UNCERTAINTY IN THE TBLC AND BULK DISSIPATION CORRECTIONS FOR THE ONE LITRE SPHERES NPL1 AND NPL2, FILLED WITH ARGON AT T = 273.16 K.

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JUNE 2010
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Gavin Sutton
Engineering Measurement

ABSTRACT

The uncertainty in the calculated Thermal Boundary Layer Correction (TBLC) / Bulk dissipation correction due to the uncertainty in the input parameters is calculated for the one litre spheres NPL1 / NPL2 filled with argon gas at $T = 273.16$ K. For typical measurement uncertainties, we find that the combined fractional uncertainty is a maximum at the lowest measurement pressure ($50$ kPa) and varies from:

- $0.24$ PPM to $0.09$ PPM for $dF/F$ moving from the (0,2) to (0,10) mode.
- $0.24$ PPM to $0.14$ PPM for $G/F$ moving from the (0,2) to (0,10) mode.

At $P = 100$ kPa, both uncertainties are less than $0.17$ PPM.
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NOMINAL PARAMETERS:

\[
\begin{align*}
T &= 273.16 \text{ K} \\
a &= 0.062 \text{ m}^* \\
m &= 0.039947706 \text{ kg.mol}^{-1} \\
h &= 1 \text{ (thermal acc. coef.)} \\
\end{align*}
\]

* note: was 0.050 m for similar TCU1 analysis.

The uncertainty in the calculated TBLC / Bulk dissipation correction due to the uncertainty in the input parameters is considered. Table 1 shows the typical uncertainties used in this study. Other parameters such as density, \(C_p\), \(C_v\), \(\gamma = (C_p/C_v)\), \(\beta(2^{\text{nd}} \text{ acoustic virial coefficient})\) and \(\delta(3^{\text{nd}} \text{ acoustic virial coefficient})\) are derived from these parameters and are not considered individually.

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Description</th>
<th>Parameter uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(T)</td>
<td>Temperature</td>
<td>20 mK*</td>
</tr>
<tr>
<td>2</td>
<td>(P)</td>
<td>Pressure</td>
<td>20 Pa**</td>
</tr>
<tr>
<td>3</td>
<td>(m)</td>
<td>Molar mass</td>
<td>0.1%</td>
</tr>
<tr>
<td>4</td>
<td>(a)</td>
<td>Radius</td>
<td>0.1%</td>
</tr>
<tr>
<td>5</td>
<td>(\lambda)</td>
<td>Thermal conductivity</td>
<td>0.1%</td>
</tr>
<tr>
<td>6</td>
<td>(\eta)</td>
<td>Viscosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>7</td>
<td>(B)</td>
<td>2(^{\text{nd}}) virial coefficient</td>
<td>1.0%</td>
</tr>
<tr>
<td>8</td>
<td>(dB/dT)</td>
<td>1(^{\text{st}}) derivative of 2(^{\text{nd}}) virial coefficient w.r.t T</td>
<td>1.0%</td>
</tr>
<tr>
<td>9</td>
<td>(d^2B/dT^2)</td>
<td>2(^{\text{nd}}) derivative of 2(^{\text{nd}}) virial coefficient w.r.t. T</td>
<td>1.0%</td>
</tr>
<tr>
<td>10</td>
<td>(C)</td>
<td>3(^{\text{rd}}) virial coefficient</td>
<td>10.0%</td>
</tr>
<tr>
<td>11</td>
<td>(dC/dT)</td>
<td>1(^{\text{st}}) derivative of 2(^{\text{nd}}) virial coefficient w.r.t T</td>
<td>10.0%</td>
</tr>
<tr>
<td>12</td>
<td>(d^2C/dT^2)</td>
<td>2(^{\text{nd}}) derivative of 2(^{\text{nd}}) virial coefficient w.r.t. T</td>
<td>10.0%</td>
</tr>
</tbody>
</table>

Table 1 input parameter uncertainties used to determine the uncertainty in the TBLC / Bulk dissipation correction. Note: * was 50 mK for similar TCU1 analysis. ** was 10 Pa for similar TCU1 analysis.

PROCEDURE

- The TBLC/Bulk dissipation correction is first calculated for the nominal parameters to give a baseline value. Radial acoustic modes \((0,2)\) to \((0,10)\) are considered.
- The value of each parameter, in turn, is increased by its estimated uncertainty (last column in Table 1) and the TBLC/Bulk dissipation correction recalculated.
- The difference between the two values is plotted against pressure from 50 kPa to 750 kPa and indicates the uncertainty of the correction due to the uncertainty in the input parameter.
- Finally, the uncertainty contributions are summed in quadrature to give an overall uncertainty in the TBLC/Bulk dissipation correction.
NOMINAL CORRECTION

Figures 1 and 2 show the TBLC/Bulk dissipation perturbation for the frequency, $F$ and the halfwidth, $G$ respectively. The correction reduces as the mode number increases and the pressure increases.
1. U[TBLC/BULK DIS] DUE TO $U(T) = 20$ MK

With the TBLC / Bulk dissipation correction calculated at the measurement temperature, we expect $U(T) < 1$ mK. We chose a worst-case scenario in which $U(T) = 20$ mK but the uncertainty in the correction is still very small.

![Uncertainty in $dF/F$ (TBLC + BULK DIS) for $\Delta T = 20$ mK](image3)

![Uncertainty in $G/F$ (TBLC + BULK DIS) for $\Delta T = 20$ mK](image4)
2. Uncertainty in $\Delta P = 20$ Pa

![Graph showing uncertainty in $\frac{dF}{F}$ and $\frac{G}{F}$ for $\Delta P = 20$ Pa.](image)

Figure 5

Figure 6
3. \( U[\text{TBLC/BULK DIS}] \) DUE TO \( U(m)/m = 0.1\% \)

Figure 7

Uncertainty in \( dF/F \) (TBLC + BULK DIS) for \( \Delta m/m = 0.1 \% \)

Figure 8

Uncertainty in \( G/F \) (TBLC + BULK DIS) for \( \Delta m/m = 0.1 \% \)
4. U[TBLC/BULK DIS] DUE TO $U(a)/a = 0.1\%$

**Figure 9**

Uncertainty in $dF/F$ (TBLC + BULK DIS) for $\Delta a/a = 0.1 \%$

**Figure 10**

Uncertainty in $G/F$ (TBLC + BULK DIS) for $\Delta a/a = 0.1 \%$
5. U[TBLC/BULK DIS] DUE TO $U(\lambda)/\lambda = 0.1\%$

**Figure 11**

Uncertainty in $dF/F$ (TBLC + BULK DIS) for $\Delta \lambda/\lambda = 0.1\%$

**Figure 12**

Uncertainty in $G/F$ (TBLC + BULK DIS) for $\Delta \lambda/\lambda = 0.1\%$
6. U[TBLC/BULK DIS] DUE TO $U(\eta)/\eta = 0.1\%$

![Graph showing uncertainty in G/F (TBLC + BULK DIS) for $\Delta \eta/\eta = 0.1\%$.](image-url)

Figure 13

<table>
<thead>
<tr>
<th>$P$ / kPa</th>
<th>$U(\bar{G}/\bar{F})$ [parts in $10^6$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.025</td>
</tr>
<tr>
<td>100</td>
<td>0.02</td>
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<tr>
<td>200</td>
<td>0.015</td>
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<tr>
<td>300</td>
<td>0.01</td>
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<tr>
<td>400</td>
<td>0.005</td>
</tr>
<tr>
<td>500</td>
<td>0.0025</td>
</tr>
<tr>
<td>600</td>
<td>0.0015</td>
</tr>
<tr>
<td>700</td>
<td>0.001</td>
</tr>
<tr>
<td>800</td>
<td>0.0005</td>
</tr>
</tbody>
</table>
7. \( U[\text{TBLC/BULK DIS}] \) DUE TO \( U(B)/B = 1.0\% \)

Uncertainty in \( \frac{dF}{F} \) (TBLC + BULK DIS) for \( \Delta B/B = 1.0\% \)

![Figure 14](image1)

Uncertainty in \( \frac{G}{F} \) (TBLC + BULK DIS) for \( \Delta B/B = 1.0\% \)

![Figure 15](image2)
8. U[TBLC/BULK DIS] DUE TO $U(\frac{dB}{dT})/ dB/dT = 1.0\%$

![Figure 16](image1)

Uncertainty in $dF/F$ (TBLC + BULK DIS) for $\Delta dB/dT = 1.0 \%$

![Figure 17](image2)

Uncertainty in $G/F$ (TBLC + BULK DIS) for $\Delta dB/dT = 1.0 \%$
9. U[TBL/C/BULK DIS] DUE TO $U(\frac{d^2 B}{dT^2})/ \frac{d^2 B}{dT^2} = 1.0\%$

**Figure 18**

**Figure 19**
10. U[TBLC/BULK DIS] DUE TO $U(C)/C = 10\%$

**Figure 20**

Uncertainty in $dF/F$ (TBLC + BULK DIS) for $\Delta C/C = 10.0 \%$

*Graph showing the uncertainty in $dF/F$ for different values of $\Delta C/C$.*

**Figure 21**

Uncertainty in $G/F$ (TBLC + BULK DIS) for $\Delta C/C = 10.0 \%$

*Graph showing the uncertainty in $G/F$ for different values of $\Delta C/C$.*
11. U[TBLC/BULK DIS] DUE TO $U(dC/dT)/dC/dT = 10\%$

Uncertainty in $dF/F$ (TBLC + BULK DIS) for $\Delta(dC/dT) = 10.0 \%$

Figure 22

Uncertainty in $G/F$ (TBLC + BULK DIS) for $\Delta(dC/dT) = 10.0 \%$

Figure 23
12. $U[\text{TBLC/BULK DIS}] \text{ DUE TO } U(d^2C/dT^2)/d^2C/dT^2 = 10\%$

Figure 24

Figure 25
Assuming all uncertainties are independent, we calculate the combined uncertainty to the TBLC / Bulk dissipation correction by quadrature summation:

Combined fractional uncertainty in frequency correction:

\[
\Delta \left( \frac{df}{f} \right)_{TBLC\ Bulk\ Diss} = \sqrt{\sum_{i=1}^{n} \Delta_i^2 \left( \frac{df}{f} \right)_{TBLC\ Bulk\ Diss}}
\]  

(1)

Combined fractional uncertainty in the halfwidth correction:

\[
\Delta \left( \frac{g}{f} \right)_{TBLC\ Bulk\ Diss} = \sqrt{\sum_{i=1}^{n} \Delta_i^2 \left( \frac{g}{f} \right)_{TBLC\ Bulk\ Diss}}
\]  

(2)

Figures 26 and 27 show the combined fractional uncertainties for the frequency and halfwidth corrections respectively.
Points to note:

- The largest uncertainties occur at the lowest pressure. This may be expected since the fractional TBLC / Bulk dissipation correction is largest at low pressures.
- The uncertainties are largest for the (0,2) mode reducing for higher order modes. Again, this may be expected since the fractional TBLC / Bulk dissipation correction decreases as the mode number increases.
- At the lowest measurement pressure (50 kPa) the combined fractional uncertainty in TBLC / Bulk dissipation correction varies from:
  - 0.24 PPM to 0.09 PPM for $dF/F$ moving from the (0,2) to (0,10) mode.
  - 0.24 PPM to 0.14 PPM for $G/F$ moving from the (0,2) to (0,10) mode.
- The reason the uncertainty on $G/F$ is slightly larger than that on $dF/F$ is due to the fact that the bulk dissipation correction only affects $G/F$.
- At $P = 100$ kPa, both uncertainties are less than 0.17 PPM.