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Requirements for the Characterisation of Non-Linear Effects in Optical Fibre Amplifiers

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Summary

This report presents a summary of the issues relating to the impact of non-linear effects occurring within erbium-doped optical fibre amplifiers (EDFAs). Non-linearities are defined here as those effects that occur in passive optical fibre, such as four-wave mixing (FWM), cross-phase modulation (XPM) and stimulated Raman scattering (SRS). The origins of these effects are explained and the relevant fibre parameters for each case are highlighted including: second-order refractive index, effective fibre core area and chromatic dispersion.

Some characteristics of optical fibre amplifiers, such as the variation in gain and noise figure with input power and wavelength are often also referred to as being non-linear. However, these parameters are already accommodated by other parts of the NMS Photonics programme and are not considered here.

It is concluded that non-linearity within lumped C-band EDFAs - those consisting of tens of metres of highly-doped fibre - is not currently an important measurement issue due to their short length. Newer lumped amplifiers designed to serve the longer-wavelength L-band consist of longer samples of doped fibre (typically 100-200 m). Non-linearity in these fibres is more significant than in C-band amplifiers. However, the relevance compared to the non-linearity in the transmission fibre is still debatable and does not warrant experimental investigation under the current photonics programme.

Distributed EDFAs have been proposed for which the interaction lengths will be orders of magnitude greater than lumped EDFAs. The most likely non-linear effect in these fibres is Raman amplification of the signals due to the intense pump wave near 1480 nm. However, the Raman gain spectrum of this lightly-doped fibre is very similar to that of conventional silica fibre, for which characterisation techniques are already established.

Contents

1. Introduction

The use of high optical powers over long lengths of transmission fibre can be partly attributed to the development of erbium doped fibre amplifiers (EDFAs) over the past ten years. These devices use a comparatively short length (tens of metres) of optical fibre doped to provide gain at signal wavelengths when pumped with a high power source at a shorter wavelength. The use of increasing powers in optical fibre communications systems has led to growing concern about non-linear effects in the transmission fibre. Effects such as cross-phase modulation, four-wave mixing and stimulated Brillouin scattering are now serious considerations for system designers. Care must be taken to ensure that the data rate in an optical communication system is not significantly impaired by these effects.

The optical power level at the output of the optical fibre amplifier is the same as the input power to the following transmission fibre. It therefore seems logical to ask whether significant non-linearity can occur within the amplifier itself. Due to their comparatively short length, non-linear effects occurring within the amplifiers themselves are not usually considered. This is evident from the very few research papers devoted to this aspect of optical amplifiers. Recently, however, longer L-band amplifiers have been developed and these have generated interest in non-linear effects within EDFAs. The possibility of distributed amplification throughout the transmission fibre also raises questions about the non-linear parameters of doped optical fibre.

We begin by clarifying what the term "non-linear" is taken to mean within the context of this report and then identify some of the key measurable parameters. The published work on this topic is reviewed and conclusions drawn regarding the needs for characterisation of optical fibre amplifiers with respect to their non-linearity. Appendices A and B contain background information on non-linear effects in transmission fibres.

2. Non-linearity in Optical Fibre Amplifiers

2.1 Scope

Many of the characteristics of optical fibre amplifiers are non-linear in the sense that the amplified output signal is not simply proportional to the input. Non-linearities of the active characteristics of the amplifier include the dependence of gain on output power and wavelength and also the reduction of gain due to spectral hole burning (SHB). Without pumping, the amplifier becomes a passive fibre in which the same non-linear effects may occur as for standard communications fibre. These effects arise due to the high optical intensity in the fibre core and potentially include an intensity-dependent refractive index, stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS). It is the non-linear effects that can occur in doped fibre amplifiers without pumping that are considered here. However, although this criterion is used as a definition of non-linear effects within OFAs, their occurrence and behaviour when the fibre is pumped to produce amplification are also considered.

In summary, the optical fibre amplifier characteristics commonly referred to as non-linear, but which are not considered within the scope of this study are:

- Gain slope or dependence of profile on gain.
- Gain vs. pump power.
- Spectral hole burning.
- Intermodulation distortion arising from source chirp, OFA gain slope and chromatic dispersion in the transmission fibre (i.e. composite second order, CSO, and composite triple beat, CTB, products in analogue sub-carrier multiplexed (SCM) systems).

These parameters are already covered within existing programmes at NPL to measure the spectral gain characteristics of EDFAs.

The effects that are considered are:

Kerr-type non-linearities, i.e. self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM).

- Stimulated scattering effects, i.e. stimulated Brillouin and stimulated Raman scattering.

2.2 Relevant Parameters

2.2.1 Effective Area

All non-linear effects are dependent upon the intensity of the electromagnetic field in the medium. However, it is the total optical power entering and leaving the fibre that is usually measured. Some method is required for converting between the two when comparing theoretical and experimental results. The measured optical power leaving a fibre is simply the integral of the intensity distribution over the entire fibre cross section. For a uniform intensity distribution, I, over a core of area A_{core}, the intensity could be calculated from the measured power, P_{meas}, using:

$$I = \frac{P_{meas}}{A_{core}} \qquad \text{2-1}$$

However, the field in a single mode fibre is not evenly distributed or even fully contained within the core. It is larger at the fibre axis than near the core-cladding interface and extends into the cladding to a degree depending on the actual refractive index profile. Calculating a uniform intensity in the core using equation 2-1 will underestimate the value on the axis of the fibre and overestimate the value near the core-cladding interface.

The effective area parameter has been defined for the purposes of calculating non-linear effects [1]. It is a single value, based on the modal field distribution, and can be used in equation 2-1 instead of A_{core} to calculate a value for the optical intensity. The effective area is defined as:

$$A_{eff} = \frac{2\pi\left(\int_0^\infty |E_a(r)|^2 r dr\right)^2}{\int_0^\infty |E_a(r)|^4 r dr} = \frac{2\pi\left(\int_0^\infty I(r) r dr\right)^2}{\int_0^\infty I^2(r) r dr} \qquad \text{2-2}$$

where $E_a(r)$ is the amplitude and $I(r)$ is the intensity of the near-field of the fundamental mode at radius r from the axis of the fibre. Typical values of the effective area for transmission fibres are in the range 50-80 μm^2. Erbium-doped fibre amplifiers use a larger core-cladding refractive index difference and smaller core diameters than most transmission fibre in order to achieve greater confinement of the signal mode-field. This should lead to a smaller effective area than typically found in conventional transmission fibres. However, Jiang *et al.* quote a value of 64 μm^2 for an erbium fibre laser [2], which is certainly within the range of transmission fibre values.

2.2.2 Second-Order Refractive Index/Non-Linear Coefficient

A critical parameter in characterising the non-linearity of silica-based transmission fibre is the second-order refractive index, n_2, which adds an intensity-dependent term to the refractive index of the fibre core. The aggregate local core refractive index, $\bar{n}(I)$, is then expressed as:

$$\bar{n}(I) = n + n_2 I, \qquad \text{2-3}$$

where I is the local intensity of the optical radiation. Since intensity is related to the total optical power divided by the effective area of the fibre and it is normally the total power in the fibre that is known, n_2 is often replaced by a non-linear coefficient, γ, given by [12]:

$$\gamma = \frac{n_2 \omega}{c A_{eff}}$$

Here ω is the angular frequency of the signal and c is the speed of light in vacuum. A value of $n_2 = 3 \times 10^{-20} \, \mathrm{m^2 / W}$ at 1550 nm has been quoted for EDF, which is ~20% higher than the typical range of values for dispersion-shifted transmission fibre [3,4].

2.2.3 Effective Length

The effective length, L_{eff}, of an optical fibre is defined as the length of lossless (or gainless) fibre that will generate the same amount of non-linearity as the fibre in question. For transmission fibres, the effective length is always less than the actual length, L, because of attenuation of the optical signal and L_{eff} is given by:

$$L_{eff} = \frac{(\exp(-\alpha L) - 1)}{\alpha}$$

where α is the linear gain coefficient in neper/km and is related to α', the gain coefficient in dB/km, by:

$$\alpha(\mathrm{neper/km}) = \frac{\ln(10)}{10} \alpha'(\mathrm{dB/km}). \qquad \text{2-6}$$

(Note that the gain coefficient is negative for a lossy transmission fibre.)

The gain coefficient of an un-pumped EDFA can be less than -70 dB. Therefore, without pumping, the optical power in an Erbium-doped fibre decreases rapidly with propagation length. The optical power in the fibre after propagation distance L will be:

$$P(L) = P_{in} \exp(\alpha L),$$

where P_{in} is the power at the input end of the fibre.

When the amplifier is active, the gain or loss of the medium at the signal wavelength will become a complicated function of the position along the fibre. The effective length of an EDFA is given by the general equation [12]:

$$L_{eff} = \frac{1}{P_{in}} \int_0^{L_{EDF}} P(z) dz \qquad \text{2-8}$$

where $P(z)$ is the signal power at location z along the optical fibre. Inserting equation 2-7 into 2-8 gives 2-5. Also, if the fibre is lossless, as would be the case for a fibre with distributed amplification, then $P(z) = const$ and $L_{eff} = L$ as expected from the definition of effective length. Finally, if we make the simplified assumption that the signal power in an amplifier increases by 20 dB as:

$$P(z) = P_{in} \exp\left(\frac{z}{L}\ln(100)\right) \qquad 2\text{-}9$$

then the effective length in this case is then 21.5 times the actual length of the EDF. For a 20 m long EDFA, this give an effective length of 430 m, which is still considerably shorter than the typical effective length for a transmission fibre.

2.2.4 Chromatic Dispersion and Amplifier Length

The chromatic dispersion of the fibre determines the group velocity mismatch between signals at different wavelengths. Two pulses that are initially synchronised at the fibre input will slip out of synchronisation at a rate determined by their relative group velocities. Effects such as cross-phase modulation and four-wave mixing, which occur due to the interaction between co-propagating signals, are more efficient if the signals walk-off slowly with respect to each other. In long transmission fibres, the length over which signals will interact is typically limited by the distance taken for signals to walk-off. Lumped erbium-doped optical fibre amplifiers (LEDFAs) use comparatively short lengths of fibre (10s of metres) and consequently the length of the EDF itself is likely be the limiting factor, rather than its dispersion. Jiang *et al.* [2] quote a value for the dispersion of 4 m of EDF of 60,000 fs^2 - equivalent to approximately -0.007 ps/nm at 1550 nm or -1.9 ps/nm.km. Longer LEDFAs, such as L-band amplifiers (typically ~100 m), are expected to introduce proportionally more non-linearity than shorter devices. Distributed erbium-doped optical fibre amplifiers (DEDFAs) are longer than LEDFAs by two or three orders of magnitude. Although these amplifiers are much longer than LEDFAs, the optical power is maintained at a lower level and non-linearities due to interaction between the signals can be avoided. However, the fibre length and pump-signal channel separation may be suitable for Raman amplification of the channels at the expense of accelerated pump depletion [5]. Raman assistance in DEDFAs is discussed further in section 2.3.3.

2.3 Implications for Optical Communications Systems

There has been relatively little literature published on the subject of non-linearities within rare earth-doped fibre amplifiers. This can be seen as an indication that these effects are not currently a high priority for investigation within the telecommunications industry. However, a debate has recently emerged as to the significance of non-linearities within the newer L-band amplifiers owing to their greater length compared with the conventional, C-band LEDFAs. In addition to the potentially detrimental effects of cross-phase modulation in L-band LEDFAs, we also discuss gain and SNR improvements that can result from stimulated Raman scattering.

2.3.1 Cross-Phase Modulation in EDFAs

Two-Channel Experiments

Experiments in which a modulated pump and CW probe channel with closely-spaced wavelengths were amplified using an L-band EDFA have shown that crosstalk can occur due to cross-phase modulation within the amplifier itself [6,7]. The magnitude of the XPM was determined from the standard deviation of intensity fluctuations imposed on the CW probe signal after it had propagated through a length of dispersive fibre. The length of the doped fibre is small compared to the walk-off length between channels (see section 4.2). This means that the efficiency of the XPM process is limited by the EDFA length for all wavelength separations within the 32 nm gain bandwidth of the amplifier. Under these circumstances, XPM in the transmission fibre is predicted to be the dominant source of crosstalk between channels < 3 nm apart. However XPM in the EDFA should be taken into consideration when calculating crosstalk between channels with larger separation. This is especially true if standard transmission fibre is used, where crosstalk due to FWM will be minimised.

Full WDM System Loading Experiments

The experimental results described above for two channels propagating through an L-band EDFA have been contradicted by measurements performed using 20 co-propagating channels in a 100 GHz WDM system simulation [8]. The amplifier used in this case consisted of a shorter length of doped fibre (145 m cf. 257 m) and the output power from the EDFA was ~6.3 dBm per pump channel and ~-1 dBm in the probe channel compared to 22.3 dBm and 16.3 dBm respectively in [7]. In this case, the XPM induced by the amplifier alone was negligible compared to that in 20 km of dispersion-shifted fibre and a factor of 9 less than that in 160 km of conventional fibre.

The differences in the techniques and EDFAs used in the two experiments make it difficult to draw comparisons between their results. All that can be said is that the XPM in a lumped EDFA seems to be negligible compared to dispersion-shifted fibre and at worst is comparable to that in conventional fibre.

2.3.2 Four-Wave Mixing in EDFAs

Discussions with members of the optical fibre telecommunications industry have identified that four-wave mixing effects within lumped C-band EDFAs are far less significant than the FWM that occurs within the subsequent section of fibre. Testing is usually performed on active amplifiers with full channel loading using multiple DFB laser sources on the ITU WDM grid. One channel is then blocked at source and the FWM power at its wavelength is measured to give an indication of the FWM crosstalk from the neighbouring channels. The efficiency of the four-wave mixing process is highest for closely-spaced channels near the zero dispersion wavelength of the transmission medium. Distributed EDFAs aim to sustain the signal channels at a power level low enough to avoid undesirable non-linearity over long distances without loss and with minimal aggregate chromatic dispersion. Four-wave mixing may need to be considered if the WDM channels are densely-packed and the fibre dispersion is maintained at a constantly small value.

2.3.3 Raman Gain in DEDFAs

Distributed EDFAs consist of long lengths of EDF (>10s of km) of lightly-doped fibre designed to compensate for attenuation with gain and provide a lossless transmission fibre without the large power excursions introduced by LEDFAs. The signal powers can be maintained low enough to avoid non-linearity or high enough to sustain it - whichever is required. However, even though the signal powers may be at low enough powers not to introduce Kerr effect non-linearities (see Appendix A) the pump at 1480 nm is able to amplify channels near 1550 through Raman gain. The transfer of optical power from the pump to the signals through stimulated Raman scattering depletes the available power for pumping the erbium atoms. However, theoretical investigations have shown that there exists an optimum condition in which reduced erbium-based amplification is more than compensated for by the Raman gain [5,9].

2.3.4 Stimulated Brillouin Scattering in EDFAs

The SBS threshold power, P_{th}, for conventional optical fibre (in which the optical signal power decreases with propagation distance due to attenuation) can be estimated from [10]:

$$P_{th} \cong 19\frac{K_{SBS}A_{eff}}{g_{SBS}L_{eff}} \qquad \text{2-10}$$

Here K_{SBS} is a polarisation factor and g_{SBS} is the peak value of the SBS spectral gain coefficient. The peak value of the Brillouin gain coefficient, g_{SBS}, is dependent on the material properties of the fibre and also the spectral width of the input signal relative to the Brillouin linewidth. No equivalent expression has been found for optical fibre amplifiers. The main differences between SBS in EDFAs and conventional fibres are:

- The physical length of the scattering medium is significantly shorter than in conventional fibre i.e. the "travelling Bragg grating" is limited in length

  The SBS pump (i.e. the signal) increases in power with propagation length rather than decreasing due to attenuation
- The Stokes signal is amplified by the stimulated emission process in the EDFA as well as SBS
- The SBS process can be seeded by ASE at the Stokes wavelength - including Fresnel-reflected ASE from the output end of the amplifier

Equation 2-10 is derived for conventional fibre with the assumption that there is no initial backward-propagating power at the Stokes wavelength. To derive a similar relation for EDFAs would require consideration of the above points. The absence of research papers into SBS within EDFAs indicates that it is not currently seen as a problem in lumped C-band EDFAs. However, lossless distributed fibre amplifiers with effective lengths equal to the full span length and significant seeding through ASE may present suitable conditions for SBS. Care must then be taken to ensure that the optical bandwidth of the signal significantly exceeds the Brillouin linewidth - as in soliton systems - or that the signal power is maintained at a sufficiently low level in WDM systems.

3. Conclusions

There is some debate as what should be considered to be a non-linear aspect of erbium-doped optical fibre amplifiers. Characteristics such as gain and noise figure can depend on the input signal power and wavelength and may not have a linear dependence. Also, intermodulation distortions such as composite triple beat (CTB) and composite second order (CSO) are sometimes referred to as non-linear [11]. However, these arise from the combination of source chirp wavelength-dependent amplifier gain and dispersion in the transmission fibre rather than an intrinsic non-linearity of the optical amplifiers. Since the necessary information for calculating these distortions is already covered in the NMS photonics programme, we restricted our investigation to non-linear effects that can affect passive fibre. This essentially means Kerr effects and stimulated scattering.

There currently appears to be no requirement for the characterisation of the non-linear parameters of lumped C-band erbium-doped fibre amplifiers. This is due to their short interaction lengths and only slightly elevated non-linear coefficient compared to the kilometres of transmission fibre that typically follow the amplifier.

EDFAs for the longer wavelength L-band region can exhibit measurable non-linearity due to their longer interaction lengths. However, the significance of their contribution relative to that of the transmission fibre is debatable. Experimental evidence currently suggests that the effect is negligible in a WDM system. We suggest that this area be monitored as L-band systems are developed and deployed over the next few years but do not recommend any further work under the current programme.

The most likely area requiring non-linear parameter characterisation is in distributed EDFAs, where the interaction length is limited by the same parameters as in transmission fibre i.e. fibre attenuation/gain and chromatic dispersion. For soliton system applications, the signal powers will need to be maintained at high enough levels to sustain self-phase modulation. Under these circumstances, the non-linear coefficient and Raman gain spectrum of the fibre will be crucial parameters. In WDM systems, interaction between channels will be minimised by keeping their power levels in the linear regime. However, Raman amplification due to high pump powers at 1480 nm can affect channels in the C- or L-band. Therefore the Raman gain coefficient of distributed EDFAs may be an important parameter, whether they are sustaining linear or non-linear channel propagation. In contrast, stimulated Brillouin scattering will not affect soliton systems with distributed amplification but may limit power levels in low-bandwidth WDM systems. SBS will be enhanced by the extended effective length of the DEDFA as well as ASE at the Stokes wavelength.

Difficulties arise in determining non-linear parameters of EDFAs while active due to the wide variety of available pumping and gain regimes. Measurements performed on the fibre without pumping are likely to be the most repeatable but the extremely high attenuation of un-pumped EDF is high and likely to prohibit these measurements for lumped EDFAs. The lower dopant concentrations in EDF for distributed amplifiers may, however, permit measurements of non-linear parameters by adapting the techniques established for transmission fibre.

4. Appendix A - Kerr Effect Non-Linearities

When radiation is incident upon a medium, the oscillating electromagnetic field interacts with electric dipoles in the molecules of the medium and causes them to oscillate. The result is a time-varying local electric polarisation in the medium. This oscillating electric field then re-radiates the electromagnetic field and the incident wave is considered to propagate through the medium via a series of such absorption and re-radiation processes. The polarisation vector, $\mathbf{P}$, induced by an electric field with amplitude vector $\mathbf{E}$ can be expressed as a general series expansion of the form [12]:

$$\mathbf{P} = \varepsilon_0 \left(\chi \cdot \mathbf{E} + \chi_2 : \mathbf{EE} + \chi_3 \vdots \mathbf{EEE} + \ldots \right) \qquad 4\text{-}1$$

where:

ε_0 is the electric permittivity of a vacuum,

χ is the linear susceptibility tensor of the medium and

χ_2 and χ_3 are second and third order susceptibility tensors.

If the induced polarisation has a purely linear dependence on the applied electric field then the re-radiated electric field will be identical to the incident field. However, when second or higher-order susceptibility terms are non-zero, harmonics begin to appear in the radiated field that were not present in the incident field.

For materials that have a symmetrical molecular structure, the polarisation induced by an incident electric field is symmetrical. The susceptibility of these materials contains only odd expansion terms, as opposed to anti-symmetric molecules for which even terms such as χ_2 may be non-zero.

The dominant susceptibility term is the linear term, χ, which determines the linear refractive index of the medium, n, and the absorption attenuation coefficient, α. The orders of the non-zero expansion terms determine the type of non-linearity to which the medium is susceptible. In materials which lack a centre of symmetry, such as quartz, KDP and ADP, the second order susceptibility is responsible for *second harmonic generation,* in which an intense wave at angular frequency ω_1 generates another wave at twice this frequency, $2\omega_1$. Another effect which is possible in these materials is *sum-frequency generation,* in which two waves at ω_1 and ω_2 interact to produce waves at $\omega_1 + \omega_2$, $2\omega_1 + \omega_2$ and $\omega_1 + 2\omega_2$.

Silica (SiO_2), the basic constituent of transmission optical fibres is a symmetric molecule and consequently, χ_2 vanishes and second-order non-linear effects are not normally observed. Rather, it is the third-order term in equation 4-1 that is responsible for non-linear behaviour, which includes *self-phase modulation* (SPM), *cross-phase modulation* (XPM) and four-wave mixing (FWM). SPM and XPM can be viewed as processes that result from the refractive index of the fibre material being a function of the intensity of the electromagnetic field. These effects will occur under fairly broad ranging conditions provided that the intensity of the optical field is high enough, whereas FWM requires the satisfaction of a stringent *phase matching* condition in order to occur efficiently.

4.1 Self-Phase Modulation

Self-phase modulation refers to the case where a signal produces a non-linear refractive index response in the fibre core material that modifies the signal itself. Soliton propagation arises through the interplay between the self-phase modulation of an intense optical pulse and the group velocity dispersion (GVD) of the transmission fibre. The parameters that determine the degree of self-phase modulation in a transmission fibre are the non-linear coefficient of the fibre, the effective length of the fibre and the peak power of the input pulse. The maximum phase shift within the pulse occurs for the temporal peak. In this case, the phase shift is given by [12]:

$$\phi_{\max} = \frac{n_2}{A_{eff}} P_0 L_{eff} , \qquad \text{4-2}$$

where $\phi_{\max}$ is the maximum non-linear phase shift and P_0 is the peak power of the pulse. The varying phase shifts within the pulse lead to spectral broadening, which can lead to pulse compression or pulse spreading, depending on the GVD of the transmission fibre.

4.2 Cross-Phase Modulation

Cross-phase modulation occurs if two or more high-power optical pulses co-propagate through a non-linear medium. The aggregate optical power at any point in the fibre determines the refractive index at that point and hence channels can interfere by modulating each others' phase. Therefore, in contrast to SPM, where a channel must carry a high optical power to induce non-linearity by itself, XPM can occur when a large number of channels co-propagate but none have sufficient individual power to induce non-linearity. This is often the case in WDM systems, where the aggregate power of a large number of channels is sufficient to cause a non-linear response in the transmission fibre.

An important effect in considering XPM in digital communications systems is the walk-off between the digital data streams in the optical channels. Cross-phase modulation will be most severe when all the channels in the fibre are carrying a digital "1". In this case, the optical power in the fibre is maximised and all channels have a signal that can be phase modulated. However, GVD in the fibre causes the channels to propagate at different speeds and the data streams gradually slip with respect to each other. Over a sufficient length of fibre, pulses corresponding to digital "1"s will slip until the time lag between their arrival at a particular point is equal to the temporal pulse width. The length of fibre required for this walk-off to happen, L_W , is given by [4]:

$$L_W = \frac{T_0}{\left| 1/v_g(\lambda_1) - 1/v_g(\lambda_2) \right|}, \qquad 4\text{-}3$$

where T_0 is the pulse width of a digital "1" and $v_g(\lambda)$ is the group velocity for a pulse at wavelength λ. The walk-off length determines the length over which cross-phase modulation between WDM channels is assumed to occur. In transmission fibres, this is typically a fraction of the total fibre length. However, the doped fibres used in amplifiers are short enough that the actual fibre length can determine the amount of XPM.

4.3 Four-Wave Mixing

Four-wave mixing leads to the appearance or amplification of radiation at a number of frequencies due to the incidence of radiation at two or more other frequencies onto a non-linear material. The generation of a new frequency of radiation due to FWM has applications in the development of tunable sources and wavelength conversion [13] in all-optical routing systems whereas the amplification of an existing signal can be used for optical amplification and demultiplexing at bit rates of up to 500 Gb/s [14]. However, frequency generation and amplification can be disadvantageous in systems where many frequencies are co-propagating but each one must remain independent. The most important example of this situation is in wavelength division multiplexed (WDM) optical communication systems.

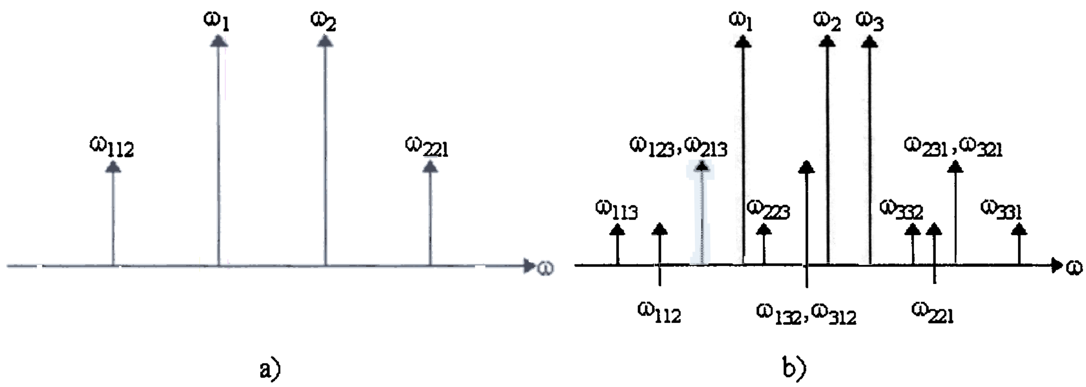

Figure 1. Additional frequencies generated through FWM in the partially degenerate (a) and non-degenerate case (b).

Figure 1 illustrates how four-wave mixing can lead to generation of a number of extra frequencies from the interaction between light at two or three incident frequencies. The relation that gives the frequency of the generated wave, ω_{ijk}, is:

$$\omega_{ijk} = \omega_i + \omega_j - \omega_k \qquad 4\text{-}4$$

If two of the three waves have the same frequency and therefore there are only two distinct frequencies initially in the fibre (Figure 1a), then the effect is known as partially degenerate four-wave mixing (PDFWM). In this case, one of the incident waves assumes the role of both ω_i and ω_j in equation 4-4 and only two new frequencies are generated - one for $\omega_i = \omega_j = \omega_1$ and one for $\omega_i = \omega_j = \omega_2$. The non-degenerate case (NDFWM), in which three unique frequencies are initially present, is illustrated in Figure 1b. It can appreciated from this diagram that a large number of generated frequencies are possible through different permutations of ω_i, ω_j and ω_k.

Equation 4-4 gives the frequency of radiation that would be generated by FWM, should the process occur. It does not provide any information as to whether the process will occur or not, this is dictated by the *phase matching condition*. Phase matching requires that the propagation constants of the waves satisfy a relation similar to 4-4 that ensures momentum is conserved before and after the interaction. In order to achieve phase matching in an optical fibre, the dispersion of the fibre must be such that the frequencies involved in FWM also have the correct propagation constants to satisfy the condition:

$$\Delta\beta = \beta_1 + \beta_2 - \beta_3 - \beta_4 = 0 .$$

How closely equation 4-5 is satisfied plays an important role in determining the efficiency of the four-wave mixing process. Expanding the propagation constants β as Taylor expansions around a central frequency, ω_m, gives the following expression for the phase mismatch [15]:

$$\Delta\beta = 2\pi c \frac{\Delta\omega_{13}\Delta\omega_{23}}{\omega_m^2} D(\omega_m) ,$$

where:

$$\omega_m = \frac{(\omega_1 + \omega_2)}{2}$$

$$\Delta\omega_{ij} = \omega_i - \omega_j$$

D is the fibre dispersion, $D(\omega) = -\frac{\omega^2}{2\pi c}\frac{d^2\beta}{d\omega^2}$.

It can be seen that phase matching can be maximised if the frequencies of the signals are closely spaced and near the zero dispersion wavelength of the fibre.

5. Appendix B - Stimulated Scattering

Raman and Brillouin scattering are inelastic processes in which part of the power is lost from an optical wave and absorbed by the transmission medium. The remaining energy is then re-emitted as a wave of lower frequency. The processes can be thought of as the conversion of an incident photon into a lower energy scattered photon plus a phonon of vibrational energy. Total energy and momentum before and after scattering must be equal, i.e. the incident photon energy is shared between the phonon and the scattered photon. Since the frequency of an optical wave is proportional to its energy, the photon produced by the scattering event has a lower frequency than the incident photon. This frequency downshifted wave is commonly referred to as the *Stokes wave.*

Spontaneous Raman and Brillouin scattering have been observed and measured in bulk samples of material such as quartz and silica [16, 17, 18]. The intensity of the scattered wave is strongly dependent on the angle of scattering and the optical power density in the material. The growth of the Stokes wave is proportional to the product of the scattering gain coefficient, the intensity of the pump wave and the intensity of any Stokes wave present. In bulk media the Stokes wave quickly disperses as it propagates away from the point of generation. However, single mode optical fibres will support low-loss propagation for waves travelling almost parallel to the fibre axis. Consequently, scattered radiation in either the forward or backward directions relative to the incident wave will be guided within the fibre and will co-propagate with the pump wave over long distances. Under these circumstances, it is possible for the Stokes wave to continue to interact efficiently with the pump wave and exponential growth in the downshifted optical power occurs. For a given length of fibre, gradually increasing the pump power launched into one end will lead to a gradual increase in Stokes power through spontaneous scattering. If the pump power is then increased further, exponential growth in the Stoke power may occur. The input pump power at which the Stokes wave increases rapidly as a function of pump power is termed the stimulated scattering threshold.

A major difference between Brillouin scattering and Raman scattering lies in the type of phonon generated - high-energy optical phonons in SRS and lower-energy acoustical phonons in SBS. The difference in frequency between the pump and Stokes waves is therefore much greater in SRS than in SBS. Typical values of the pump-Stokes frequency difference are 10-GHz (~0.1-nm at 1550-nm) for SBS and 13-THz (~110-nm at 1550-nm) for SRS. Another key distinction between the two effects is that the scattered wave due to SBS travels predominantly *backwards.* The SBS Stokes wave emerges from the input end of the fibre whereas the Stokes wave due to SRS travels forwards with the pump wave.

Both SBS and SRS have so-called *threshold powers* above which power transfer to the Stokes wave increases rapidly. In SBS this means that the amount of optical power leaving the far end of the fibre no longer increases linearly with the input power. The maximum launch power becomes clamped and excess power is simply reflected back out of the fibre. For long distance and highly-branched fibre links, it is important that as much power as possible can be launched into the fibre to compensate for attenuation and power splitting. Limits on the maximum launch power due to SBS must therefore be avoided.

The Stokes wave due to Raman scattering can be shifted from the pump wave by typically 10 to 100-nm and continues to propagate forwards along the fibre together with the pump wave. If the pump is actually one channel of a multi-wavelength communication system, then its Stokes wave may overlap with other channels at longer wavelengths - leading to crosstalk and *Raman amplification*. In Raman amplification, the shorter wavelength channel experiences power depletion and acts as a pump for amplification of the longer wavelength channel.

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