High Temperature Friction:
A Simplified Approach for Estimating Uncertainty of Measurement

by

Malcolm S Loveday & Matt Brooks

NPL Materials Centre,
National Physical Laboratory,
Teddington, TW11 0LW, UK.

ABSTRACT

As part of a DTI Funded Project concerned with the determination of Friction Coefficients for use as input data for process control models, the issues concerning the estimation of uncertainty of measurement have been addressed. Estimates are presented for the measurement uncertainty for three methods used for the determination of interfacial friction namely a) the Cockcroft – Male Ring Test, b) the NPL Big Friction Rig and c) the NPL Pin-on-Disc Rig.

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National Physical Laboratory
Teddington, Middlesex TW11 0LW, UK

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Approved on behalf of Managing Director, NPL, by Dr C Lea,
Head, NPL Materials Centre.

CONTENTS

1. INTRODUCTION

1.1 Background

It is now widely recognised that all experimental measurements should be accompanied by an estimate of the Uncertainty of Measurement as outlined in the ISO TAG4 *'Guide to the expression of uncertainty in measurement,'* (1994), colloquially know as the 'GUM'.

The approach adopted here for a Friction Measurement Uncertainty Budget is similar to that proposed for a creep testing uncertainty budget used in association with the Creep Certified Reference Material, CRM 425, (Loveday, 1996) and Room temperature Tensile Testing (Loveday 1999). Further information and guidance on estimating measurement uncertainties are available elsewhere including a *'Beginners Guide'* (Bell, 1999) as well as more comprehensive documents (UKAS, M3003, 1997 and Kandil *et al*, 2000)

It should be noted that a simplified approach has been adopted in this Measurement Note without the use of the Welch-Satterthwaite equation, or consideration of correlated parameters or degrees of freedom. However it is unlikely that consideration of these additional parameters will significantly alter the approximate uncertainty estimates presented here relating to friction measurements.

The friction measurement methods considered here have been developed or evaluated in two DTI funded projects concerning measurement of friction and heat transfer coefficients for process modelling and control. Further details are given elsewhere (Loveday, Gee & Brooks, 2000). In addition, information concerning the development of measurement methods for procuring reliable data for process modelling can be found at the following web site: **http://www.npl.co.uk/npl/cmmt/procmetal/rpfpm.html**

1.2 An Overview of Uncertainty Estimation Based Upon the GUM

The "**Guide to the expression of uncertainty in measurement**", which will be referred to hereafter as GUM, was published jointly by several authoritative standards bodies, namely BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML. It is a comprehensive document of over 90 pages based upon rigorous statistical methods for the summation of uncertainties from various sources. Its complexity has provided the driving force for a number of organisations to produce simplified versions of the GUM, eg the National Institute of Science and Technology (NIST) in the USA (Taylor and Kuyatt, 1993), the National Measurement Accreditation Service (NAMAS) in the UK (M3003, 1997). These documents all give guidance on how to estimate uncertainty of measurement based upon an "uncertainty budget" concept.

The total uncertainty of a measurement is determined by summing all the contributing components in an appropriate manner. It is necessary to quantify all the contributions, and at the preliminary evaluation stage to decide whether some contributions are negligible and therefore not worth including in the subsequent calculations. For most practical measurements, in the materials field the definition of negligible may be taken as a component smaller than one-fifth of the largest component. The GUM categorises two ways of evaluating uncertainties, A and B. Type A determination is by repeated observation and provided sufficient readings are available, say greater than 9, then conventional statistical analysis can be used to determine the standard deviation $s(q_k)$.

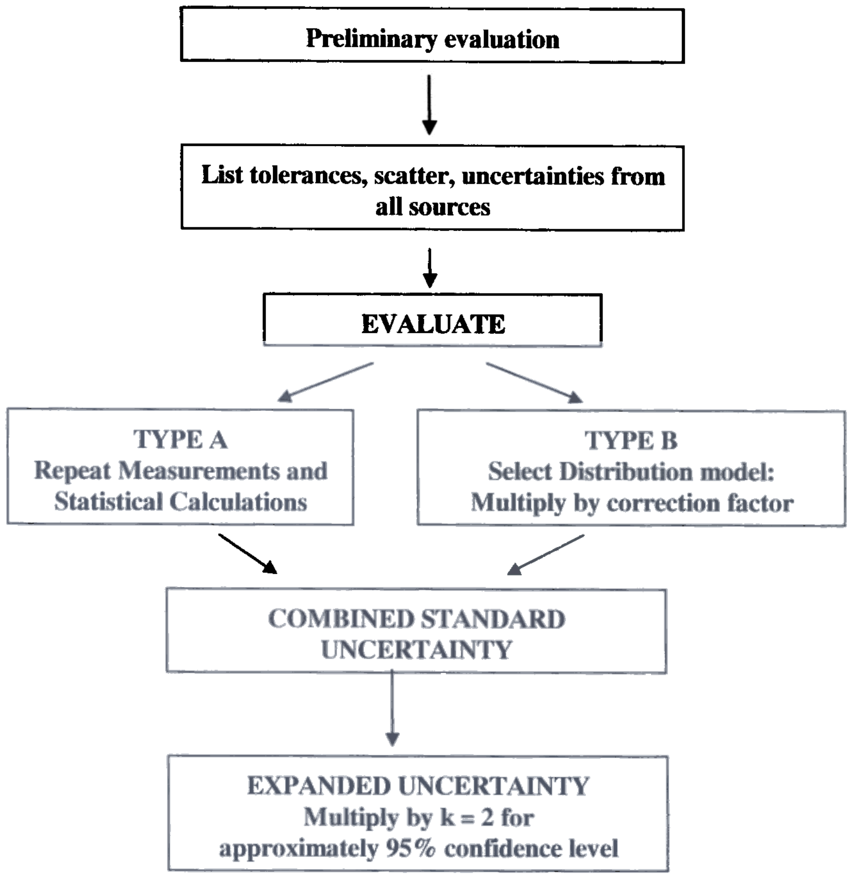


Figure 1. GUM: Outline of Procedure for Estimation of Uncertainty

Type B evaluation is by means other than Type A and makes use of, for example, tolerances specified in standards, measured data, manufacturers specifications and calibration certificates. In most cases the knowledge of a simple model of the relationship between the various components, and of the likely distribution model of the components is also used. If for example the tolerance specified in a Standard is ± a, then in absence of any other knowledge, it may be appropriate to assume a rectangular distribution model in which case, the uncertainty becomes $u_s = \frac{a}{\sqrt{3}}$. If better knowledge is available, it may be that a triangular distribution would be more appropriate, then $u_s = \frac{a}{\sqrt{2}}$, (see GUM), where u_s denotes a **Standard Uncertainty** obtained by multiplying U by an appropriate factor. The next step is to determine the **Combined Standard Uncertainty, u_c** by summing the standard uncertainties, usually by using the root sum square method. The **Expanded Uncertainty** U_E, is then obtained by multiplying u_c by a coverage factor, k. At approximately the 95% confidence level, this equates to k = 2. Thus, $U_E = 2u_c$. this procedure is shown schematically in Figure 2.

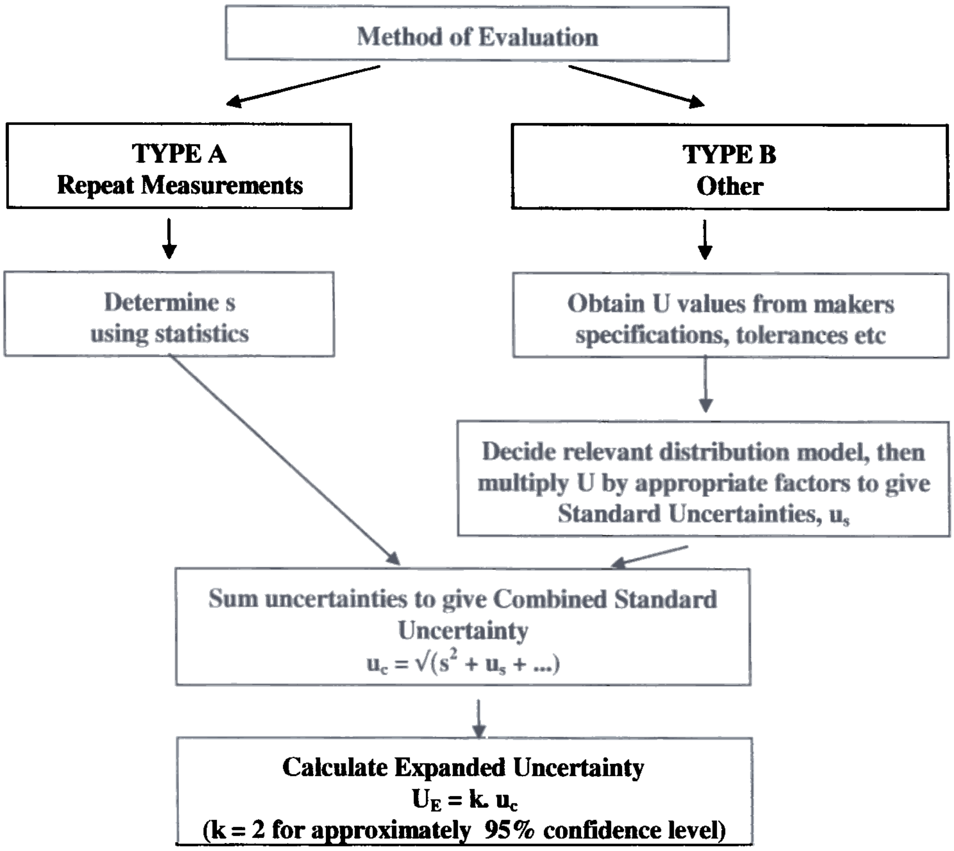


Figure 2 Procedure for estimating uncertainty in accordance with the GUM

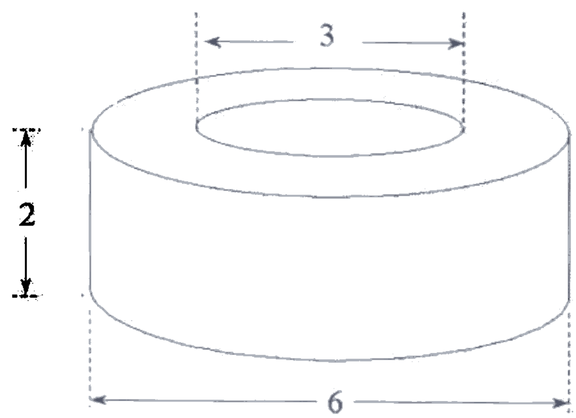
2. THREE FRICTION MEASUREMENT METHODS

2.1 The Cockcroft -Male Ring Test

One of the most important pieces of data for modelling forging and forming processes is the friction coefficient, which describes the amount of friction between the deforming metal work-piece and the tool. These coefficients can be estimated using the ring up-setting test, generally known as the Cockcroft and Male Ring Test (Male & Cockcroft, 1964), although it earlier been proposed by Kunogi (1954) and Kudo (1955). However, a number of simplifying assumptions have to be made which can lead to large errors and uncertainties, (Fletcher *et al*,1998). Moreover, since the values vary with other parameters such as relative velocity and contact pressure, any estimate of friction can only be an average. In the ring test an annular sample of specified dimensions is deformed between platens. From a measurement of the changes of the internal diameter of the testpiece it is possible to deduce a 'friction factor', m, by interpolation of theoretically calculated calibration curves. If the friction is high the central hole is reduced in diameter when the ring is subjected to compression. Low friction results in the inner diameter increasing. Friction coefficients, μ, may then be derived from the empirical relationship

$$\mu = m / (2\sqrt{3})$$ [Male & Depierre,1970] ..(Eqn 1)

Although this test is widely used as a simple ranking test for assessing lubricants or surface finishes of tools, it is not ideal for generating reliable quantitative data for modelling purposes since the values obtained are highly dependent on the assumptions made for calculating the calibration curves. The shape of the curves dictate that at low values of deformation, say less than ~ 20%, the method is not sufficiently sensitive, and results are highly dependent upon the accuracy of the thermo-mechanical material description,(Fletcher *et al*, 1998).



$D_o : D_i : H = 6 : 3 : 2$

Figure 3. Schematic Diagram of Cockcroft – Male Ring

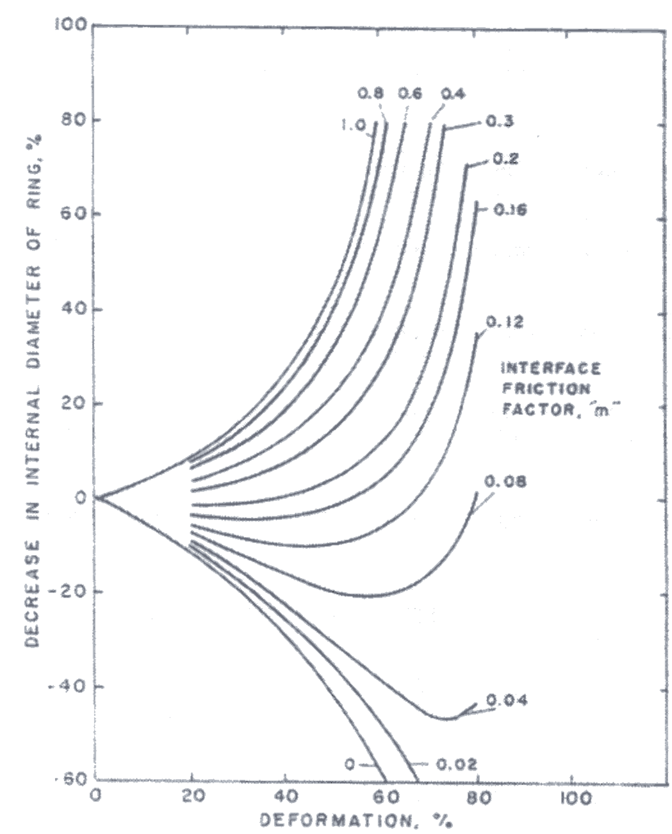


Figure 4 Calibration curves for ring test . (Male, A.T. and Depierre, V. (1970)

Typically the preferred ring geometry is shown in Figure 3, and an example of the theoretical calibration curves is shown in Figure 4. It should be noted that if different theoretical assumptions are made concerning the generation of such curves then there can be significant differences in the shape of the curves and hence the magnitude of the value of the friction factor subsequently determined (Fletcher *et al*, 1998).

2.2 The NPL Big Friction Rig

The technique incorporates a 250kN Instron servo hydraulic high rate actuator mounted on a NPL designed rig, used in conjunction with a 7MN (~700ton) press, Figure 5. The rig can be used with two sizes of cylindrical billets, of 30 and 50 mm in diameter, which are pre-heated in a furnace up to 1000°C and then transferred manually into the "grabber arm" on the friction rig, Figure 6. An axial load is applied using the press, and the load is monitored using a three column load cell designed by NPL. As the billets are compressed between platens they are dragged horizontally up to 100mm between the platens, at rates up to 100 mm/s, and using the 250kN actuator controlled by an Instron 8500 Series controller. By recording the vertical and horizontal loads, the coefficient of friction can then be determined, an allowance being made for the fact that both the upper and lower surfaces of the billet are in contact with tool steel platens. The loads, displacement and temperature of the billet are recorded using a PC based data logger, written LabView software. Further details of the design and calibration of the Big Friction Rig are given elsewhere (Loveday *et al*, 2000)

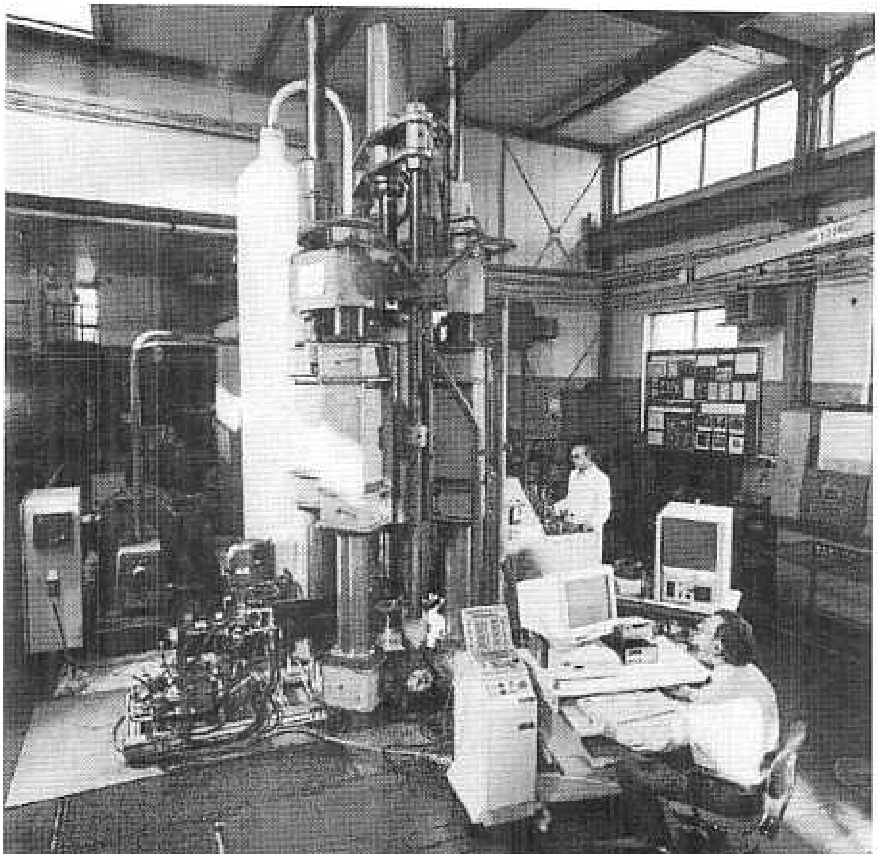


Figure 5. The NPL Big Friction Rig

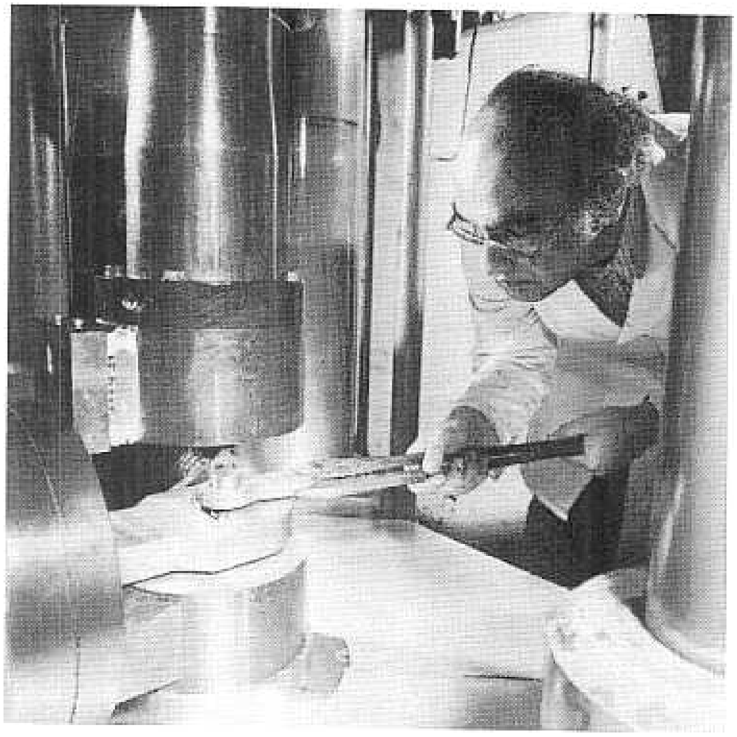


Figure 6. Hot billet being transferred from a furnace to the grabber arm on the Big Friction Rig.

2.3 The NPL Pin-on-Disc Friction Rig

This apparatus can heat strip samples up to in excess of 1000°C in a few seconds using a DC electrical current of up to 200 A, [Gee *et al,* 1997]. The sample may then be rapidly brought into contact with a rotating disc of tool steel using a motorised drive mechanism, with transducers measuring the resultant vertical and horizontal forces. A schematic diagram of the apparatus is shown in Figure 7 and a photograph in Figure 8. To prevent the thin strip sample collapsing it is supported on a zirconia thermal insulator and its temperature monitored using a 0.1 mm Type R thermocouple. In principle this rig could be fully encased so that measurements may be carried out in an inert atmosphere. The sample size in contact with the disc is typically 5 mm x 5 mm x 2 mm, while the disc is 100 mm in diameter. Contact speeds up to 2 m/s are achievable.

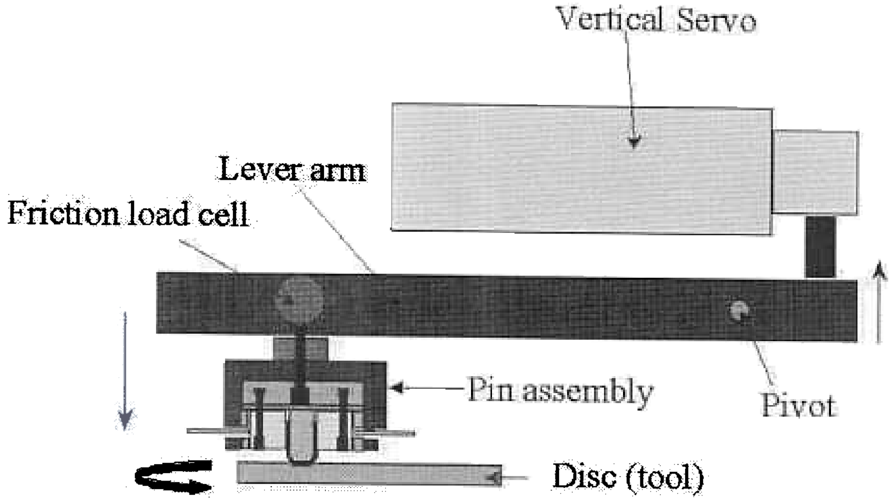


Figure 7 Schematic layout of Pin-on-Disc Rig.

Figure 8. Pin-on-Disc Rig

3 MEASUREMENT UNCERTAINTIES

3.1 Uncertainty Estimates for Cockcroft -Male ring test

3.2.1 Introduction

Examples of Cockcroft –Male rings of a steel deformed at high temperature are shown in plan view and cross section in Figure 9. In general, the measurements of the internal diameter of the hole are made at a number of positions and the average value calculated. Measurements may be undertaken using a vernier calliper or a shadowgraph projector. However it should be recognised that reproducible measurements involve operator skill which can lead to a systematic error between one laboratory and another. Thus although the technique is suitable for evaluating the relative performance of different lubricants or surface finish of dies within a single laboratory, it will be shown that the quantitative assessment of friction factors is prone to a large measurement uncertainty.

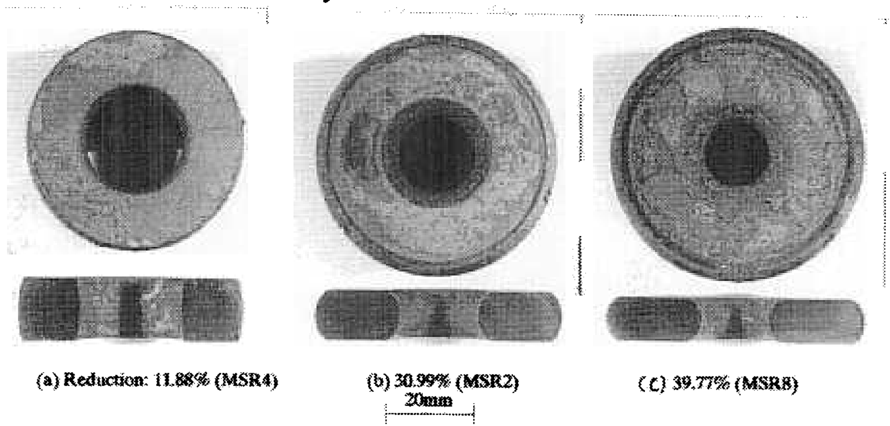


Figure 9 Photograph of rings , before and after deformation.
(Courtesy of Y H Li, MSc Thesis, Sheffield University 1996)

3.2.2 Assessment of Uncertainty of Measured parameters d & h

The sources of error associated with the diameter and height measurements for Cockcroft – Male rings are tabulated in Tables 3.2.1., 3.2.2. & 3.2.3. The measured values in mm have been expressed as a % for the rings with a geometry shown in Figure 3, with h_o = 16mm and D_i = 24mm. The assumed probability distributions are also indicated following the procedure laid down in the GUM, and the Expanded Uncertainty, U_e, is given. It has been assumed that although an oxide coating on those materials prone to oxidation can have significant effect on the diameter measurements, the height measurement is relatively un-affected by oxidation since the top and bottom surfaces of the rings are abraded by the platens, and in general the samples are cooled fairly rapidly after testing so that further oxidation does not occur.

Symbol	**Source of Uncertainty (y)**	**Value +/- mm**	**Value +/- %**	**Probability Distribution**	**Divisor**	**$u_i(y)$ +/- %**
äd_o	Initial diameter repeatability	0.1	0.4	Normal	2	0.2
äd_f	Final diameter repeatability	0.2	0.8	Normal	2	0.4
äd_c	Vernier calliper Calibration uncertainty	0.02	0.08	Normal	2	0.04
u_c	Combined Uncertainty	-	-	-	-	0.449
U_e	**Expanded Uncertainty (k=2, 95% confidence)**	-	-	-	-	**0.9**

Table 3.2.1 Uncertainty in Diameter Measurements ***(without oxide)***

Symbol	**Source of Uncertainty (y)**	**Value +/- mm**	**Value +/- %**	**Probability Distribution**	**Divisor**	**$u_i(y)$ +/- %**
äd_o	Initial diameter repeatability	0.1	0.4	Normal	2	0.2
äd_f	Final diameter repeatability	1	5	Normal	2	2.5
äd_c	Vernier calliper Calibration uncertainty	0.02	0.08	Normal	2	0.04
u_c	Combined Uncertainty	-	-	-	-	2.51
U_e	**Expanded Uncertainty (k=2, 95% confidence)**	-	-	-	-	**5**

Table 3.2.2 Uncertainty in Diameter Measurements ***(with oxide)***

Symbol	Source of Uncertainty (y)	Value +/- mm	Value +/- %	Probability Distribution	Divisor	$u_i(y)$ +/- %
$\ddot{a}h_o$	Initial height, repeatability	0.05	0.3	Normal	2	0.15
$\ddot{a}h_f$	Final height, repeatability	0.2	2.5	Normal	2	1.25
$\ddot{a}h_c$	Micrometer, Calibration uncertainty	0.002	0.025	Normal	2	0.013
u_c	Combined Uncertainty	-	-	-	-	1.26
U_e	**Expanded Uncertainty (k=2, 95% confidence)**	-	-	-	-	**2.52**

Table 3.2.3 Uncertainty in Height Measurements

3.2.3 Uncertainty in friction factor, 'm' due to uncertainty in d and h measurements

For the purpose of this analysis it has been assumed that the overall compression of the ring is nominally 50%, which is a reasonable reduction consistent with industrial forging. In reality different regions of a forged component may experience forging strains ranging from zero upto ~ 200% or even larger. Because of the complex shape of the Friction Calibration curves, ideally it would be necessary to estimate the measurement uncertainty for the entire range of friction factor, m, experienced locally at different points by the components, however for the purpose of the analysis the variation of 'm' attributable to the measurement uncertainties associated with the diameter and height measurements given above are tabulated for various values of 'm' and then an nominal average value of äm used in the final uncertainty budget given below.

Friction Factor m	Lower Value of m at $U_{e,h}$ = - 2.5%	Upper value of m at $U_{e,h}$ = + 2.5%	Difference in m , +/-	% Difference in m
1	0.8	1.2 ?	0.2	20
0.6	0.75	0.55	0.2	30
0.3	0.28	0.34	0.06	20
0.1	0.1	0.1	0	0
0.04	0.035	0.045	0.01	25

Table 3.2.4 Variation in 'm' due to estimated uncertainty in height measurement of U_e = +/- 2.5%; values obtained from calibration curves.

The nominal mean value of the variation of m ~ +/- 25%. This value is used as an input value in the final uncertainty budget below in Table 3.2.6.

Friction Factor m	Lower Value of m at $U_{e,d}$ = - 2.5%	Upper value of m at $U_{e,d}$ = + 2.5%	Difference in m , +/-	% Difference in m
0.8	0.76	0.84	0.08	10
0.37	0.35	0.39	0.04	11
0.18	0.178	0.182	0.004	2
0.08	0.077	0.083	0.006	7.5
0.005	0.003	0.007	0.004	80

Table 3.2.5 Variation in 'm' due to estimated uncertainty in diameter measurement (without oxide) of U_e = +/- 0.9%; values picked off calibration curves.

Ignoring the very large value of +/- 80% at the very low friction conditions (m = 0.005) the nominal mean value of the variation of m ~ =/- 10 %. This value is used as an input value in the uncertainty budget given below in Table 3.2.6 (a) .

In the case of materials prone to oxidation, the post test internal diameter measurement is subject to a considerably larger uncertainty as indicated in Table 3.2.2 where it is shown that the uncertainty is ~ +/- 5% which is approximately 5 times larger than the measurement without oxide significantly influencing the results. Thus the values given in Table 3.2.5 may merely be multiplied by a factor of 5 to give the variation in 'm' with oxide present, hence a mean value of +/- 50% may be used as the uncertainty in 'm' as the input parameter in Table 3.2.6 (b) .

Table 3.2.6 Final Uncertainty Budget for variation of Friction Factor, m, attributable to the measured uncertainties in height and diameter measurements

Symbol	Source of Uncertainty (y)	Value +/- %	Probability Distribution	Divisor	$u_i(y)$ +/- %
$äm_h$	Uncertainty in m due to height measurements ,	25	Normal	2	12.5
$äm_d$	Uncertainty in m due to diameter measurements,	10	Normal	2	5
u_c	Combined Uncertainty	-	-	-	13.5
U_e	**Expanded Uncertainty (k=2, 95% confidence)**	-	-	-	**27**

a) *Without Oxide*

Symbol	Source of Uncertainty (y)	Value +/- %	Probability Distribution	Divisor	$u_i(y)$ +/- %
$äm_h$	Uncertainty in m due to height measurements	25	Normal	2	12.5
$äm_d$	Uncertainty in m due to diameter measurements	50	Normal	2	25
u_c	Combined Uncertainty	-	-	-	28
U_e	**Expanded Uncertainty (k=2, 95% confidence)**	-	-	-	**56**

b) *With Oxide*

3.3 Uncertainty Estimates for Big Friction Rig

As explained in Section 2.2, friction coefficients are determined using the Big Friction Rig from the ratio of the horizontal and axial loads, measured using load cells whilst a hot billet is dragged horizontally between compression platens and the billet is deformed. The approach adopted here for estimating the uncertainty of measurement is to utilise the uncertainty values for the two load cells determined during calibration to provide traceability to the National Measurement System (NMS). The uncertainty attributable to repeatability determined from the type of curves shown in Figure 10, which have all been classified as 'Class 1' data (Brooks & Loveday, 2000), giving a value of ± 3.3% has also been included.

The reproducibility has been evaluated based on replicate measurements shown in Figure 10 where results on different testpieces tested under the same testing conditions have been measured.

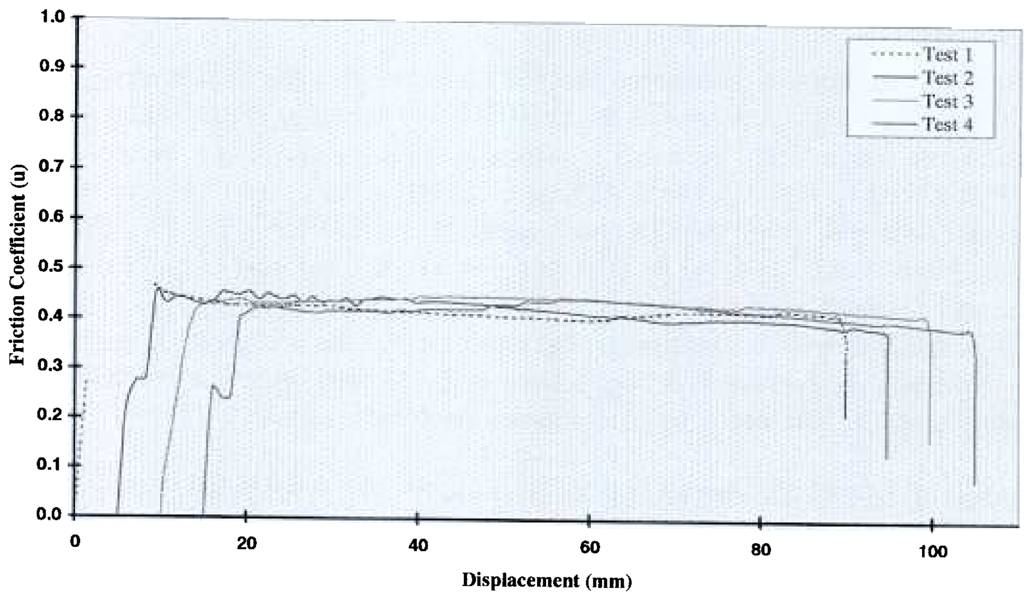


Figure 10 Repeat test measurements for the Big Friction Rig

For each individual test, a statistical analysis of the recorded data points is undertaken over the portion of the friction coefficient / displacement curve where friction has achieved steady-state conditions. I.e. the initial rise as the horizontal grabber arm accelerates up to the specified speed is ignored and likewise the falling tail on the curve, indicating that the grabber arm has stopped moving.

Thus, the mean value of friction coefficient is determined together with its associated standard deviation from the ratio of the horizontal dragging force to the axial force, an allowance being made for both the upper and lower contact faces of the billet. For the measurements shown in Figure 10 the mean results are tabulated in Table 3.3.1, together with the mean of the means and its associated standard deviation which has been used as a quantitative indication of the repeatability. The reproducibility between tests has been calculated from the standard deviation of the means divided by √4 to give a standard error.

Test No	Average friction coefficient, μ	Standard Deviation
1	0.42	0.011
2	0.42	0.020
3	0.43	0.010
4	0.42	0.015
Mean Value	0.4225	0.014
Standard Deviation of Mean	0.005	
Standard Error	**$0.005/\sqrt{4} = 0.0025$**	

Table 3.3.1 Summary of repeat tests carried out on the Big Friction Rig using Mild Steel testpieces at 590°C at 10mm/s under an axial stress of 107 MPa.

Thus the average standard deviation for the individual tests expressed as a percentage corresponds to (0.014/0.4225) x 100 = 3.3%, representing the repeatability of the tests. The reproducibility for the four replicated tests corresponds to (0.0025/0.4225) x 100 = 0.6%. These values are then used in the uncertainty budget given on the next page.

Details of the calibration procedures adopted for calibrating the load cells used in the Big Friction rig are given in greater detail elsewhere (Loveday *et al*, 2000), however essentially the large 7MN load cell is removed from the press and mounted in the 12MN machine in Force Section, NPL and calibrated as a force proving device in accordance with EN 10002 Part 3 (now superseded by ISO EN 376). The uncertainty associated with this process has been assessed by an ISO Working Group ISO TC 164 WG 3 and the associated uncertainty for a Class 1 device is given as +/- 0.62 % . It should be noted that in this particular application, since the compressive load and the horizontal dragging load are both applied in a single defined direction, the influence of reversibility on the two load cells performance has been ignored.

In the case of the 250 kN load cell incorporated in the Big Friction Rig in line with the grabber arm, it is calibrated using a force proving device mounted in a special reaction frame which butts up against the front of the Big Friction Rig. The load cell is then calibrated in accordance with EN10002 Part2 (now superseded by ISO EN 7500 Part1) . The force proving device is demounted from the calibration reaction frame and is verified in the NPL 1.2 MN dead weight machine.

The various sources of uncertainty are listed in Table 3.3.2 and the Combined and Expanded Uncertainties are calculated using the conventional root mean square technique.

Symbol	Source of Uncertainty (y)	Value +/- %	Probability Distribution	Divisor	$u_i(y)$ +/- %
$U_{e\ cal}$	Calibration of 250 kN INSTRON Load cell in BFR	0.62	Normal	2	0.31
U_{std}	Verification of 7MN load cell.	0.24	Normal	2	0.12
r	Repeatability friction test (standard deviation)	3.3	Normal	1	3.3
R	Reproducibility between tests (standard deviation)	0.6	Normal	1	0.6
u_c	Combined Uncertainty	-	-	-	3.37
U_e	**Expanded Uncertainty (k=2, 95% confidence)**	-	-	-	**6.74**

Table 3.3.2 Uncertainty Budget for Large Friction Rig (BFR).

3.4 Uncertainty Estimates for Pin-on-Disc Friction Rig

Because of the geometry of the Pin-on Disc Friction rig, direct *in-situ* calibration of the load measurement system to follow the traceability route to the NMS, used for the Big Friction Rig, has not been possible to date. However, a bench comparison with a calibrated load cell has been undertaken to provide a spot check, and thus a nominal value of +/- 1% has been used in the uncertainty budget below based upon the manufacture's specification for the load cells. Since the uncertainty associated with the load cells is appreciably less than that due to the repeatability of the test results, it is unlikely that errors in the load cell uncertainties will have a significant affect on the overall uncertainty estimate. A typical example of recorded raw data from the Pin-on-Disc rig is shown in Figure 11.

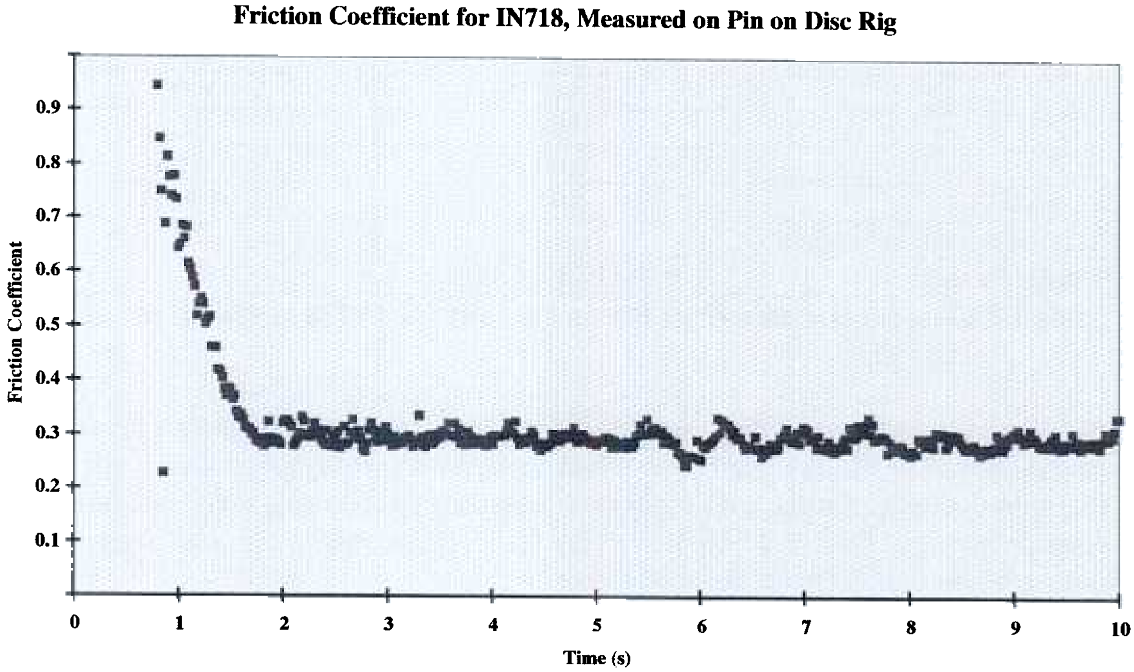


Figure 11. Typical example of a friction curve measured using the Pin-on Disc Rig

Results of repeat tests are shown in Figure 12 for IN 718 at 400°C at 60mm/s under a nominal axial stress of 80 MPa.

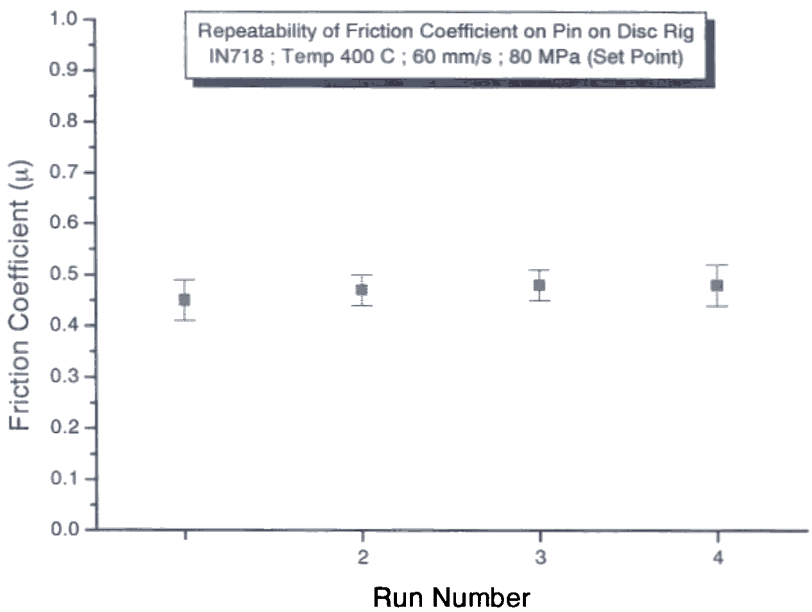


Figure 12. Repeat tests on the Pin on Disc friction rig.

The mean values of the friction coefficients from the curves has been determined in a similar manner to that described above for the Big Friction Rig, and the results are given in Table 3.4.1

Test No	Average friction coefficient, μ	Standard Deviation
1	0.45	0.04
2	0.47	0.03
3	0.48	0.03
4	0.48	0.04
Mean Value	0.47	0.035
Standard Deviation of Mean	0.0141	
Standard error	**0.0141/√4 = 0.0071**	

Table 3.4.1 Repeat test data from Pin on Disc Rig for IN718 at 400°C, 60 mm/s 80 MPa.

Thus the average standard deviation for the individual tests expressed as a percentage corresponds to (0.035/0.47) x 100 = 7.45%, representing the repeatability of the tests. The reproducibility for the four replicated tests carried out using different testpieces corresponds to (0.0071/0.47) x 100 = 1.5 %. These values are then used in the uncertainty budget given on the next page.

Symbol	Source of Uncertainty (y)	Value +/- %	Probability Distribution	Divisor	$u_i(y)$ +/- %
$U_{e\,cal}$	Specification of 1kN horizontal strain gauge load cell	1	Rectangular	√3	0.577
U_{std}	Specification of 1kN piezo electric axial load cell.	1	Rectangular	√3	0.577
r	Repeatability friction tests (standard deviation)	7.45	Normal	1	7.45
R	Reproducibility between tests (standard deviation)	1.5	Normal	1	1.5
u_c	Combined Uncertainty	-	-	-	7.64
U_e	**Expanded Uncertainty (k =2, 95% confidence)**	-	-	-	**15.3**

Table 3.4.2 Uncertainty Budget for Pin on Disc Friction Rig .

4. DISCUSSION

The measurement uncertainties estimated above are summarised in Table 4.1 for two techniques developed at NPL for the measurement of friction coefficients, μ, and for the Cockcroft-Male ring test which is used for the determination of a friction factor, m.

Test Method	Measured Parameter	Uncertainty * + /- %
Big Friction Rig	μ	7
Pin on Disc Friction Rig	μ	15
Cockcroft-Male Ring Test (without oxide)	m	28
Cockcroft-Male Ring Test (with oxide)	m	56

Table 4.1 Summary of Friction Measurement Uncertainties
(* at the 95% confidence level)

It should be noted that in the calculations for uncertainty for friction coefficients using the two NPL techniques an allowance was made for the errors associated with a series of repeat measurements; in the case of the Cockcroft- Male tests no such allowance was made because suitable data was not available to the authors at the time of writing. Thus the measurement uncertainty values presented here for the Cockcroft –Male test are likely to be an underestimate.

It was mentioned in section 2.1 that an empirical relationship has been reported which correlates the friction factor, m , with friction coefficients, μ, see Equation 1, for 50% deformation. This relationship is based on a correlation of results on a variety of materials published by a number of authors and was presented by Male & Depierre (1970) and is shown here in Figure 13. It can be seen from the scatter in the data that if this correlation is used then it would not be unreasonable to include an additional ~10% to the to the uncertainties quoted above for the Cockcroft-Male test.

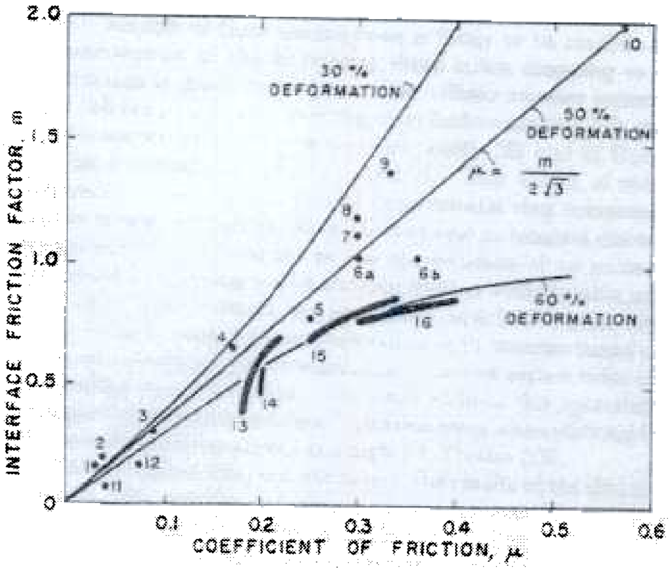


Figure 13. Correlation of μ and m. [Male and Depierre, 1970]

The estimated uncertainties for the various tests are presented schematically in Figure 14 where the uncertainties are shown as error bars for a nominal friction value of 0.45.

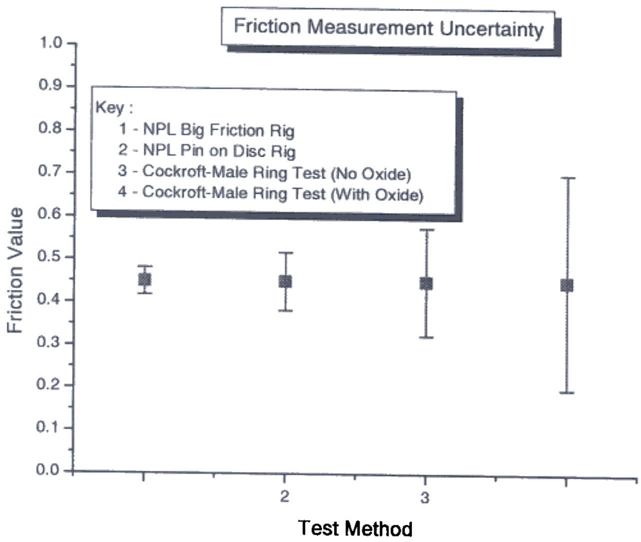


Figure 14. Schematic Representation of Friction Measurement Uncertainty for Various Techniques. Error bars represent the 95% confidence level.

5. CONCLUSIONS

It has been shown that the new friction measurement techniques developed under the DTI funded programme are significantly more accurate than the traditional Cockcroft –Male ring test and can thus provide more reliable data for use in hot forging process control and modelling.

6. Symbols

k	Coverage factor used to calculate expanded uncertainty U_e
m	Friction factor
μ	Friction Coefficient
r	Repeatability of measurement results
R	Reproducibility of measurement results
s	Estimate of the standard deviation, σ, of the population of values of a random variable *q* based on a limited sample of n results from that population
	Repeatability of measurement of initial diameter of Cockcroft-Male ring
	Repeatability of measurement of final diameter of Cockcroft-Male ring
δd_o	Vernier calliper calibration uncertainty
δh_c	Repeatability of measurement of initial height of Cockcroft-Male ring
δh_f	Repeatability of measurement of final height of Cockcroft-Male ring
δh_o	Vernier calliper calibration uncertainty
δm_d	Uncertainty in Friction factor, m, due to diameter measurements
δm_h	Uncertainty in Friction factor, m, due to height measurements
u_c	Combined standard uncertainty estimate
U_e	Expanded uncertainty estimate
$U_{e\ cal}$	Expanded uncertainty estimate associated with the calibration of the 250 kN load cell within the horizontal grabber arm of the Big Friction Rig
	Expanded uncertainty estimate for the diameter of a deformed Cockcroft – Male ring.
	Expanded uncertainty estimate for the height of a deformed Cockcroft – Male ring.
	Individual uncertainty estimate for parameter Y
u_s	Standard uncertainty
U_{std}	Expanded uncertainty estimate associated with the verification of the 7MN Force proving device within the Big Friction Rig.

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