients of variation of sensitivity with temperature are not accurately known. It is therefore sensible to assign an uncertainty to the
correction which is equal to the correction itself (so a 1% correction would contribute 1% to the uncertainties). For example if the water temperature is 15°C, the correction is 0.2 dB, and 0.2 dB is added to the calibration uncertainty.

The second approach is to state the uncorrected sensitivity along with the temperature at which measurements were made, and this is the approach adopted by NPL. This has the advantage that no uncertainty contribution need be included since the calibration is valid strictly only for the temperature at which the measurements were performed. This must be remembered when comparing calibrations; a calibration using an outdoor facility in winter may have been performed at only 4°C whereas a calibration in an indoor laboratory tank may have been performed at a temperature some 20 degrees higher. Fortunately, the temperature of the water in the NPL tank stabilises at around 20±3°C, so the seasonal variation is relatively small, and the actual temperature is recorded using a digital thermometer calibrated by a NAMAS accredited laboratory to an accuracy of ±0.2°C.

In general, more information is needed regarding the variation of hydrophone sensitivity with temperature, since transducer manufacturers appear unable to make definitive statements. However, to make measurements at many temperatures is time-consuming and requires low calibration uncertainties in order that the variation can be accurately determined.

4.2.11 Electrical grounding problems

It is well known that electrical problems can be experienced during measurements due to there being several different electrical paths to ground. Most electronic instruments have a low or ground input terminal which is connected to earth potential via the earth connection of the mains plug. However, the hydrophone also makes contact, either resistively or capacitively, with the surrounding water which may have a different earth potential. This closed ground loop can pick up electromagnetic interference and, more importantly, ground currents may flow between the different earth connections. This problem can be particularly difficult to eliminate at high frequencies.

With the exception of the digitising oscilloscope and power amplifiers, the equipment used at NPL has its signal ground connector isolated from the mains earth. This allows a single earthing point to be used at the preamplifier input. In addition, the water in the tank is connected to a high quality earth with copper earthing braid. Furthermore, a differential input on the digitising oscilloscope is used to discriminate against any ground currents that may be present in the measuring circuit. Because the current probe is used for measuring the drive current, no direct contact is made between the ground of the power amplifier circuit and the signal ground on the measuring side of the circuit.

At high frequencies (above 200 kHz), electromagnetic pickup of the drive signal is observable on the oscilloscope when viewing the hydrophone signal. In general, this is not a problem since it occurs at a different position in time and is easily gated out of the measurement. However, at very low frequencies, the separation distance must be large enough to ensure that the end of the current waveform does not overlap the start of the hydrophone signal on the
oscilloscope (ie the propagation delay must be longer than the duration of the tone-burst). Any overlap allows the possibility of electrical interference between the signals.

Some hydrophones have the signal ground connected to the cable shield, whereas others have the cable shield floating with respect to ground. In each case, the electrical conditions are slightly different since the cable shield may make contact with the water, perhaps via a metal case. Where it is possible to operate a transducer in either of these two configurations, the chosen configuration should be stated when the calibration is reported.

4.2.12 Noise.

If a significant amount of unwanted noise is present with the signals, this can lead to errors in the measurements. The noise can have several sources and can be classified in a number of ways: eg thermal noise, acoustical noise and electrical noise. Thermal noise in the water is really only significant at the highest frequencies considered here and since the acoustic pressures generated at high frequencies are relatively large, it does not pose a serious problem. Acoustic noise generated in the environment surrounding the tank and transmitted into the water can be a problem if the laboratory is noisy. Equipment, air-conditioning fans, road traffic and overflying aircraft all generate noise which can be detected in the NPL tank. Fortunately, this noise occurs at low frequencies, often in the audio range and sometimes at subsonic frequencies. Little acoustic noise can be detected above 10 kHz.

The electrical noise present with the signal itself originates from a number of sources. There is the self-noise of the transducers, amplifiers and equipment; the pick-up of the electrical supply (mains hum); and also, there is electromagnetic pick-up from other equipment and even from local radio stations (at high frequencies).

During calibrations of reference hydrophones, noise is usually only a problem at low frequencies where small hydrophones are inefficient projectors and so generate low acoustic pressures. To eliminate the problem of noise at NPL, various steps are taken. Shielded cables are used for all electrical connections and electrical filtering is used on the measured signals. Generally, a bandwidth of one octave is used since the use of too narrow a bandwidth can increase the Q of the measuring system and have an adverse effect on the transient on the tone-burst, reducing the number of cycles of steady state signal available for measurement.

When calibrating a hydrophone of low sensitivity, noise can be a significant problem and the contribution to the uncertainties due to this effect must be assessed for each case individually.

4.2.13 Sound absorption by the water medium

Another cause of deviation from the ideal spherical spreading loss is the absorption of sound by the water medium. This depends on the square of the signal frequency and is only significant at the very highest frequencies considered here. For example, at 500 kHz the attenuation of the acoustic signal due to absorption in fresh water is 0.00055 dB cm\(^{-1}\), giving a reduction in the measured signal of 0.6% for a separation distance of 1 m. A correction can
be applied for absorption in the calibration software. An uncertainty contribution equal to the correction is introduced but the value is fairly small.

4.2.14 Loading corrections

The sensitivity of a hydrophone is most often specified as the end-of-cable open-circuit sensitivity. This is the sensitivity of the hydrophone at the end of its cable when not connected to an electrical load. When a specific electrical load (such as that presented by the input of an oscilloscope or an amplifier) is connected to the output of the hydrophone, a correction must be applied to take account of the electrical loading on the hydrophone. Since the impedance of a transducer is highly reactive (it is mainly capacitance), any amplifier connected to the transducer must have a very high input impedance (low capacitance) if there is not to be significant loading and consequent loss of sensitivity. The correction to obtain the end-of-cable loaded sensitivity of the hydrophone can be determined using the method described in Appendix 2.

At NPL, the hydrophone voltage is measured after amplification by a preamplifier. The measured voltage is then corrected to the equivalent open-circuit voltage by the calibration software using the method of Appendix 2. The correction is different for each hydrophone used and varies with frequency. To determine the correction requires that the impedance of the hydrophone and amplifier are known. The input impedance of the amplifier is given by the manufacturer’s specification, but the impedance of the hydrophone must be measured. At NPL, the hydrophone impedances are measured at the frequencies required for calibration using an impedance analyser under software control, and then the values are stored for automatic correction during calibration. In general, the corrections applied are less than 1% of the measured voltage.

The accuracy of the correction depends on the accuracy of the impedance measurements. These in turn depend on the accuracy of the impedance analyser (the accuracy varies depending on frequency and amplitude ranges), and the influence of the lack of free-field conditions on the impedance measurements (the impedance analyser drives the transducers in a continuous wave-mode and reflections from tank boundaries may influence the results). This latter effect is important when operating near to the resonance of a transducer where the radiation impedance contributes a significant part of the measured impedance. For this reason, the impedance measurements are performed at several positions in the water tank to determine the influence of the tank boundaries. If necessary the impedance can be determined indirectly from measurements of the voltage and current under pulsed conditions. A typical uncertainty of between ±0.5 and 1% is included in the overall uncertainty budget, though this may have to be increased when operating close to a transducer resonance or if a large correction is applied for a very low capacitance hydrophone.

Often during calibration, extra cable is connected to the hydrophone in addition to the integrated cable so that a connection can be made to equipment several metres away. The cable will have an impedance which will load the hydrophone, and a correction must be made using the method described above.
If the hydrophone contains an integrated preamplifier, any loading corrections should be negligible since the preamplifier will act as an impedance converter and present a low output impedance to any connecting cable and equipment. However, it is sometimes desirable to know the sensitivity of the hydrophone element alone in the absence of the preamplifier. In this case, an insert voltage calibration\(^4\) can be performed if the amplifier has the necessary connections (this is often referred to as a "voltage coupling loss" measurement). With modern electronic circuitry, the preamplifier is usually as stable as the hydrophone element itself and so the end-of-cable sensitivity of the hydrophone/preamplifier combination is the more commonly quoted sensitivity.

### 4.3 RANDOM UNCERTAINTY

As has been stated at the beginning of Section 4, a measure of random uncertainty may be made by performing repeated measurements of a particular quantity and observing the variation.

For a calibration of a hydrophone, four independent determinations of the sensitivity are performed from four independent sets of measurements, each individual set involving the measurement of the transfer impedances between the pairs of transducers. For truly independent sets of measurements, the transducers are removed from the water between sets, remounted, returned to the water and realigned. Each set of measurements is also performed at a different separation distance.

The mean of the four values for the sensitivity is calculated along with the standard deviation. The random uncertainty is then calculated and expressed for an estimated confidence probability of not less than 95%.

The size of the random uncertainty depends upon the transducers under test. In general, for a given transducer it will also vary with frequency. For reference hydrophones, the random uncertainty (expressed at a 95% confidence level) typically varies between 1 and 3%.
5 UNCERTAINTY BUDGET

5.1 ASSESSING THE UNCERTAINTIES

Table 1 gives the best estimates of each uncertainty contribution at a number of frequencies and for a number of typical reference hydrophones. In the table, a cross-reference is given to the relevant sub-section of Section 4 of this report where more information on the derivation of the uncertainty can be found. It should be understood that the values in the table are for well-behaved well-characterised devices and it is difficult to make generalisations regarding other hydrophones. Similarly, the values in the table are for calibrations performed in the NPL tank. For calibrations using other facilities, a similar analysis may be applied but the value of the uncertainties may be quite different.

As stated in the analysis given in Section 4, many of the uncertainty contributions are device dependent. Consequently, the uncertainty contributions must be assessed for each transducer from observations taken during the calibration. For example, if a transducer is of low sensitivity, the contribution due to noise may have to be increased appropriately. Similarly, if the response of a device is strongly directional and is aligned using a reference mark, the contribution due to misalignment may have to be increased substantially. Furthermore, a high Q transducer will not reach steady-state response quickly and this will necessitate a larger contribution due to the transient response of the device.

In addition to the above, a number of checks are performed during the calibration to enable an estimate of the accuracy to be made. The reciprocity check is performed at all frequencies and any deviation from the desired agreement is automatically highlighted by the computer software. Similarly, the four repeated measurements are performed at different separation distances which allows a check to be made for the correct spherically-spreading acoustic field (distance-loss checks). These two procedures provide a check on the performance of the NPL reference transducers in the calibration, and any deviation from expected performance may require an increase in the appropriate uncertainty contribution.

5.2 COMBINING UNCERTAINTIES

When examining how each of the individual systematic uncertainties contributes to the overall systematic uncertainty for a calibration, it is important to consider the equation for the sensitivity:

\[ M_h^2 = \frac{2}{\rho f} \frac{d_1 d_3}{d_2} \frac{Z_{PH} Z_{TH}}{Z_{PT}} \]  \hspace{1cm} (14)

The values for systematic uncertainty are estimated as semi-ranges, that is the correct value is estimated to lie in the range \(+a\) to \(-a\) where \(a\) is the estimate of uncertainty. In general, these have been combined by the method described in NAMAS document NIS 3003.\(^{15}\) Essentially, this method uses the Central Limit Theorem to combine the semi-ranges and derive a total systematic uncertainty, \(U_s\), expressed for a confidence level of 95% according
to the following equation:

\[ U_s = k \left( \frac{a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2}{3} \right)^{1/2} \]  \hspace{1cm} (15)

where \( a_1, a_2 \) etc are the individual contributions of systematic uncertainty and \( k = 1.96 \).

In Table 1, for each transfer impedance measurement, the individual contributions are summed according to Equation 15 to give a total uncertainty expressed at the 95\% confidence level. The uncertainties for the attenuator and current probe calibration contribute once only since Equation (14) involves a ratio of transfer impedances and some cancellation of errors takes place. Both the attenuator and current probe calibration contributions are already expressed at the 95\% confidence level (they derive from calibrations by NAMAS accredited laboratories).

The contribution due to the reciprocity parameter includes contributions for water density, acoustic frequency and non-reciprocal behaviour, and these are also expressed as semi-ranges, as is the contribution for the distance measurement. Three distance measurements are included in Equation (14) in the form of a ratio, but since they are all measured by the same system (and have nominally the same value), some error cancellation will occur and only one contribution is included. The individual contributions from these sources are combined according to Equation (15) to give a total uncertainty expressed at the 95\% confidence level.

The 95\% confidence level systematic uncertainties derived from each of the three transfer impedances, the attenuator and current probe calibrations, and the reciprocity parameter and distance measurement are then combined in quadrature to give a total systematic uncertainty for \( M_H^2 \), also expressed at the 95\% confidence level. The systematic uncertainty in \( M_H^2 \) is then equal to half the systematic uncertainty in \( M_H^2 \).

The random uncertainty, \( U_R \), calculated from the four repeated calibrations (typically about 2.5\% expressed at a 95\% confidence level) is then combined with the total systematic uncertainty as follows:

\[ U_T = (U_R^2 + U_s^2)^{1/2} \] \hspace{1cm} (16)

to give an overall uncertainty, \( U_T \), expressed for a confidence level of 95\%.
Table 1  Uncertainties (in %) at three frequencies for each of three hydrophones.

<table>
<thead>
<tr>
<th>Source</th>
<th>Frequency (kHz)</th>
<th>Sec</th>
<th>B&amp;K 8104</th>
<th>B&amp;K 8103</th>
<th>Reson TC 4034</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>For each of $Z_{out}$, $Z_{in}$, $Z_{ot}$:</td>
<td>Transient effects:</td>
<td>4.2.4</td>
<td>5.0</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Alignment (directivity):</td>
<td>4.2.1</td>
<td>---</td>
<td>0.3</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Non-spherical field:</td>
<td>4.2.6</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>Amplifier loading:</td>
<td>4.2.14</td>
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<td>1.0</td>
<td>1.0</td>
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<td></td>
<td>Noise:</td>
<td>4.2.12</td>
<td>1.0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Wetting</td>
<td>4.2.8</td>
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<td>0.5</td>
<td>0.5</td>
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<tr>
<td></td>
<td>Electrical grounding effects:</td>
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<td>---</td>
<td>---</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Total for each Z (95%):</td>
<td></td>
<td>6.0</td>
<td>2.8</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>Attenuator calibration (95%):</td>
<td>4.1.3</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Current probe calibration (95%):</td>
<td>4.1.3</td>
<td>0.6</td>
<td>0.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Measurement of distance and J:</td>
<td>Acoustic frequency:</td>
<td>4.1.1</td>
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<td>0.2</td>
<td>0.2</td>
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<td></td>
<td>Water density:</td>
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<td>0.2</td>
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<td></td>
<td>Non-reciprocal behaviour:</td>
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<td>1.0</td>
<td>2.0</td>
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<td>Distance measurement:</td>
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<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Total for J and d (95%):</td>
<td></td>
<td>1.3</td>
<td>1.3</td>
<td>2.4</td>
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<tr>
<td>Total systematic uncertainty for $M_{in}$ (95%):</td>
<td></td>
<td></td>
<td>10.5</td>
<td>5.0</td>
<td>8.1</td>
</tr>
<tr>
<td>Total systematic uncertainty for $M_{in}$ (95%):</td>
<td></td>
<td></td>
<td>5.3</td>
<td>2.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Total random uncertainty for $M_{in}$ (95%):</td>
<td></td>
<td></td>
<td>4.3</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Overall uncertainty for $M_{in}$ (95%):</td>
<td></td>
<td></td>
<td>5.8</td>
<td>3.5</td>
<td>4.7</td>
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</table>
6 COMPARISON WITH OPTICAL INTERFEROMETRY

To validate a calibration method, it is essential to compare it with other independent methods of calibration. At high frequencies, an intercomparison can be made between the free-field reciprocity calibration method and optical interferometry. At NPL, a laser interferometer is operated as the primary standard method of calibrating hydrophones at frequencies between 500 kHz and 20 MHz, mainly for the calibration of the miniature PVDF membrane and needle-probe hydrophones used in medical ultrasonics. However, calibrations are possible using the interferometer at frequencies as low as 200 kHz, and with some modification to the calibration procedure it is possible to calibrate hydrophones of the type used in underwater acoustics using this facility.

6.1 THE NPL LASER INTERFEROMETER

Hydrophone calibration by laser interferometry has been described in detail previously\textsuperscript{26,27} and only a brief description is given here. In the method, the acoustic field produced by a transducer is detected by a thin plastic membrane (known as a pellicle) which is coated on one side with gold, rendering it optically reflecting. The pellicle is thin enough (5 \(\mu\)m) to be acoustically transparent so that it follows the particle displacement of the acoustic wave. The displacement of the pellicle is measured using a specially-designed Michelson interferometer and the acoustic pressure is calculated from the measured displacement. The hydrophone is then substituted for the pellicle with the acoustic centre of the hydrophone positioned at the same point in the acoustic field as was interrogated by the laser beam. The hydrophone is then calibrated by measuring the output voltage corresponding to the known acoustic pressure. The method has the advantage of being directly traceable to primary standards of length measurement, and has been validated for frequencies above 500 kHz by international intercomparison with other national standards laboratories\textsuperscript{28}.

6.2 MEASUREMENTS

In order to calibrate successfully the type of hydrophone used in underwater acoustics using the interferometer, certain modifications to the measurement procedure were necessary, some of which degraded the accuracy of the method slightly. The water tank used with the interferometer is only 1 metre long with a cross-section of 0.5 x 0.5 metres, and it is this relatively small tank size which gave rise to the measurement problems.

As usual when making measurements in reverberant tanks, gated tone-burst signals were used. However, when making displacement measurements, the distance from the steel pellicle mount to the point of measurement was only 5 cm which left a measurement window of only 30 \(\mu\)s (equivalent to 6\(\lambda\) at 200 kHz) before the onset of reflections. Fortunately, both the interferometer response and that of the source transducers used are low Q, allowing the displacement signal to reach steady state in only 2-3 cycles. However, when measuring the hydrophone voltage, significant transients were observed on the signal which therefore required more than three cycles to reach steady state. Consequently, it was not possible to make the displacement and hydrophone voltage measurements on the same cycles of the tone-
burst, and the hydrophone and source transducer had to be translated away from the pellicle mount to allow for a longer tone-burst to be used without interference from reflections. The above situation is not ideal and causes some degradation in accuracy compared to calibrations of PVDF membrane hydrophones where the same mount can be used for both pellicle and hydrophone. This has led to higher random uncertainties than is usual for calibrations using the interferometer, and an extra contribution to the systematic uncertainties is also required.

Two hydrophones were calibrated using the interferometer, a Brüel and Kjær 8103 and a Reson System TC4034. Both the hydrophones chosen here are being used above their resonance frequency which is not ideal, but there is a lack of more suitable alternative devices for use up to 500 kHz. Both hydrophones were calibrated in the same mount as had been used previously for reciprocity calibration. The mounting poles were attached to optical mounting stages which in turn sat on an optical bench which straddled the tank. This allowed translation in three orthogonal directions, one of which was arranged to be the axis of the source transducer. Each hydrophone was aligned with its reference mark pointing toward the source transducer along its axis. A miniature rotation stage allowed the hydrophones to be rotated about their own axes. Two plane-piston transducers were used as the sources to cover the frequency range, calibrations being performed well into the far-field of the transducers.

![Calibration graph](image)

**Figure 20.** Calibration of a B&K 8103 hydrophone by laser interferometry (×) and by the free-field reciprocity method (continuous line).

6.3 RESULTS

The results of the intercomparison can be seen in Figures 20 and 21 where the sensitivity of
each hydrophone is plotted versus frequency. Since the reciprocity calibrations were performed at intervals of only 5 kHz, the points have been connected together with a continuous line. Also shown on the plots are typical overall uncertainties for the reciprocity method.

Figure 21 Calibration of a Reson TC 4034 hydrophone by laser interferometry (×) and by the free-field reciprocity method (continuous line).

Figures 22 and 23 show the differences between the results of the two methods for the B&K 8103 and the TC 4034 respectively. The error bars shown are for the random uncertainties derived from four independent measurements and expressed at the 95% confidence level. The assessment of systematic uncertainties when using the interferometer to calibrate this type of hydrophone at these frequencies requires some further work and so only random uncertainties are shown. Due to the problems described above, the random uncertainties are as high as 7 or 8% at some frequencies which is considerably larger than the 2% obtained when calibrating membrane hydrophones using the interferometer.

It can be seen that most of the measurements agree within the combined random uncertainties (they at least overlap). This is remarkably good agreement when it is considered that the two methods are entirely independent and do not have any common sources of systematic uncertainty. It is interesting to note that the interferometer calibrations are lower at all frequencies except 200 kHz where they are higher. It should be noted that the greatest difficulties were experienced at 200 kHz when using the interferometer, so it may not be correct to read too much into the higher values for the interferometer at 200 kHz.

Although the implementation is somewhat complex, the fundamental principle of calibration
by laser interferometry is essentially simple, and the method has potential for high accuracy. With further work to modify the physical constraints imposed by the tank and mounting arrangements, it should be possible to considerably reduce the random uncertainty in the measurements. However, to extend the frequency range down below 200 kHz would require a radical redesign of the tank and mount.

Figure 22 Difference between calibrations performed by interferometry (×) and those performed by free-field reciprocity (●) for the TC 4034 hydrophone.
Figure 23  Difference between calibrations performed by interferometry (×) and those performed by free-field reciprocity (●) for the B&K 8103 hydrophone.
7 DISCUSSION AND CONCLUSIONS

As demonstrated herein, with careful implementation it is possible to calibrate reference hydrophones by the method of three-transducer spherical-wave reciprocity in the frequency range 10-500 kHz with an accuracy of ±0.5 dB or better. The major sources of uncertainty are the lack of ideal free-field conditions, the problems associated with making measurements using bursts of sound, and the difficulties encountered when operating at high frequencies where hydrophones are generally less well-behaved. The following conclusions may be drawn.

The use of a hydrophone at frequencies above its resonance frequency is not desirable. The performance is much less well-behaved than at frequencies below resonance and the sensitivity is generally lower and is changing rapidly with frequency. At 500 kHz, the acoustic wavelength is only 3 mm which is considerably smaller than the typical transducer size which can lead to acoustical problems due to the finite size of the transducers. This also leads to a greater variation of sensitivity with direction and so the orientation of the transducer during measurements must be specified very accurately. Ideally, new miniature transducers need to be developed for use as small omnidirectional hydrophones at frequencies of 300-600 kHz.

A parallel can be drawn with air-borne acoustics, in particular with the calibration of microphones. In microphone calibration, methods based on the principle of acoustic reciprocity are also used, requiring similar measurements of electrical voltages and currents, but the accuracies achieved are considerably better. However, microphone calibrations are not usually performed under pulsed conditions nor are they performed at such high frequencies where the directional response of the microphone is of crucial importance. Moreover, in the calibration of microphones the conditions of measurement are more constrained or at least specified more precisely. The most accurate calibrations require the use of standard microphones which have been designed specifically for use in accurate calibration.

In terms of metrology, the degree of specification and standardisation in microphone calibration is well ahead of the situation that exists in hydrophone calibration, and in underwater acoustics in general. Further standardisation would be beneficial, for example in the specification of an agreed configuration for mounting and a procedure for the wetting of reference hydrophones. Ideally, further work is also required to measure the variation of sensitivity with temperature for each type of reference hydrophone to enable calibrations to be corrected to a standard temperature.

The practical lower frequency limit for the calibration method described here is about 1-2 kHz, since even in the largest tanks measurements using pulse techniques become virtually impossible at frequencies below this. In the frequency range from 20 Hz to 2 kHz, determination of the pressure sensitivity by the method of coupler reciprocity is preferable as a primary standard method. This method has the additional advantage that it is possible to perform calibrations at high static pressure, something that is difficult using free-field techniques since it requires a very large pressurised vessel.

Allowing for the inevitable degradation in accuracy that occurs in the dissemination process,
measurements in the field should be possible to accuracies of ±1 dB. In practice, measurements in the field (for example in the ocean) are prone to errors due to the lack of controlled conditions and uncertainties in acoustic propagation which can be considerably larger than the calibration uncertainties.

Thus, the calibration method described in this report meets most of the requirements provided that the device calibrated is the one which is then used in the field. Improved accuracy of primary standards may be necessary if intermediate stages are used in the dissemination of standard calibrations, as may occur when faster and more versatile methods are used to calibrate measuring hydrophones by comparison with secondary standard hydrophones.
REFERENCES


APPENDIX 1  MEASUREMENTS IN REVERBERANT TANKS USING PULSE TECHNIQUES

Figure 24 shows a typical arrangement for measurements in a tank of dimensions \( l \times b \times h \) with the transmitting and receiving transducers a distance \( d \) apart. Also shown are the paths of reflected signals (echoes) between the transducers.

![Diagram of water tank showing the sources of reflections.](image)

**Figure 24**  Schematic diagram of water tank showing the sources of reflections.

To avoid interference, the pulse duration should be short enough so that reflected pulses arrive at the receiver after the termination of the direct signal. Therefore, by simple geometrical considerations, the pulse duration, \( \tau \), must meet the following criteria:

\[
\tau \leq \frac{l-d}{c} \tag{17}
\]
\[ \tau \leq \frac{2d}{c} \quad (18) \]

\[ \tau \leq \frac{\sqrt{b^2 + a^2 - d}}{c} \quad (19) \]

where \( c \) is the speed of sound in water. Equation (17) is concerned with reflections from the end walls of the tank, Equation (18) with reflections between transducers and Equation (19) with reflections from the side walls of the tank. A similar equation can be written for reflections from the water surface and bottom of the tank, but since the NPL tank has a square cross section, so that \( h = b \), the equation is in fact identical to Equation (19) and has been omitted.

The pulse duration governs the lower limiting frequency for which calibrations can be made. If the lowest frequency of interest is 10 kHz, then the pulse duration should be at least 0.1 ms if there is to be at least one cycle in the signal. However, transducers are resonant devices and take typically Q cycles of their resonance frequency to reach a steady-state response, where Q is the quality factor describing the sharpness of resonance. This means that the minimum pulse length required is transducer-dependent. Fortunately, the transducers used as reference hydrophones generally have low Q-factors (three or less) and resonances at frequencies above 50 kHz. As a general rule therefore, the pulse length must contain at least Q full cycles of signal to account for the transient response of the transducers. Hence,

\[ \tau \geq \frac{Q}{f_m} \quad (20) \]

where \( f_m \) is the lowest frequency of interest. If \( f_m \) is 10 kHz and \( Q = 3 \), then \( \tau \geq 0.3 \) ms.

The distance between the transmitting device and the receiving device must be large enough to minimise errors due to the finite size of the transducers. For this purpose, the separation must be larger than the maximum dimension of the largest transducer, and the hydrophone must be in the far-field of the source. For two transducers with maximum dimensions of sensitive areas \( a_1 \) and \( a_2 \) respectively, the distance, \( d \), must be chosen so that:

\[ d > \frac{a_1^2 + a_2^2}{\lambda} \quad (21) \]

and simultaneously:

\[ d > a_1 \quad (22) \]

and

50
\[ d > a_2 \] (23)

in order for the error due to the lack of spherical divergence to be less than 2\%. Again, this condition is transducer dependent. If a typical hydrophone dimension of 2 cm is taken, and the upper frequency limit is 500 kHz, then Equation (21) approximates to \( d > 0.3 \text{ m} \).

The mathematical conditions described above are plotted in Figure 25 for the NPL tank. The region over which the tank can be used is shown as the shaded area. The conditions described by equations (17)-(19) (the solid lines on the diagram) are governed by the tank dimensions, whereas the other conditions (broken lines) are governed by the upper and lower frequency limits, and characteristics of the transducer such as its size and Q-factor. It can be seen that for a given size of tank, there will be a frequency below which it is not possible to make measurements, even using pulse techniques. From Figure 25, it appears that for the NPL tank it is possible to attempt measurements at frequencies as low as 5 kHz. However, there are other factors to consider which can degrade the measurement accuracy at low frequencies. For example, the signal-to-noise ratio is poor for small hydrophones as they are inefficient projectors.

![Figure 25](image-url)

**Figure 25** Restrictions on pulse duration and transducer separation for the NPL tank with dimensions \( l=2 \text{ m}, h=b=1.5 \text{ m} \).

The maximum pulse repetition rate that can be used is governed by the reverberation time of the tank. This is dependent on the frequency used, the amount of absorption that takes place in the tank walls and the size and shape of the tank. It can be determined using an oscilloscope.
by measuring the time it takes for the amplitude of the reflected signals to reduce to the level of the background noise. For the NPL tank, the reverberation time measured using the above definition is 60 ms at 10 kHz. This allows for a maximum pulse repetition rate of 17 Hz.
APPENDIX 2  ELECTRICAL LOADING CORRECTIONS

The sensitivity of a hydrophone is often specified as the end-of-cable open-circuit sensitivity. This is the sensitivity of the hydrophone at the end of its cable when not connected to an electrical load. When a specific electrical load, such as an oscilloscope or an amplifier, is used at the output of the hydrophone, the end-of-cable loaded sensitivity of the hydrophone can be determined using the following method.

Consider the general case in which the hydrophone is considered as a two-terminal network of complex impedance \( Z \). Consider also that the hydrophone is connected to an electrical load of complex impedance \( Z_d \). From electrical network theory, the end-of-cable loaded sensitivity of the hydrophone, \( M_L \), when connected to the specified load is related to the end-of-cable open-circuit sensitivity, \( M_e \), by

\[
M_L = M_e \left( \frac{\text{Re} (Z_d)^2 + \text{Im} (Z_d)^2}{[\text{Re} (Z_d) \cdot \text{Re} (Z)]^2 + [\text{Im} (Z_d) \cdot \text{Im} (Z)]^2} \right)^{1/2} \quad (24)
\]

where \( \text{Re} \) and \( \text{Im} \) denote the real and imaginary parts of the relevant complex impedance.

Often, the electrical load can be assumed to be a parallel combination of a resistance \( R_d \) and capacitance \( C_d \). In this case, \( \text{Re}(Z_d) \) and \( \text{Im}(Z_d) \) are given by

\[
\text{Re} (Z_d) = \frac{R_d}{1 + \omega^2 C_d^2 R_d^2} \quad (25)
\]

and

\[
\text{Im} (Z_d) = \frac{-\omega C_d R_d^2}{1 + \omega^2 C_d^2 R_d^2} \quad (26)
\]

where \( \omega \) is the angular frequency.

A further simplification is possible if the impedances of both the hydrophone and the load can be assumed to be capacitive. In this case, if \( C \) is the end-of-cable capacitance of the hydrophone including any integral cable and connector, Equation (24) reduces to

\[
M_L = M_e \left[ \frac{C}{C + C_d} \right] \quad (27)
\]

Note: \( C \) can be approximated by \( 1/(\omega \text{Im}(Z)) \).